Transformed Regression-based Long-Horizon Predictability Tests*

Matei Demetrescu[a], Paulo M. M. Rodrigues[b] and A. M. Robert Taylor[c]

[a] Institute for Statistics and Econometrics, University of Kiel

[b] Banco de Portugal and NOVA School of Business and Economics

[c] Essex Business School, University of Essex

May 28, 2021

Abstract

We propose new tests for long-horizon predictability based on IVX estimation (see Kostakis et al., 2015) of transformed regressions. These explicitly account for the over-lapping nature of the dependent variable which features in a long-horizon predictive regression arising from temporal aggregation. Because we use IVX estimation we can also incorporate the residual augmentation approach recently used in the context of short-horizon predictability testing by Demetrescu and Rodrigues (2020) to improve efficiency. Our proposed tests have a number of advantages for practical use. First, they are simple to compute making them more appealing for empirical work than, in particular, the Bonferroni-based methods developed in, among others, Valkanov (2003) and Hjalmarsson (2011), which require the computation of confidence intervals for the autoregressive parameter characterising the predictor. Second, unlike some of the available tests, they allow the practitioner to remain ambivalent as to whether the predictor is strongly or weakly persistent. Third, the tests are valid under considerably weaker assumptions on the innovations than extant long-horizon predictability tests. In particular, we allow for quite general forms of conditional and unconditional heteroskedasticity in the innovations, neither of which are tied to a parametric model. Fourth, our proposed tests can be easily implemented as either one or two-sided hypotheses tests, unlike the Bonferroni-based methods which require the computation of different confidence intervals for the autoregressive parameter depending on whether left or right tailed tests are to be conducted (see Hjalmarsson, 2011). Finally our approach is straightforwardly generalisable to a multi-predictor context. Monte Carlo analysis suggests that our preferred test displays improved finite properties compared to the leading tests available in the literature. We also report an empirical application of the methods we develop to investigate the potential predictive power of real exchange rates for predicting nominal exchange rates and inflation.

Keywords: long-horizon predictive regression; IVX estimation; (un)conditional heteroskedasticity; unknown regressor persistence; endogeneity; residual augmentation.

JEL classifications: C12, C22, G17.

---

*The authors thank Tassos Magdalinos for many useful discussions on this work. Rodrigues gratefully acknowledges financial support from the Portuguese Science Foundation (FCT) through project PTDC/EGE-ECO/28924/2017, and (UID/ECO/00124/2013 and Social Sciences DataLab, Project 22209), POR Lisboa (LISBOA-01-0145-FEDER-007722 and Social Sciences DataLab, Project 22209) and POR Norte (Social Sciences DataLab, Project 22209). Taylor gratefully acknowledges financial support provided by the Economic and Social Research Council of the United Kingdom under research grant ES/R00496X/1.

1 Introduction

Since the seminal work of Fama and French (1988) and Campbell and Shiller (1988) (see Fama and French, 2018, for an overview) there has been substantial interest in testing for long-horizon predictability, most notably in stock returns, exchange rates and the term structure of interest rates; see, *inter alia*, Campbell and Shiller (1987), Campbell and Shiller (1988), Fama (1998); Campbell and Cochrane (1999); Campbell and Viceira (1999); Menzly et al. (2004); Mishkin (1990); Boudoukh and Matthew (1993) and Chang et al. (2018). Boudoukh et al. (2008) argue that although predictability might be relatively weak for a short-horizon it has the potential to be much stronger for a long-horizon due to the persistence of the predictors typically used, such as dividend yields and dividend price ratios, among others.

Empirical evidence on the short- or long-horizon predictability of returns largely derives from inference obtained from predictive regressions and, as such, the size and power properties of tests from these regressions are of fundamental importance. Many early studies are based on the assumption that the predictor is weakly persistent and are therefore based on the use of standard OLS t and F-type regression statistics, constructed using either Newey-West or Hodrick type standard errors (see, for example, Weigand and Irons, 2007). However, data analysis presented in, among others, Campbell and Yogo (2006a) and Welch and Goyal (2008) suggests that many of the variables used in predictive regressions are strongly persistent with autoregressive roots close to unity, and that a large negative correlation often exists between the series we are attempting to forecast (e.g. returns) and the predictor's innovations, such that the predictive regressor is endogenous. In such cases these methods, developed for use with weakly persistent regressors, are theoretically invalid and this can lead to sizable finite sample bias in the estimates of the coefficients from the predictive regression (Stambaugh, 1986 and Mankiw and Shapiro, 1986) and, correspondingly, to significant over-rejections of the null hypothesis of no predictability (both in short- and long-horizon contexts), thereby significantly increasing the likelihood that any finding of long-horizon predictability is spurious; see, *inter alia*, Valkanov (2003), Cochrane (2011), and Phillips (2015)).[1]

As a result, more recently a number of procedures for testing for short- and long-horizon predictability have been developed in the literature which are designed to be robust as to whether the predictors are weakly or strongly persistent; see, in particular, Gonzalo and Pitarakis (2012), Phillips and Lee (2013), Phillips (2014), Elliott et al. (2015), Lee (2016), Kostakis et al. (2015), Breitung and Demetrescu (2015), Demetrescu et al. (2020), Demetrescu and Hillmann (2020) and Demetrescu and Rodrigues (2020). Many of these procedures are based on the extended instrumental variable estimation [IVX] method of Phillips and Magdalinos (2009) which has gained widespread popularity in this literature and which will form the basis of the tests which we propose in this paper. The IVX approach consists of filtering putative predictors such that, where these are strongly (weakly) persistent, the filtered series are approximately mildly integrated (weakly dependent) variables. These filtered variables are then used to instrument the predictor in the predictive regression of interest. As a result of the reduced persistence of the instrument when compared to the original variable when the latter is strongly persistent, the resulting predictability test will follow a standard limit distribution (e.g. Gaussian or chi-squared) irrespective of whether the predictors are strongly

[1] The standard errors proposed by Hodrick (1992), which exploit the moving-average structure of the temporally aggregated error term under the null hypothesis of no predictability, perform slightly better than Newey-West standard errors in finite samples (see Ang and Bekaert, 2007) but are, nonetheless, still invalid under endogeneity and strong persistence.

or weakly persistent.

An additional complication, relative to the case of short-horizon predictability testing, arises when looking to develop tests for long-horizon predictability. Specifically, serial correlation is induced into the error term in the long-horizon predictive regression, arising from the temporal aggregation of the dependent variable (which therefore contains overlapping observations). To address this issue, Valkanov (2003) and Hjalmarsson (2011) propose using the conventional OLS t-statistic but scaled by a constant to reflect the inflation of the standard errors as the prediction horizon increases. The methods developed in Valkanov (2003) and Hjalmarsson (2011) are, however, somewhat restrictive in practice as they are based on the assumption that the predictor is strongly persistent. Tests for multiple-horizon predictability designed to be asymptotically valid regardless of whether the predictors are strongly or weakly persistent and for handling the issues arising from temporal aggregation are also considered by Phillips and Lee (2013) who develop tests from a reversed predictive regression framework, estimated by IVX. Their approach consists of switching from a predictive regression from the h-period returns on a predetermined variable to a predictive regression of single period returns on the same predetermined variable aggregated over h-periods. Xu (2020) proposes an alternative approach, which allows the predictors to be either weakly or strongly persistent, and builds on an implied estimator obtained from the short-horizon predictive regression model. The implied estimator concept dates back to Campbell and Shiller (1988) and Hodrick (1992), and was used by Cochrane (2008) and Lettau and Van Nieuwerburgh (2008). Xu (2020) derives the asymptotic distribution of the implied test statistic and proposes the use of a Bonferroni-type approach along the lines of Phillips (2014) as well as a wild bootstrap approach to compute the necessary critical values. The resulting test aims to control for size under various degrees of persistence.

In this paper we add to the corpus of available tests for long-horizon predictability in the literature. The tests we will develop are designed to be valid under weaker conditions than the leading long horizon predictability tests in the literature, all of which either assume the strength of the persistence of the predictor is known (some assume it is weakly persistent, some that it is strongly persistent) and/or assume that the innovations are conditionally homoskedastic. In particular, our proposed tests can be validly implemented without knowledge of whether the predictors are weakly or strongly persistent, and have pivotal limiting null distributions under quite general patterns of unconditional time heteroskedasticity in the innovations, allowing for time-varying innovation variances but also the possibility of time-varying correlations between the innovations, and very general forms of conditional heteroskedasticity. Moreover, the practitioner is not required to assume a parametric model for either the conditional or unconditional time-variation in the innovations. Unlike Bonferroni-based tests, our tests can be easily implemented to test either one or two-sided hypotheses and can be straightforwardly generalised to allow for multiple predictors. In a detailed Monte Carlo experiment we also compare the finite size and power properties of our proposed tests with the best-performing robust long-horizon predictability tests in the literature, namely the implied test of Xu (2020), the Bonferroni-based approach of Hjalmarsson (2011), and the reversed regression-based test of Phillips and Lee (2013). These results suggest that our proposed tests overall display superior finite sample properties to the extant tests.

The tests we propose are developed within a transformed regression framework which explicitly accounts for the serial correlation induced by temporal aggregation in the error in the original long-horizon regression. We estimate the parameters of the transformed regression using the IVX approach of Kostakis et al. (2015). The use of IVX estimation has the advantage that it also allows us to implement a feasible form of residual augmentation which cannot be employed where the predictive regression is estimated by OLS. This approach, discussed in Demetrescu and Rodrigues (2020) in the context of the IVX one-step ahead (short-horizon) predictive regression, consists of augmenting the transformed predictive regression with an additional regressor, constructed as the residuals obtained from fitting an autoregression to the predictor. Residual augmentation, at least for the case of a known degree of persistence, can be traced back to at least Phillips (1991), and augmenting regression models with residuals or nonlinear functions thereof is known to be an effective way of increasing efficiency; see, for example, Im and Schmidt (2008). In the context of the short-horizon predictive regression, Demetrescu and Rodrigues (2020) show that this approach is particularly effective for strongly persistent predictors. We will demonstrate that the estimation effect from fitting this autoregression to the predictor is asymptotically negligible in the set-up we consider and leads to more efficient estimation of the transformed predictive regression model on which our long-horizon tests are based. In particular, akin to Amihud and Hurvich (2004), this form of residual augmentation eliminates endogeneity in the limit, such that the bias of the IVX slope coefficient estimator is reduced compared to the corresponding IVX estimation from the transformed regression without this additional regressor.[2]

The remainder of the paper is organised as follows. Section 2 introduces the long-horizon predictive regression testing framework and outlines the assumptions on the model under which we work. In Section 3 we briefly review the leading tests in the literature: namely, Bonferroni-based approaches to testing for long-horizon predictability, focussing on the tests of Hjalmarsson (2011), the reversed regression based approach of Phillips and Lee (2013), and the implied testing approach of Xu (2020). In section 4 we detail our proposed transformed regression based tests for long-horizon predictability testing, and here we also discuss their large sample properties. For expositional purposes, the material in sections 2-4 assumes the case of a single predictor. The case of multiple predictors is discussed in section 5. Section 6 analyses the finite sample properties of the procedures in an in depth Monte Carlo study. In section 7 we report an empirical application of the methods developed in the paper to exchange rate predictability. Section 8 concludes. An on-line supplementary appendix collects all technical proofs of the results stated in the paper together with some additional supporting Monte Carlo results and technical derivations.

2 The Long-horizon Predictive Regression Framework

2.1 The Model and Assumptions

The long-horizon predictability testing framework in general has, as its backbone, the recursive system typically encountered in the short-run (one period) predictive regression context; that is,

$$y_{t+1} = \alpha_1 + \beta_1 x_t + u_{t+1}, \qquad t = 1, \ldots, T-1, \tag{2.1}$$

$$x_{t+1} = \mu_x + \xi_{t+1}, \qquad \text{and} \qquad \xi_{t+1} = \rho\xi_t + v_{t+1}, \tag{2.2}$$

[2] This bias reduction improves the MSE of the forecasts generated using the fitted residual augmented long-horizon regression; see the evidence provided by Demetrescu and Rodrigues (2020) for the one-step ahead case.

where y_{t+1} is, for example, a continuously compounded excess return of an asset or the variation of a nominal exchange rate from t to $t+1$ and x_{t+1} is some (putative) predictor variable. The errors u_t are assumed to form a martingale difference [MD] sequence; precise details will be given below. In our main exposition and technical analysis we will follow the bulk of this literature and focus attention on the case of a single predictor; that is, where x_t in (2.1) is a scalar variable. Extensions to the case where the predictive regression contains multiple predictors will be discussed in section 5.

Our interest in this paper centres on testing the null hypothesis, H_0, that $(y_{t+1} - \alpha_1)$ is a MD sequence and, hence, that y_{t+1} is not predictable by x_t which entails that $\beta_1 = 0$ in (2.1).[3] The alternative hypothesis is that y_{t+1} is predictable by x_t, in which case $\beta_1 \neq 0$. As discussed in the Introduction it is important for practical purposes to allow for the possibility of strong persistence in the predictor variable x_t and to allow the shocks driving the predictor, v_t in (2.2), to be contemporaneously correlated with the unpredictable component of y_t; that is, u_t in (2.1). We will allow for both of these through Assumptions 1–4 which follow. First, with respect to the degree of persistence in x_t, this is controlled via the parameter ρ. We allow x_t to be either weakly or strongly persistent through the following assumption.

Assumption 1 *The data are generated according to (2.1) and (2.2) with initial condition ξ_1 which is bounded in probability.*

Assumption 2 *Exactly one of the two following conditions holds true:*

i) ***Strongly persistent predictors:*** *The autoregressive parameter ρ in (2.2) is local-to-unity with $\rho := 1 - c/T$ where c is a fixed constant.*

ii) ***Weakly persistent predictors:*** *The autoregressive parameter ρ in (2.2) is fixed and bounded away from unity, $|\rho| < 1$.*

Remark 1. Many commonly used predictors are strongly persistent, exhibiting sums of sample autoregressive coefficients which are close to or only slightly smaller than unity. Near-integrated asymptotics have been found to provide better approximations for the behaviour of test statistics in such circumstances; see, *inter alia*, Elliott and Stock (1994). However, not all (putative) predictors are strongly persistent and a large part of the literature works with models which take x_t to be generated from a stable autoregressive process; see, for example, Amihud and Hurvich (2004). We therefore allow for either of these possibilities to hold for x_t. It is important to stress that the long-horizon predictability tests developed in Valkanov (2003) and Hjalmarsson (2011) are only valid for the case where x_t is strongly persistent. ◇

To complete the specification of our predictive regression model, we make the following assumptions with regard to the error terms, u_t and v_t, which are designed to allow for empirically relevant features frequently found in economic and financial time series.

Assumption 3 *The errors u_t and v_t in (2.1) and (2.2), respectively, are characterized as*

$$u_t = \gamma \nu_t + \varepsilon_t, \qquad t \in \mathbb{Z} \tag{2.3}$$

[3] All of the tests we discuss in this paper could equally well be used to test the null hypothesis $H_0 : \beta_1 = \beta_0$, but as the focus in equity forecasting is on testing the null hypothesis of a zero coefficient on the predictor we will restrict our discussion to $\beta_0 = 0$.

$$v_t \quad = \quad a_1 v_{t-1} + \ldots + a_{p-1} v_{t-p+1} + \nu_t, \tag{2.4}$$

where $(\varepsilon_t, \nu_t)'$ is serially uncorrelated, satisfying the conditions of Assumption 4 below, and the lag polynomial $A(L) := 1 - a_1 L - \ldots - a_{p-1} L^{p-1}$ is invertible. For further reference we define $\omega := \left(1 - \sum_{k=1}^{p-1} a_k\right)^{-1}$ and we denote by ϕ_k the coefficients of the lag polynomial $(1 - \rho L)A(L)$; in case of weak persistence, let b_k denote the coefficients of the (infinite-order) MA representation of the process ξ_t, $\sum_{k \geq 0} b_k L^k = ((1 - \rho L) A(L))^{-1}$.

Assumption 4 *Let*

$$\begin{pmatrix} \varepsilon_t \\ \nu_t \end{pmatrix} := \begin{pmatrix} \sigma_{\varepsilon t} \zeta_{\varepsilon t} \\ \sigma_{\nu t} \zeta_{\nu t} \end{pmatrix}$$

where $\boldsymbol{\zeta} := (\zeta_{\varepsilon t}, \zeta_{\nu t})'$ is a uniformly L_4-bounded stationary and ergodic martingale difference [MD] sequence satisfying $\mathrm{E}\left(\boldsymbol{\zeta}_t \boldsymbol{\zeta}_t'\right) = \mathbf{I}_2$ and $\mathrm{E}\left(\left\|\mathrm{E}_0\left(\sum_{t=1}^{T}(\boldsymbol{\zeta}_t\boldsymbol{\zeta}_t' - \mathbf{I}_2)\right)\right\|^2\right) = O(T^{2\epsilon})$ for some $\epsilon < \frac{1}{2}$, with $\mathrm{E}_0(\cdot)$ denoting expectation conditional on $\{\boldsymbol{\zeta}_{-i}\}_{i=0}^{\infty}$ and $\mathbf{I}_k$ the $k \times k$ identity matrix. Furthermore, let $\sigma_{\varepsilon t} := \sigma_\varepsilon\left(\frac{t}{T}\right)$ and $\sigma_{\nu t} := \sigma_\nu\left(\frac{t}{T}\right)$, where $\sigma_{\cdot}(\cdot)$ are piecewise Lipschitz-continuous bounded, non-stochastic functions on $(-\infty, 1]$, which are bounded away from zero.

Remark 2. Assumption 3 imposes, through (2.4), the condition that the errors v_t driving ξ_t in (2.2) follow a finite-order autoregression (AR) such that the predictor x_t is an $AR(p)$ process with $p \geq 1$; Valkanov (2003) makes the same assumption. The finite-order AR assumption is required for the tests developed in section 4.2 which make use of the residual augmented regression approach of Demetrescu and Rodrigues (2020). Here the transformed long-horizon predictive regression is augmented by the residuals from fitting an $AR(p)$ model to the predictor x_t. We conjecture that these tests would also be asymptotically valid under a linear process type assumption on v_t, provided the truncation lag for the fitted autoregression is allowed to increase at a suitable rate with the sample size, T. It is, however, important to note that the long-horizon predictability tests developed in both Hjalmarsson (2011) and Xu (2020) are based on the considerably more restrictive assumption that $A(L) = 1$, such that v_t is serially uncorrelated and, hence, that x_t follows an $AR(1)$. $\diamondsuit$

Remark 3. Assumption 4 is similar to Assumption 3 of Demetrescu et al. (2020) and we refer the reader to Demetrescu et al. (2020) for a detailed discussion of these conditions. Briefly, it allows for unconditional time heteroskedasticity of quite general form in the innovations $(\varepsilon_t, \nu_t)'$ through the functions $\sigma_\varepsilon(\cdot)$ and $\sigma_\nu(\cdot)$ which allow both ε_t and ν_t to display time-varying unconditional variances and for both contemporaneous and time-varying (unconditional) correlation between ε_t and ν_t. Empirically plausible models of single or multiple (co-) variance shifts, (co-)variances which follow a broken trend, and smooth transition (co-)variance shifts are all permitted under this assumption. The MD structure placed on $\boldsymbol{\zeta}$ allows for conditional heteroskedasticity of a general form obviating the need to choose a specific parametric model by instead adopting an explicit assumption of martingale approximability whereby $\mathrm{E}\left(\left\|\mathrm{E}_0\left(\sum_{t=1}^{T}(\boldsymbol{\zeta}_t\boldsymbol{\zeta}_t' - \mathbf{I}_2)\right)\right\|^2\right) = O(T^{2\epsilon})$ for some $\epsilon < \frac{1}{2}$, where ϵ controls the degree of persistence permitted in the conditional variances. Stationary vector GARCH processes with finite fourth-order moments satisfy these condition with $\epsilon = 0$, although Assumption 4 is considerably more general as it also allows for asymmetric effects in the conditional variance. Stationary autoregressive stochastic volatility processes as, for example, are assumed in Johannes et al. (2014) are also permitted. $\diamondsuit$

Remark 4. Assumption 4 is considerably weaker than the corresponding conditions imposed by the leading tests for long-horizon predictability in the literature. Valkanov (2003), Phillips and Lee (2013) and Xu (2020) all impose conditional (and, hence, unconditional) homoskedasticity on the innovations. In Remark 12, page 4414, Xu (2020) suggests the possibility that his approach could be modified (but does not actually develop such a modification) to allow for the case where the innovations can be conditionally heteroskedastic satisfying essentially the same conditions as are imposed in Assumption INNOV of Kostakis et al. (2015, p. 1512) for their short-horizon predictability tests. These conditions are, however, still considerably more restrictive than Assumption 4 as, in additional to imposing unconditional homoskedasticity, they also impose the condition that the error term in (2.1) is generated according to a stationary finite-order GARCH(p, q) model with finite fourth moments. Hjalmarsson (2011) allows for conditional heteroskedasticity but again assumes unconditional homoskedasticity; notice, however, that Hjalmarsson (2011) does not allow for the case where x_t is weakly persistent, which as discussed in Remark 12 of Xu (2020), is the case where allowing for conditional heteroskedasticity is most problematic. ♢

Remark 5. The error term u_t from (2.1) is formulated as a linear combination of the uncorrelated innovations ε_t and ν_t. It is then seen that the degree of endogeneity present is controlled through the parameter γ. Where $\gamma = 0$, $u_t = \varepsilon_t$ and, hence, the error term in (2.1) is uncorrelated with the innovation driving the predictor. The degree of endogeneity present is measured by the correlation between u_t and ν_t. This is given by $\phi_t := \gamma\sigma_{\nu t}/\sigma_{ut}$ which can therefore be either constant or time-varying under Assumption 4. It is worth stressing that Assumption 3 restricts γ in (2.3) to be time-invariant. We need to make this assumption to establish the large sample validity of the residual augmentation method we make use of in section 4.2. It might be possible to relax this assumption by using local (nonparametric) estimation of γ, but we leave such developments for further research. The restriction that γ is constant is common to all of the existing long-horizon predictability tests discussed above. ♢

2.2 The Long-Horizon Predictive Regression Specification

The most common long-horizon predictive regression specification used in empirical analysis results from the h-period, $h \geq 1$, temporal aggregation of (2.1) and is given by

$$y_{t+h}^{(h)} = \alpha_h + \beta_h x_t + error_{t+h}, \; t = 1, \ldots, T-h \tag{2.5}$$

where $y_{t+h}^{(h)} := \sum_{j=1}^{h} y_{t+j}$ is the h-period cumulative variable to be predicted. Notice that for $h = 1$, (2.5) is simply the short-horizon predictive regression in (2.1). To gain further insight into the specific features of (2.5), let us examine the h-horizon cumulated dependent variable $y_{t+h}^{(h)}$ closer. From (2.1), this can be written as,

$$y_{t+h}^{(h)} = h\alpha_1 + \beta_1 \sum_{j=0}^{h-1} x_{t+j} + u_{t+h}^{(h)}, \tag{2.6}$$

where, based on (2.3), $u_{t+h}^{(h)} := \sum_{j=1}^{h} u_{t+j} = \gamma v_{t+h}^{(h)} + \varepsilon_{t+h}^{(h)}$ with $v_{t+h}^{(h)}$ and $\varepsilon_{t+h}^{(h)}$ defined implicitly. Notice, moreover, that as the autoregressive representation of the predictor in (2.2) is given as

$x_{t+1} = \mu_x(1-\rho) + \rho x_t + v_{t+1}$, then by recursive substitution we have that,

$$\sum_{j=0}^{h-1} x_{t+j} = I_{h\geq 2} \sum_{j=1}^{h-1} \sum_{i=1}^{j} \rho^{i-1} \mu_x(1-\rho) + \sum_{j=0}^{h-1} \rho^j x_t + I_{h\geq 2} \sum_{j=1}^{h-1} \sum_{i=1}^{h-j} \rho^{i-1} v_{t+j} \tag{2.7}$$

where $I_{h\geq 2}$ is an indicator variable which takes the value 1 when $h \geq 2$ and 0 otherwise.

Consequently, replacing $\sum_{j=0}^{h-1} x_{t+j}$ in (2.6) by the expression on the right-hand side of (2.7), the general representation of the long-horizon predictive regression model specification is obtained,

$$y_{t+h}^{(h)} = \alpha_h + \beta_h x_t + w_{t+h}^{(h)} \tag{2.8}$$

where $\alpha_h := h\alpha_1 + \beta_1 I_{h\geq 2} \sum_{j=1}^{h-1} \sum_{i=1}^{j} \rho^{i-1} \mu_x(1-\rho)$, $\beta_h := \beta_1 \sum_{j=0}^{h-1} \rho^j$ and $w_{t+h}^{(h)} := u_{t+h}^{(h)} + \beta_1 I_{h\geq 2} \sum_{j=1}^{h-1} \sum_{i=1}^{h-j} \rho^{i-1} v_{t+j}$. Equation (2.8) gives a concrete formulation to (2.5) and corresponds to the long-horizon framework that will be considered for analysis throughout this paper.

As will subsequently become clear, this representation is particularly convenient for the case where x_t is strongly persistent. We note, however, that any serial correlation in v_t induces regressor endogeneity which, under weak persistence, leads to inconsistent estimation. In such cases is more suitable to write

$$\sum_{j=0}^{h-1} x_{t+j} = I_{h\geq 2} \sum_{j=1}^{h-1} \left(1 - \frac{\theta_j}{\theta_0}\right) \mu_x + \sum_{j=0}^{h-1} \frac{\theta_j}{\theta_0} x_t + I_{h\geq 2} \sum_{j=1}^{h-1} v_{t+j}^{\perp}$$

where $v_{t+j}^{\perp} := \xi_{t+j} - \frac{\theta_j}{\theta_0}\xi_t$, or $x_{t+j} = \mu_x\big(1 - \frac{\theta_j}{\theta_0}\big) + \frac{\theta_j}{\theta_0} x_t + v_{t+j}^{\perp}$ with $\theta_j := \sum_{k\geq 0} b_k b_{k+j}$, $j = 0, \ldots, h-1$, where b_k are the coefficients of the (infinite-order) MA representation of the process ξ_t. Under unconditional homoskedasticity, the quantities $v_{t+j}^{\perp}$ are projection errors from an orthogonal projection of ξ_{t+j} on ξ_t, while, under time-varying volatility, they can be interpreted as local counterparts thereof. Importantly, they are uncorrelated in the limit with the predictor x_t and can thus be seen as components of the h-step ahead forecast error. For weak persistence, this implies the long-horizon coefficient β_h will differ from $\beta_1 \sum_{j=0}^{h-1} \rho^j$ if v_t exhibits serial correlation; we may therefore switch to $\beta_h = \beta_1 \sum_{j=0}^{h-1} \frac{\theta_j}{\theta_0}$ whenever appropriate.

Equation (2.8) shows that $\beta_h \neq \beta_1$ for $h > 1$ when $\beta_1 \neq 0$. The coefficient β_h in (2.8) is therefore empirically useful, as a finding of statistical significance from an estimate of β_h can still be interpreted as evidence of long-horizon predictability, given that if there is no short-run predictability ($\beta_1 = 0$) then there is also no predictability at other horizons ($h \geq 1$). Consequently, under suitable assumptions, the null hypothesis of no-predictability, H_0, can be tested using statistics computed from (2.5). Under the assumption that x_t is weakly persistent tests can be based on conventional regression t-statistics, provided h is fixed. However, care is needed because the dynamics of the error term $w_{t+h}^{(h)}$ in (2.8) differ according to whether there is predictability or not. In particular, if $\beta_1 = 0$ (and, hence, $\beta_h = 0$), then this error term is an $MA(h-1)$ process. Where $\beta_1 \neq 0$, any serial correlation in v_t will change the dynamics of $w_{t+h}^{(h)}$; for example, if v_t is an $MA(1)$ process, then $w_{t+h}^{(h)}$ will follow an $MA(h)$ process. To account for these dynamics the t-statistic needs to be based on either HAC (Newey and West, 1987) or Hodrick (1992) standard errors. Although these are asymptotically equivalent, simulation evidence presented in Ang and Bekaert (2007) suggests the latter deliver tests with better finite sample behaviour. Moreover, Nelson and Kim (1993) show that finite sample biases present in the OLS estimate, $\hat{\beta}_1^{OLS}$ say, of β_1 from the short-horizon predictive regression in (2.1) (which are larger, other things equal, the greater the persistence of the predictor and the higher the endogeneity correlation between the innovations) are exacerbated by the long-horizon aggregation. Consequently, several bias correction approaches have been suggested for the case where x_t is weakly persistent; see for instance, Stambaugh (1999), Lewellen (2004), Amihud and Hurvich (2004), Amihud et al. (2009, 2010) and Kim (2014).

The standard t-tests and bias-correction methods discussed above are, however, not valid when x_t is strongly persistent. In particular, the limiting null distribution of the t-statistic is not pivotal because the endogeneity present in the model is not accounted for. In the next section we will briefly review tests which have been developed in the literature to allow for strong persistence in x_t.

3 Extant Tests for Long-Horizon Predictability allowing for Strongly Persistent Predictors

In this section we present a brief overview of test procedures for long-horizon predictability which allow for strongly persistent predictors. Specifically, we will outline the Bonferroni-based approach of Hjalmarsson (2011), the implied test of Xu (2020), and the reversed regression-based test of Phillips and Lee (2013). A Monte Carlo study comparing the finite sample performance of these tests with the tests proposed in this paper will be provided in section 6.

3.1 Bonferroni-based Tests

Assuming x_t is a strongly persistent (near-integrated) predictor, Hjalmarsson (2011) builds on the approach of Amihud and Hurvich (2004) to compute a second-order bias corrected estimate of β_h in order to develop a feasible long-horizon predictability test. In the context of (2.8), this is based on the infeasible augmented regression,

$$y_{t+h}^{(h)} = \alpha_h + \beta_h x_t + \gamma \nu_{t+h}^{(h)} + \varepsilon_{t+h}^{(h)} + r_{t+h}, \; t = 1, ..., T-h, \tag{3.1}$$

where $r_{t+h} := w_{t+h}^{(h)} - u_{t+h}^{(h)} = \beta_1 I_{h\geq 2} \sum_{j=1}^{h-1} \sum_{i=1}^{h-j} \rho^{i-1} v_{t+j}$, and, from Assumption 3 and (2.2),

$$\nu_{t+h}^{(h)} := \sum_{j=1}^{h} \nu_{t+j} = \sum_{j=1}^{h} \left[(x_{t+j} - \mu_x) - \sum_{k=1}^{p-1} \phi_k (x_{t+j-k} - \mu_x) \right].$$

The inclusion of $\nu_{t+h}^{(h)}$ in (3.1) serves to remove the endogeneity bias present in standard OLS estimation of (2.8) . Assuming $(u_{t+1}, v_{t+1})'$ is an unconditionally homoskedastic MD process, Hjalmarsson (2011) shows that, for fixed h, the infeasible scaled OLS estimator from (3.1), $\hat{\beta}_h^I$ say, when divided by h has a mixed normal null limiting distribution whose variance does not depend on h.

In order to obtain a feasible version of (3.1), Hjalmarsson (2011) adopts an approach based on Bonferroni-bounds. This involves computing a first-stage confidence interval for the local to unity parameter c which is then used to develop a test for long-horizon predictability based on a bias reduced estimate of β_h (see also Campbell and Yogo, 2006a). Denoting this confidence interval, with confidence level $100(1-\lambda_1)\%$, by $[\underline{c}_{\lambda_1}, \overline{c}_{\lambda_1}]$, feasible, yet conservative, versions of tests for $H_0 : \beta_h = 0$ against $H_A : \beta_h > 0$ and $H_0 : \beta_h = 0$ against $H_A : \beta_h < 0$, which we will generically define as t_h^{Bonf}, are, respectively,

$$h^{-1/2} t_{h,\tilde{c}^*}^{min} := \min_{\tilde{c} \in [\underline{c}_{\lambda_1}, \bar{c}_{\lambda_1}]} h^{-1/2} t_{h,\tilde{c}}^{OLS} > z_{\lambda_2} \tag{3.2}$$

and

$$h^{-1/2} t_{h,\tilde{c}^*}^{max} := \max_{\tilde{c} \in [\underline{c}_{\lambda_1}, \bar{c}_{\lambda_1}]} h^{-1/2} t_{h,\tilde{c}}^{OLS} < z_{\lambda_2}, \tag{3.3}$$

with $t_{h,\tilde{c}}^{OLS}$ being the OLS t-ratio for $\beta_h = 0$ computed from a feasible version of (3.1) where $\hat{\nu}_{t+h}^{(h)}$ is obtained based on $\hat{\rho} := 1 - \tilde{c}/T$ with $\tilde{c} \in [\underline{c}_{\lambda_1}, \bar{c}_{\lambda_1}]$, and z_{λ_2} is the standard normal critical value associated with the significance level λ_2 of the test, such that $\lambda_1 + \lambda_2 = \lambda$, where λ is the desired significance level of the test. In other words, a rejection occurs for the Bonferroni bounds test only if it occurs for every possible value of c in the first stage confidence interval. The requirement that $\lambda_1 + \lambda_2 = \lambda$ can lead to overly conservative tests and, in practice, adjustments to λ_1, to shrink the coverage rates of the confidence intervals for c are typically recommended; see Cavanagh et al. (1995) and Campbell and Yogo (2006b). In the linear predictive regression context, Hjalmarsson (2012) finds that his test has better power properties than the earlier test of Valkanov (2003). It is important to stress that these Bonferroni-based tests are developed under the assumption that x_t is strongly persistent and are not valid if x_t is weakly persistent. As we will see from the simulation results in section 6, these tests do indeed not perform well when x_t is weakly persistent. Moreover, it is important to note that Hjalmarsson (2011)'s approach is based on the assumption that $A(L) = 1$ in Assumption 3, such that x_t follows an $AR(1)$.

3.2 Xu (2020)'s Implied Test

Xu (2020) develops an alternative approach to testing for long-horizon predictability which allows for the case where the predictor, x_t, is either strongly or weakly persistent based on the computation of the implied long-horizon coefficients from short-horizon regression estimates; see, among others, Campbell and Shiller (1987), Kandel and Stambaugh (1996), Hodrick (1992) and Bekaert and Hodrick (1992). This choice of estimator is motivated by the observation that short-horizon estimation is often more efficient than long-horizon estimation; see, for example, Boudouk and Richardson (1994). Xu (2020) bases his test on the implied estimator of β_h, $\tilde{\beta}_h := \hat{\beta}_1^{OLS} \sum_{j=0}^{h-1} \hat{\rho}^j$ where $\hat{\beta}_1^{OLS}$ and $\hat{\rho}$ are the OLS estimates obtained from (2.1) and (2.2), respectively.

The implied long-horizon predictability test of Xu (2020), for the null hypothesis $H_0 : \beta_h = 0$, is based on the statistic

$$t_h^{Xu} = \upsilon_{IM}^{-1} \tilde{\beta}_h \tag{3.4}$$

where $\upsilon_{IM}^2 := \hat{\boldsymbol{q}} \hat{\boldsymbol{\Omega}} (\sum_{t=1}^{T-1} \bar{x}_t) \hat{\boldsymbol{q}}'$ with $\hat{\boldsymbol{q}} := (\hat{q}_1, \hat{q}_2)$, where $\hat{q}_1 := \sum_{j=0}^{h-1} \hat{\rho}^j$ and $\hat{q}_2 := \hat{\beta}_1^{OLS} \sum_{j=0}^{h-1} j \hat{\rho}^{j-1}$, and where the vector of OLS residuals, $\hat{\mathbf{e}}_{t+1} := (\hat{u}_{t+1}, \hat{v}_{t+1})'$, computed from (2.1) and (2.2), is used to estimate the covariance matrix of $\mathbf{e}_{t+1}$, $\hat{\boldsymbol{\Omega}} := \sum_{t=1}^{T-1} \hat{\mathbf{e}}_{t+1} \hat{\mathbf{e}}_{t+1}'$.

Under the assumption of conditionally homoskedastic MD innovations, Xu (2020) shows that under H_0: (i) if x_t is strongly persistent, $t_h^{Xu} \xrightarrow{d} \phi \left[(\int_0^1 \bar{J}_c^2(s))^{-1/2} \int_0^1 \bar{J}_c(s) \mathrm{d}W(s) \right] + (1 - \phi^2)^{1/2} \mathcal{Z}$, where ϕ denotes the (time-invariant) correlation between the innovations u_{t+1} and ν_{t+1} in (2.1) and (2.2) (see Assumption 3), J_c an OU process driven by the standard Wiener process W and $\mathcal{Z}$ is a standard normal variate independent of W; and (ii) if x_t is weakly persistent, $t_h^{Xu} \xrightarrow{d} N(0, 1)$. These results show that the limiting null distribution of the test statistic changes depending on the persistence of the predictor and the magnitude of ϕ. To account for this, Xu (2020) proposes two alternative ways to compute the necessary critical values. One is based on a Bonferroni procedure and the other uses a bias-corrected wild bootstrap approach (residual-based with recursive design), although Xu (2020) does not formally establish the asymptotic validity of the latter. Out of the two approaches Xu (2020) recommends the latter, which is the one we will consider in our Monte Carlo analysis. It is important to note that the asymptotic validity of Xu (2020)'s test, like that of Hjalmarsson (2011), relies on the assumption that x_t is an $AR(1)$ process, so that $A(L) = 1$ in Assumption 3. The assumption of no serial correlation in v_t is essential for Xu (2020)'s approach under weak persistence, as in this case we have that $\beta_h = \beta_1 \sum_{j=0}^{h-1} \frac{\theta_j}{\theta_0}$ (see section 2.2), implying that $\beta_1 \sum_{j=0}^{h-1} \rho^j$ is not the correct quantity to base a test on.

3.3 Reversed Regression-based Tests

An alternative to the use of HAC or Hodrick (1992) standard errors to account for the serial correlation in the error term in the long-horizon predictive regression model in (2.8) discussed in section 2.2 is to use an alternative regression specification that is designed to explicitly account for the overlapping data issue. One such approach is to use so-called *reverse regressions*; see, among others, Jegadeesh (1991) and Cochrane (1991). This approach, instead of being based on the regression from the h-period returns on a predetermined variable, as in (2.5), is based on a regression of single period returns on the same predetermined variable but aggregated over h-periods. Specifically, this *reverse regression* formulation is given by,

$$y_{t+h} = \alpha_h^{rev} + \beta_h^{rev} x_{t+h-1}^{(h)} + u_{t+h},\ t = 1, ..., T-h \tag{3.5}$$

where $x_{t+h-1}^{(h)} := \sum_{j=0}^{h-1} x_{t+j}$. See also Hodrick (1992), Maynard and Ren (2014), Ang and Bekaert (2007), and Wei and Wright (2013), *inter alia*. It is seen from (3.5) that the error term is u_{t+h} which is serially uncorrelated. An implication of this is that the IVX estimation and hypothesis testing methods developed in Kostakis et al. (2015) can be directly applied to (3.5), which is not the case for (2.8) because of the induced serial correlation in $w_{t+h}^{(h)}$.

The OLS estimate of β_h^{rev} from (3.5) is given by $\hat{\beta}_h^{rev} := (\sum_{t=1}^{T-h} \bar{x}_{t+h-1}^{(h)} \bar{y}_{t+h})/(\sum_{t=1}^{T-h} (\bar{x}_{t+h-1}^{(h)})^2)$, where for a generic sequence $\{w_t\}_{t=a}^{b}$, $\bar{w}_t := w_t - (b-a+1)^{-1} \sum_{s=a}^{b} w_s$. It is not hard to establish that, regardless of whether x_t is weakly or strongly persistent, $\hat{\beta}_h^{rev} = (\sum_{t=1}^{T-h} \bar{x}_t^2)/(\sum_{t=1}^{T-h} (\bar{x}_{t+h-1}^{(h)})^2) \hat{\beta}_h^{OLS}$ $+ o_p(1)$, where $\hat{\beta}_h^{OLS}$ is the OLS estimate of β_h from (2.5). Motivated by this, Phillips and Lee (2013) develop a long-horizon predictability test based on applying IVX estimation to the reverse regression in (3.5). Specifically, they use the IVX instrument z_t suggested by Kostakis et al. (2015), which is constructed from the predictor as,

$$z_t := (1 - \varrho L)_+^{-1} \Delta x_t = \sum_{j=0}^{t} \varrho^j \Delta x_{t-j}. \tag{3.6}$$

The persistence of z_t is controlled by setting $\varrho := 1 - \frac{a}{T^\eta}$, with $0 < \eta < 1$. If x_t is near integrated, this makes z_t approximately mildly integrated (and thus of lower persistence), while if x_t is weakly persistent then one may decompose $z_t = x_t - \mu_x + r_t$, where the rest term satisfies $r_t \to 0$ as $t \to \infty$ and can be controlled for in the relevant expressions; see e.g. Lemma S.3 in the supplementary appendix for details. Because the reversed regression in (3.5) features $x_{t+h-1}^{(h)} := \sum_{j=0}^{h-1} x_{t+j}$, the long-horizon IVX approach is based on instrumenting $x_{t+h-1}^{(h)}$ by $z_{t+h-1}^{(h)} := \sum_{j=0}^{h-1} z_{t+j}$.

Allowing the forecast horizon, h, to grow at rate $T^{1/2}T^{-\eta} + T^{\eta}h^{-1} + h/T \to 0$, such that it increases at a slower rate than the sample size T, but faster than the (user-controlled) degree of mild integration of the instrument, Phillips and Lee (2013)'s long-horizon predictability statistic is

$$t_{h,ivx}^{rev,PL} := (\mathcal{H}^{-1}\hat{\sigma}_u^2)^{-1/2}\hat{\beta}_{h,ivx}^{rev} \tag{3.7}$$

where $\hat{\beta}_{h,ivx}^{rev} := \left(\sum_{t=1}^{T-h} \bar{x}_{t+h-1}^{(h)} z_{t+h-1}^{(h)}\right)^{-1} \sum_{t=1}^{T-h} z_{t+h-1}^{(h)} \bar{y}_{t+h}, \mathcal{H} := \left[\mathcal{H}_{\bar{x}^{(h)}z^{(h)}}(\mathcal{H}_{z^{(h)}z^{(h)}})^{-1}\mathcal{H}_{x^{(h)}z^{(h)}}'\right]^{-1}$, $\mathcal{H}_{x^{(h)}z^{(h)}} := \sum_{t=1}^{T-h} x_{t+h-1}^{(h)} z_{t+h-1}^{(h)}$, $\mathcal{H}_{z^{(h)}z^{(h)}} := \sum_{t=1}^{T-h}(z_{t+h-1}^{(h)})^2$ and $\hat{\sigma}_u^2 := \frac{1}{T-1}\sum_{t=1}^{T-1}\hat{u}_{t+1}^2$. Assuming that the innovations are conditionally homoskedastic, Phillips and Lee (2013) show that $t_{h,ivx}^{rev,PL}$ has a standard normal limiting distribution under H_0. It should be noted that Phillips and Lee (2013) do not formally allow for the possibility that x_t is weakly persistent.

4 Transformed Regression-based Long-Horizon Predictability Tests

In this section we introduce our new approach to long-run predictability testing which builds on the IVX framework of Kostakis et al. (2015) and the augmented regression approach of Amihud and Hurvich (2004), Hjalmarsson (2011) and Demetrescu and Rodrigues (2020). In common with the tests of Xu (2020) the tests we develop are asymptotically valid regardless of whether the predictor, x_t, is weakly or strongly persistent, however we do not need to implement either a Bonferroni or wild bootstrap scheme to run our tests. Moreover, unlike the tests in Xu (2020) we do not need to assume that x_t follows an $AR(1)$ process. We can also allow the innovations $(u_{t+1}, v_{t+1})'$ to display the very general forms of unconditional and/or conditional heteroskedasticity specified under Assumption 4.

4.1 Transformed Regression IVX based Tests

In a recent paper Britten-Jones et al. (2011) develop a method for conducting inference in linear regression models with overlapping observations and stationary covariates. Before showing how we can apply this approach to the specific setting considered in this paper, we first briefly review the transformed regression approach. To that end, suppose we have a generic linear regression model $\mathbf{A}_h\mathbf{y} = \mathbf{X}\boldsymbol{\beta}+\mathbf{u}$, where $\mathbf{y}$ is the $(T-1)$-vector of single period returns, $\mathbf{A}_h$ is the *known* $(T-h)\times(T-1)$ aggregation matrix with entries $a_{ij} = 1$ if $i \le j \le i+h-1$ and zero otherwise, $i = 1, ..., T-h$, such that $\mathbf{A}_h\mathbf{y}$ is the vector of (overlapping) h-period returns, $\mathbf{X}$ the regressor matrix with associated vector of coefficients, $\boldsymbol{\beta}$ and $\mathbf{u}$ is the error vector. Britten-Jones et al. (2011) demonstrate that the OLS estimate of $\boldsymbol{\beta}$ from this regression, $\tilde{\boldsymbol{\beta}}$ say, is numerically identical to the OLS estimate from the transformed regression $\mathbf{y} = \tilde{\mathbf{X}}\boldsymbol{\beta} + \tilde{\mathbf{u}}$, where $\tilde{\mathbf{X}} := \mathbf{A}_h'\mathbf{X}(\mathbf{X}'\mathbf{A}_h\mathbf{A}_h'\mathbf{X})^{-1}\mathbf{X}'\mathbf{X}$. The associated estimation error from the transformed regression can then be written as $\tilde{\boldsymbol{\beta}} - \boldsymbol{\beta} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{A}_h\tilde{\mathbf{u}}$, which is seen to depend on the autocorrelation structure of $\tilde{\mathbf{u}}$, the disturbance term in the transformed (non-overlapping) regression, rather than on $\mathbf{u}$, the disturbance in the untransformed (overlapping) regression. The part of the autocorrelation in $\mathbf{u}$ induced by the temporal aggregation (through $\mathbf{A}_h$) is therefore explicitly accounted for and does not need to be estimated from the data when conducting inference on $\boldsymbol{\beta}$ via the transformed regression. In the context of the DGP in (2.1)–(2.2), a key implication of this result is that while the IVX approach of Kostakis et al. (2015) cannot be used to conduct valid inference on β_h in (2.8) under Assumption 3, because of the autocorrelation present in the error term $u_{t+h}^{(h)}$ induced by temporal aggregation, it can when applied to the transformed regression analogue of (2.8).

To that end, consider again (2.8). Using the general result above it can be shown[4] that the OLS estimator of the slope parameter β_h, $\hat{\beta}_h^{OLS} := (\sum_{t=1}^{T-h} \bar{x}_t \bar{y}_{t+h}^{(h)})/(\sum_{t=1}^{T-h} \bar{x}_t^2)$, can be written equivalently as

$$\hat{\beta}_h^{trf} := \frac{\sum_{t=1}^{T-1} \bar{x}_t^{trf,(h)} \bar{y}_{t+1}}{\sum_{t=1}^{T-h} \bar{x}_t^2} \tag{4.1}$$

where

$$\bar{x}_t^{trf,(h)} := \begin{cases} \sum_{i=1}^{t} \bar{x}_i & \text{for } t = 1, ..., h-1 \\ \bar{x}_t^{(h)} := \sum_{i=1}^{h} \bar{x}_{t-h+i} & \text{for } h \leq t \leq T-h \\ \sum_{i=t-h+1}^{T-h} \bar{x}_i & \text{for } t = T-h+1, ..., T-1 \end{cases} . \tag{4.2}$$

From (4.1) it can be observed that $\hat{\beta}_h^{trf}$ is computed from the original non-overlapping one period returns. Notice that the transformed estimator in (4.1) can also be obtained from a regression of $\bar{y}_{t+1}$ on $\tilde{\bar{x}}_{t+h-1}^{trf,(h)}$, where

$$\tilde{\bar{x}}_t^{trf,(h)} := \left(\sum_{t=1}^{T-1} \left(\bar{x}_t^{trf,(h)} \right)^2 \right)^{-1} \left(\sum_{t=1}^{T-h} \bar{x}_t^2 \right) \bar{x}_t^{trf,(h)}.$$

Interestingly, it can be shown that the OLS slope estimator from the reverse regression (3.5), $\hat{\beta}_h^{rev}$ say, and $\hat{\beta}_h^{trf}$ are linearly related; specifically,

$$\hat{\beta}_h^{rev} = \frac{\sum_{t=1}^{T-h} \bar{x}_t^2}{\sum_{t=1}^{T-h} (\bar{x}_{t+h-1}^{(h)})^2} \hat{\beta}_h^{trf} + \frac{\sum_{k=1}^{h-1} [(\sum_{t=T-h+1}^{T-k} \bar{x}_i) \bar{y}_{T-k+1} - (\sum_{i=1}^{k} \bar{x}_i) \bar{y}_{k+1}]}{\sum_{t=1}^{T-h} (\bar{x}_{t+h-1}^{(h)})^2}$$

which suggests that when h is small the performance of predictability statistics from the reversed regression and transformed regression should be very similar, but as h increases their performance will likely differ.

If we knew that the predictor, x_t, was weakly persistent then we could base tests on the OLS estimate from the transformed regression discussed above. However, as with the tests of Phillips and Lee (2013) from section 3.3, we want to allow for strongly persistent predictors. We will therefore apply the IVX framework of Kostakis et al. (2015) to the transformed regression. To that end, recall the IVX instrument z_t defined in (3.6). The transformed regression based IVX estimator is then obtained by regressing $\bar{y}_{t+1}$ on $\tilde{z}_t^{trf,(h)}$, where

$$\tilde{z}_t^{trf,(h)} := \frac{\left(\sum_{t=1}^{T-h} z_t \bar{x}_t \right) z_t^{trf,(h)}}{\sum_{t=1}^{T-1} \left(z_t^{trf,(h)} \right)^2} \tag{4.3}$$

with

$$z_t^{trf,(h)} := \begin{cases} \sum_{i=1}^{t} z_i & \text{for } t = 1, ..., h-1 \\ z_t^{(h)} := \sum_{i=1}^{h} z_{t-h+i} & \text{for } h \leq t \leq T-h \\ \sum_{i=t-h+1}^{T-h} z_i & \text{for } t = T-h+1, ..., T-1. \end{cases} . \tag{4.4}$$

[4] Full derivations for the functional forms of the estimators and statistics from the transformed regression given in this section are provided in the supplementary appendix.

Hence, we obtain the transformed regression IVX estimator,

$$\hat{\beta}_{h,ivx}^{trf} = \frac{\sum_{t=1}^{T-1} z_t^{trf,(h)} \bar{y}_{t+1}}{\sum_{t=1}^{T-h} z_t \bar{x}_t}. \tag{4.5}$$

It can be shown that

$$\hat{\beta}_{h,ivx}^{trf} = \beta_h + \frac{\sum_{t=1}^{T-1} z_t^{trf,(h)} \bar{u}_{t+1}}{\sum_{t=1}^{T-h} z_t \bar{x}_t} + o_p(1) \tag{4.6}$$

and so the IVX estimate provides the basis for inference on β_h. In particular, a test for the null hypothesis, $H_0 : \beta_h = 0$, against one or two-sided alternatives, can be obtained using the IVX based t-ratio from the transformed regression,

$$t_{h,ivx}^{trf} := \frac{\hat{\beta}_{h,ivx}^{trf}}{s.e.\left(\hat{\beta}_{h,ivx}^{trf}\right)}. \tag{4.7}$$

In the context of (4.7), in view of Assumption 4 which allows for both conditional and unconditional heteroskedasticity in the innovations, we implement our IVX-based tests with conventional White heteroskedasticity-robust standard errors; that is,

$$s.e.\left(\hat{\beta}_{h,ivx}^{trf}\right) := \left[\left(\sum_{t=1}^{T-h} z_t \bar{x}_t\right)^{-1} \sum_{t=1}^{T-1} \left(z_t^{trf,(h)} \ddot{u}_{t+1}\right)^2 \left(\sum_{t=1}^{T-h} z_t \bar{x}_t\right)^{-1}\right]^{1/2} \tag{4.8}$$

where $\ddot{u}_{t+1} := \bar{y}_{t+1} - \hat{\beta}_{h,ivx}^{trf} \bar{z}_t^{trf,(h)}$ are the residuals from the IVX estimation of the transformed regression. When testing for no predictability, one may alternatively compute the residuals under the null; that is, $\ddot{u}_{t+1} := \bar{y}_{t+1}$.

4.2 Residual Augmented Tests

Recall the infeasible augmented regression in (3.1). On first sight, one might think it is possible to implement a feasible version of (3.1) that can be estimated by OLS simply by replacing the regressor $\nu_{t+h}^{(h)}$ with an estimate of that quantity constructed from the OLS residuals, $\hat{\nu}_t$ say, obtained from fitting an $AR(p)$ model to x_t (see Equation (4.9) below). This will not, however, work. To illustrate why, consider the feasible estimator $\hat{\beta}_h^F := \left(\sum_{t=p}^{T-h} \bar{x}_t^2\right)^{-1} \sum_{t=p}^{T-h} \hat{\bar{y}}_{t+h}^{(h)} \bar{x}_t$, where $\hat{\bar{y}}_{t+h}^{(h)} := \bar{y}_{t+h}^{(h)} - \hat{\gamma}\hat{\nu}_{t+h}^{(h)}$ and $\hat{\gamma}$ is a consistent estimator of γ e.g. the fitted coefficient of an OLS regression of $\bar{y}_t$ on $\hat{\nu}_t$ when testing the null $\beta_h = 0$.[5] In the simplest possible case where no short-run dynamics are present in the predictor process, it then follows that,

$$\hat{\beta}_h^F = \hat{\beta}_h^I + \gamma\left(\hat{\rho} - \rho\right) \frac{\sum_{t=1}^{T-h} \bar{x}_t \bar{x}_{t+h-1}^{(h)}}{\sum_{t=1}^{T-h} \bar{x}_t^2} + o_p(1)$$

where $\hat{\beta}_h^I$ is the infeasible estimate of β_h from (3.1). This shows that the feasible estimate features an additional term relative to the infeasible estimator, $\hat{\beta}_h^I$, which depends on the estimation error associated with the predictor's autoregressive parameter, $(\hat{\rho}-\rho)$, weighted by $\gamma\left(\sum_{t=1}^{T-h} \bar{x}_t^2\right)^{-1} \sum_{t=1}^{T-h} \bar{x}_t \bar{x}_{t+h-1}^{(h)}$. This term can be shown to be of the same order of magnitude as $\hat{\beta}_h^I$ (see e.g. Cai and Wang, 2014,

[5]If not testing the null $\beta_h = 0$, $\hat{\gamma}$ may of course be obtained via the OLS regression of $\hat{u}_t$ and $\hat{\nu}_t$.

for the short-horizon case) which renders the limiting null distribution of $\hat{\beta}_h^F$ non-pivotal. In fact, if computing the feasible estimator for $h = 1$ by augmenting the predictive regression with the OLS residuals $\hat{\nu}_{t+1}$, it can be shown that the residuals $\hat{\nu}_{t+1}$ are exact orthogonal to the regressor, x_t, and so this version of the feasible estimator will be numerically identical to the standard OLS estimator in the short-horizon case.

As discussed in the context of short-horizon predictability testing in Demetrescu and Rodrigues (2020), the problem with implementing a feasible version of (3.1) discussed above does not arise if we estimate the residual augmented regression by IVX. Following this idea, we can apply residual augmentation to the transformed IVX estimate discussed in section 4.1 by regressing $\bar{y}_{t+1} - \hat{\gamma}\hat{\nu}_{t+1}$, rather than $\bar{y}_{t+1}$, on $\tilde{z}_t^{trf,(h)}$, where $\tilde{z}_t^{trf,(h)}$ is as defined in (4.3) and the residuals $\hat{\nu}_{t+1}$ are computed from an estimated autoregressive model of order p for the predictor x_t, *viz.*,

$$\hat{\nu}_{t+1} := \bar{x}_{t+1} - \sum_{k=1}^{p} \hat{\phi}_k \bar{x}_{t+1-k} = \nu_t - \sum_{k=1}^{p} \left(\hat{\phi}_k - \phi_k\right) \bar{x}_{t-k} \tag{4.9}$$

for $t = p, \ldots, T-1$, where $\hat{\phi}_k$, $k = 1, ..., p$ are the OLS autoregressive parameter estimates. The dependent variable, $\bar{y}_{t+1} - \hat{\gamma}\hat{\nu}_{t+1}$, is simply the OLS residual from the regression of $\bar{y}_{t+1}$ on $\hat{\nu}_{t+1}$. In practice the lag augmentation order, p, in (4.9) can be selected using a standard information criterion, setting the minimum possible lag length allowed to be one. We denote the resulting residual-augmented transformed regression IVX estimator by $\hat{\beta}_{h,ivx}^{trf,res}$.

The suitability of this approach in the IVX framework stems from the fact that the additional term attributable to the OLS estimation errors in the feasible estimation, discussed above, is asymptotically negligible in the IVX context when the predictor is strongly persistent. To see why, consider the computational form for $\hat{\beta}_{h,ivx}^{trf,res}$,

$$\hat{\beta}_{h,ivx}^{trf,res} := \frac{\sum_{t=p}^{T-1} z_t^{trf,(h)} (\bar{y}_{t+1} - \hat{\gamma}\hat{\nu}_{t+1})}{\sum_{t=1}^{T-h} z_t \bar{x}_t} \tag{4.10}$$

which can be written equivalently as

$$\hat{\beta}_{h,ivx}^{trf,res} = \frac{\sum_{t=p}^{T-1} z_t^{trf,(h)} (\beta_1 \bar{x}_t + \bar{u}_{t+1} - \hat{\gamma}\hat{\nu}_{t+1})}{\sum_{t=1}^{T-h} z_t \bar{x}_t}.$$

Using results from the proofs of Theorems 4.1 and 4.3 in the supplementary appendix, it can be shown that

$$\begin{aligned}
\hat{\beta}_{h,ivx}^{trf,res} &= \beta_h + \frac{\sum_{t=p}^{T-1} z_t^{trf,(h)} (\bar{\varepsilon}_{t+1} + \gamma\bar{\nu}_{t+1} - \hat{\gamma}\hat{\nu}_{t+1})}{\sum_{t=1}^{T-h} z_t \bar{x}_t} + o_p(1) \\
&= \beta_h + \frac{\sum_{t=p}^{T-1} z_t^{trf,(h)} (\bar{\varepsilon}_{t+1} - \gamma(\hat{\nu}_{t+1} - \bar{\nu}_{t+1}) - (\hat{\gamma} - \gamma)\hat{\nu}_{t+1})}{\sum_{t=1}^{T-h} z_t \bar{x}_t} + o_p(1) \\
&= \beta_h + \frac{\sum_{t=p}^{T-1} z_t^{trf,(h)} \bar{\varepsilon}_{t+1}}{\sum_{t=1}^{T-h} z_t \bar{x}_t} + \gamma \sum_{k=1}^{p} (\hat{\phi}_k - \phi_k) \frac{\sum_{t=p}^{T-1} z_t^{trf,(h)} \bar{x}_{t-k}}{\sum_{t=1}^{T-h} z_t \bar{x}_t} + o_p(1).
\end{aligned}$$

As will be demonstrated in the formal derivations in the appendix, the usual OLS autoregressive convergence rates on $\hat{\phi}_k$ suffice for the estimation effect to be negligible under strong persistence.

In the short-horizon case, Demetrescu and Rodrigues (2020) show, however, that the variance of $\hat{\beta}^{trf,res}_{h,ivx}$ will be affected by residual augmentation under weak persistence. For this reason they recommend computing the standard errors corresponding to the weak persistence case, and prove that the correction term this entails has an asymptotically negligible effect on the standard errors under strong persistence, such that one may conveniently use the standard errors developed for weak persistence irrespective of whether the predictor exhibits weak or strong persistence. We will adopt the same approach in the long-horizon case.

Thus, the relevant test statistic to test the null hypothesis $H_0 : \beta_h = 0$ is given by,

$$t^{trf,res}_{h,ivx} := \frac{\hat{\beta}^{trf,res}_{h,ivx}}{s.e.\left(\hat{\beta}^{trf,res}_{h,ivx}\right)} \tag{4.11}$$

where

$$s.e.\left(\hat{\beta}^{trf,res}_{h,ivx}\right) := \left(\mathcal{H}_{zx}\right)^{-1}\left[\mathcal{H}_{z^{trf,(h)}\hat{\varepsilon}z^{trf,(h)}\hat{\varepsilon}} + \hat{\gamma}^2\hat{Q}^{trf,(h)}_T\right]^{1/2}$$

with $\mathcal{H}_{zx} := \left(\sum_{t=1}^{T-h} z_t\bar{x}_t\right)$; $\mathcal{H}_{z^{trf,(h)}\hat{\varepsilon}z^{trf,(h)}\hat{\varepsilon}} := \left(\sum_{t=p}^{T-1}\left(z^{trf,(h)}_t\right)^2\hat{\hat{\varepsilon}}^2_{t+1}\right)$; and

$$\hat{Q}^{trf,(h)}_T := \mathcal{H}'_{z^{trf,(h)}\bar{\boldsymbol{x}}}\mathcal{H}^{-1}_{\bar{\boldsymbol{x}}\bar{\boldsymbol{x}}}\mathcal{H}_{\bar{\boldsymbol{x}}\bar{\boldsymbol{x}}v}\mathcal{H}^{-1}_{\bar{\boldsymbol{x}}\bar{\boldsymbol{x}}}\mathcal{H}_{z^{trf,(h)}\bar{\boldsymbol{x}}};$$

letting $\bar{\boldsymbol{x}}_t := (\bar{x}_t, \ldots, \bar{x}_{t-p+1})'$, we have further $\mathcal{H}_{z^{trf,(h)}\bar{\boldsymbol{x}}} := \left(\sum_{t=p}^{T-1} z^{trf,(h)}_t\bar{x}_t, \ldots, \sum_{t=p}^{T-1} z^{trf,(h)}_t\bar{x}_{t-p+1}\right)'$, $\mathcal{H}_{\bar{\boldsymbol{x}}\bar{\boldsymbol{x}}} := \sum_{t=p}^{T-1}\bar{\boldsymbol{x}}_t\bar{\boldsymbol{x}}'_t$; and $\mathcal{H}_{\bar{\boldsymbol{x}}\bar{\boldsymbol{x}}v} := \sum_{t=p}^{T-1}\bar{\boldsymbol{x}}_t\bar{\boldsymbol{x}}'_t\hat{\nu}^2_{t+1}$, with $\hat{\hat{\varepsilon}}_{t+1}$ the residuals from regressing y_{t+1} on $\hat{\nu}_{t+1}$ and an intercept (i.e. computed under the null hypothesis). When not testing the null $\beta_h = 0$, $\hat{\hat{\varepsilon}}_{t+1}$ should be computed as the usual residuals, i.e. including x_t as regressor. These (heteroskedasticity-robust) standard errors are designed to automatically take the estimation variability of $\hat{\phi}_k$ into account whenever needed, such that the standard errors are asymptotically correct without having to specify whether x_t is weakly or strongly persistent; cf. Demetrescu and Rodrigues (2020).

4.3 Asymptotic Theory

In this section we analyse the large sample distributions of the estimators and test statistics proposed in sections 4.1 and 4.2, when the data generating process is as in (2.1)–(2.2) under Assumptions 1–4. In this setting, it is observed that the partial sums of the innovations v_t and ε_t display joint weak convergence to time-transformed Brownian motions (see Lemma S.1 in the supplementary appendix); precisely,

$$\frac{1}{\sqrt{T}}\sum_{t=1}^{\lfloor sT\rfloor}\begin{pmatrix}\varepsilon_t\\ v_t\end{pmatrix} \Rightarrow \begin{pmatrix}\int_0^s \sigma_\varepsilon(r)\,\mathrm{d}W_\varepsilon(r)\\ \int_0^s \sigma_\nu(r)\,\mathrm{d}W_\nu(r)\end{pmatrix}$$

were "$\Rightarrow$" denotes weak convergence on the space of càdlàg real functions on $[0,1]^k$ equipped with the Skorokhod topology, and where W_ε and W_ν are independent standard Wiener processes. Moreover, under near integration (Assumption 2.(i)), it also follows that the stochastic part of the suitably normalised regressor weakly converges to an Ornstein-Uhlenbeck-type process; that is,

$$T^{-1/2}\xi_{\lfloor sT\rfloor} \Rightarrow \omega\int_0^s e^{-c(s-r)}\sigma_\nu\mathrm{d}W_\nu(r) =: \omega J_{c,\sigma}(s)\,. \tag{4.12}$$

In Theorems 4.1 and 4.2 we first establish the limiting distributions of $\hat{\beta}^{trf,res}_{h,ivx}$ and $\hat{\beta}^{trf}_{h,ivx}$ and their associated standard errors in the case where x_t is strongly persistent.

Theorem 4.1 *Under Assumptions 1, 2.(i), 3 and 4 with $\epsilon < \min\{1-\eta;\eta/2\}$ and as $h,T\to\infty$ such that $h/(\min\{T^{3\eta/2-1/2};T^{2\eta-1}\}) + T^{1/2-\eta/2}/h \to 0 \to 0$,*

$$\frac{T^{\eta/2+1/2}}{h}\left(\hat{\beta}^{trf,res}_{h,ivx}-\beta_h\right) \Rightarrow \mathcal{MN}\left(0, \frac{a\int_0^1 \sigma_\nu^2(s)\,\sigma_\varepsilon^2(s)\,\mathrm{d}s}{2\omega^2\left(J_{c,\sigma}(1)\bar{J}_{c,\sigma}(1)-\int_0^1 J_{c,\sigma}(s)\mathrm{d}J_{c,\sigma}(s)\right)^2}\right)$$

where a and η are the tuning parameters for the IVX instrument in (3.6) and $\mathcal{MN}$ denotes a mixed normal distribution, with ω defined in assumption 3, $J_{c,\sigma}(s)$ defined in (4.12) and $\bar{J}_{c,\sigma}(s) := J_{c,\sigma}(s) - \int_0^1 J_{c,\sigma}(s)\mathrm{d}s$, and

$$\frac{T^{\eta/2+1/2}}{h} s.e.\left(\hat{\beta}^{trf,res}_{h,ivx}\right) \Rightarrow \frac{\sqrt{a\int_0^1 \sigma_\nu^2(s)\,\sigma_\varepsilon^2(s)\,\mathrm{d}s}}{\sqrt{2\omega^2}\left(J_{c,\sigma}(1)\bar{J}_{c,\sigma}(1)-\int_0^1 J_{c,\sigma}(s)\mathrm{d}J_{c,\sigma}(s)\right)}.$$

Remark 6. The limiting results given in Theorem 4.1 are similar to those given in Theorem 3.2 of Demetrescu and Rodrigues (2020) for the short-horizon $h=1$ case, but hold under considerably weaker assumptions on the innovations than are allowed for in Demetrescu and Rodrigues (2020); here, we allow for conditional heteroskedasticity while Demetrescu and Rodrigues (2020) only consider heterogeneous independent error sequences. Compared to the short-horizon case, the results in Theorem 4.1 need to take account of the implied aggregation of various quantities which, although individually asymptotically negligible quantities, arise over h periods. Given that we allow for $h\to\infty$, this entails the need to place additional conditions on the persistence allowed for in the IVX instrument, as controlled by η. In particular, Theorem 4.1 requires that $\eta > 1/3$, in addition to conditions relating the persistence of z_t to the strength of the GARCH effects present in the data generating process, as controlled by ϵ. The choice of $\eta = 0.95$ for the IVX tuning parameter recommended by Kostakis et al. (2015) is permitted under our rate restrictions, as long as the serial dependence in the conditional variances is not too high. We note that the results only require that $h\to\infty$ at a minimal rate, which is quite mild when η is close to unity. ♢

Theorem 4.2 *Under the conditions of Theorem 4.1, we have that*

$$\frac{T^{\eta/2+1/2}}{h}\left(\hat{\beta}^{trf}_{h,ivx}-\beta_h\right) \Rightarrow \mathcal{MN}\left(0, \frac{a\int_0^1 \sigma_\nu^2(s)\left(\sigma_\varepsilon^2(s)+\gamma^2\sigma_\nu^2(s)\right)\mathrm{d}s}{2\omega^2\left(J_{c,\sigma}(1)\bar{J}_{c,\sigma}(1)-\int_0^1 J_{c,\sigma}(s)\mathrm{d}J_{c,\sigma}(s)\right)^2}\right)$$

and

$$\frac{T^{\eta/2+1/2}}{h} s.e.\left(\hat{\beta}^{trf}_{h,ivx}\right) \Rightarrow \frac{\sqrt{a\int_0^1 \sigma_\nu^2(s)\left(\sigma_\varepsilon^2(s)+\gamma^2\sigma_\nu^2(s)\right)\mathrm{d}s}}{\sqrt{2\omega^2}\left(J_{c,\sigma}(1)\bar{J}_{c,\sigma}(1)-\int_0^1 J_{c,\sigma}(s)\mathrm{d}J_{c,\sigma}(s)\right)}.$$

Remark 7. A comparison of the results in Theorems 4.1 and 4.2 highlights the (asymptotic) efficiency gains which arise from residual augmentation. This can be seen by noting that the asymptotic variance (conditional on $J_{c,\sigma}$) of $\hat{\beta}^{trf}_{h,ivx}$ is strictly larger than that of the residual augmented estimator, $\hat{\beta}^{trf,res}_{h,ivx}$, whenever $\gamma \neq 0$. ♢

In Theorems 4.3 and 4.4 we next establish the limiting distributions of $\hat{\beta}^{trf,res}_{h,ivx}$ and $\hat{\beta}^{trf}_{h,ivx}$ and their associated standard errors in the case where x_t is weakly persistent.

Theorem 4.3 *Under Assumptions 1, 2.(ii) 3 and 4, we have as $h, T \to \infty$ such that $h^3/T + T^{1/2-\eta/2}/h \to 0$,*

$$\sqrt{\frac{T}{h}}\left(\hat{\beta}^{trf,res}_{h,ivx} - \beta_h\right) \xrightarrow{d} \mathcal{N}\left(0, \frac{\frac{\omega^2}{(1-\rho)^2}\int_0^1 \sigma_\nu^2(s)\,\sigma_\varepsilon^2(s)\,\mathrm{d}s}{\left(\theta_0\int_0^1 \sigma_\nu^2(s)\,\mathrm{d}s\right)^2}\right)$$

where $\theta_0 := \sum_{k\geq 0} b_k^2$ is as defined in Assumption 3, and

$$\sqrt{\frac{T}{h}}s.e.\left(\hat{\beta}^{trf,res}_{h,ivx}\right) \xrightarrow{p} \frac{\omega\sqrt{\int_0^1 \sigma_\nu^2(s)\,\sigma_\varepsilon^2(s)\,\mathrm{d}s}}{(1-\rho)\,\theta_0\int_0^1 \sigma_\nu^2(s)\,\mathrm{d}s}.$$

Theorem 4.4 *Under the conditions of Theorem 4.3, we have that*

$$\sqrt{\frac{T}{h}}\left(\hat{\beta}^{trf}_{h,ivx} - \beta_h\right) \xrightarrow{d} \mathcal{N}\left(0, \frac{\frac{\omega^2}{(1-\rho)^2}\int_0^1 \sigma_\nu^2(s)\left(\sigma_\varepsilon^2(s) + \gamma^2\sigma_\nu^2(s)\right)\mathrm{d}s}{\left(\theta_0\int_0^1 \sigma_\nu^2(s)\,\mathrm{d}s\right)^2}\right)$$

and

$$\sqrt{\frac{T}{h}}s.e.\left(\hat{\beta}^{trf}_{h,ivx}\right) \xrightarrow{p} \frac{\omega\sqrt{\int_0^1 \sigma_\nu^2(s)\left(\sigma_\varepsilon^2(s) + \gamma^2\sigma_\nu^2(s)\right)\mathrm{d}s}}{(1-\rho)\,\theta_0\int_0^1 \sigma_\nu^2(s)\,\mathrm{d}s}.$$

Remark 8. An interesting implication of the results in Theorems 4.1 – 4.4 is that the convergence rates of both $\hat{\beta}^{trf,res}_{h,ivx}$ and $\hat{\beta}^{trf}_{h,ivx}$ decrease with the forecast horizon. In the strongly persistent case, however, β_h increases (approximately) linearly in h which offsets the decreased convergence rate of the estimators. In contrast, in the weakly persistent case, β_h can be seen to remain bounded leading to power losses as the horizon h increases. We will also see this difference in a comparison of the asymptotic lower power functions of the tests given in Theorems 4.5 (strongly persistent predictor) and 4.6 (weakly persistent predictor) which follow next. The Monte Carlo experiments reported in section also clearly bear out this asymptotic prediction. ♢

Theorem 4.5 *Under the conditions of Theorem 4.1 and local alternatives of the form $\beta_1 = bT^{-\eta/2-1/2}$, we have that*

$$t^{trf,res}_{h,ivx} \xrightarrow{d} \mathcal{MN}\left(b\frac{\omega\sqrt{\frac{2}{a}}\left(J_{c,\sigma}(1)\bar{J}_{c,\sigma}(1) - \int_0^1 J_{c,\sigma}(s)\mathrm{d}J_{c,\sigma}(s)\right)}{\sqrt{\int_0^1 \sigma_\nu^2(s)\,\sigma_\varepsilon^2(s)\,\mathrm{d}s}}, 1\right)$$

and

$$t^{trf}_{h,ivx} \xrightarrow{d} \mathcal{MN}\left(b\frac{\omega\sqrt{\frac{2}{a}}\left(J_{c,\sigma}(1)\bar{J}_{c,\sigma}(1) - \int_0^1 J_{c,\sigma}(s)\mathrm{d}J_{c,\sigma}(s)\right)}{\sqrt{\int_0^1 \sigma_\nu^2(s)\left(\sigma_\varepsilon^2(s) + \gamma^2\sigma_\nu^2(s)\right)\mathrm{d}s}}, 1\right).$$

Theorem 4.6 *Under the conditions of Theorem 4.3 and local alternatives of the form* $\beta_1 = bh^{1/2}T^{-1/2}$, *we have that*

$$t_{h,ivx}^{trf,res} \xrightarrow{d} \mathcal{N}\left(b\frac{(1-\rho)\,\theta_0 \int_0^1 \sigma_\nu^2(s)\,\mathrm{d}s}{\omega\sqrt{\int_0^1 \sigma_\nu^2(s)\,\sigma_\varepsilon^2 \mathrm{d}s}}; 1\right).$$

and

$$t_{h,ivx}^{trf} \xrightarrow{d} \mathcal{N}\left(b\frac{(1-\rho)\,\theta_0 \int_0^1 \sigma_\nu^2(s)\,\mathrm{d}s}{\omega\sqrt{\int_0^1 \sigma_\nu^2(s)\left(\sigma_\varepsilon^2(s) + \gamma^2\sigma_\nu^2(s)\right)\mathrm{d}s}}; 1\right).$$

Using the results given above in Theorems 4.5–4.6 we are now in a position to establish the limiting null distributions of our proposed transformed regression long-horizon predictability test statistics, $t_{h,ivx}^{trf}$ from section 4.1 and $t_{h,ivx}^{trf,res}$ from section 4.2.

Corollary 1 *Under the null hypothesis of no predictability* $H_0 : \beta_h = 0$, *we have that under Assumptions 1–4 with* $\epsilon < \min\{1-\eta; \eta/2\}$ *and as* $h, T \to \infty$ *such that* $h/\min\left\{T^{3\eta/2-1/2}; T^{2\eta-1}; T^{\eta/3}\right\} + T^{1/2-\eta/2}/h \to 0$,

$$t_{h,ivx}^{trf,res} \xrightarrow{d} \mathcal{N}(0,1) \quad \textit{and} \quad t_{h,ivx}^{trf} \xrightarrow{d} \mathcal{N}(0,1).$$

The result in Corollary 1 demonstrates the key result for practical implementation of our proposed long-horizon predictability tests, that both $t_{h,ivx}^{trf}$ and $t_{h,ivx}^{trf,res}$ admit standard normal limiting null distributions regardless of whether the predictor is weakly or strongly persistent. These results hold under the very general forms of conditional and/or unconditional heteroskedasticity permitted under Assumption 4.

5 Multiple Predictors

In empirical work one might wish to consider predictive regression models with several possible predictors. This can help avoid the problem of spurious predictive regression effects in the case where relevant strongly persistent predictors are omitted from the estimated predictive regression; cf. Georgiev et al. (2018). In this section we briefly detail how the long-horizon predictability tests developed in section 4 can be implemented with multiple predictors.

To that end consider replacing (2.8) by its multivariate counterpart

$$y_{t+h}^{(h)} = \alpha_h + \boldsymbol{\beta}_h' \boldsymbol{x}_t^\dagger + w_{t+h}^{(h)} \tag{5.1}$$

where $\boldsymbol{x}_t^\dagger := (x_{t1}, \ldots, x_{tK})'$ follows a K-dimensional vector autoregressive data generating process of order p, VAR(p); that is,

$$\boldsymbol{x}_t^\dagger = \boldsymbol{\mu}_x + \mathbf{R}\boldsymbol{x}_{t-1}^\dagger + \boldsymbol{v}_t, \text{ and } \boldsymbol{v}_t = \sum_{j=1}^{p-1} \boldsymbol{\Gamma}_j \boldsymbol{v}_{t-j} + \boldsymbol{\nu}_t \tag{5.2}$$

which is either stable or (near) integrated as before depending on the properties of the (diagonal) autoregressive coefficient matrix $\mathbf{R}$. The process $\boldsymbol{v}_t$ is assumed to follow a stable VAR($p-1$) process.

As with (2.8), the regression coefficients and error term in (5.1) can be related back to those in the corresponding short-horizon regression, $y_{t+1} = \alpha_1 + \boldsymbol{\beta}_1' \boldsymbol{x}_t^\dagger + u_{t+1}$, via the relationships, $\alpha_h := h\alpha_1 + \boldsymbol{\beta}_1' I_{h\geq 2} \sum_{j=1}^{h-1} \sum_{i=1}^{j} \mathbf{R}^{i-1} \boldsymbol{\mu}_x (\mathbf{I} - \mathbf{R})$, $\boldsymbol{\beta}_h' := \boldsymbol{\beta}_1' \sum_{j=0}^{h-1} \mathbf{R}^j$ and $w_{t+h}^{(h)} := u_{t+h}^{(h)} + \boldsymbol{\beta}_1' I_{h\geq 2} \sum_{j=1}^{h-1} \sum_{i=1}^{h-j} \mathbf{R}^{i-1} \boldsymbol{v}_{t+j}$.

Again we allow for the possibility of endogeneity in all regressors through the non-zero coefficient vector $\boldsymbol{\gamma}$ in the decomposition

$$u_{t+1} := \boldsymbol{\gamma}'\boldsymbol{\nu}_{t+1} + \varepsilon_{t+1}, \tag{5.3}$$

where the innovations $\boldsymbol{\nu}_{t+1}$ and ε_{t+1} are heterogeneous MDs, obeying a multivariate version of Assumption 4.

To implement the transformed bias reduced IVX approach introduced in this paper in the multiple predictive regression case, we first compute the vector of residuals $\hat{\boldsymbol{\nu}}_t$ from a vector autoregression model of order p of the demeaned predictors; that is, with $\bar{\boldsymbol{x}}_t^{\dagger} := (\bar{x}_{t1}, \ldots, \bar{x}_{tK})'$,

$$\hat{\boldsymbol{\nu}}_{t+1} := \bar{\boldsymbol{x}}_{t+1}^{\dagger} - \sum_{j=1}^{p} \hat{\boldsymbol{\Phi}}_j \bar{\boldsymbol{x}}_{t+1-j}^{\dagger}, \quad t = p, \ldots, T-1, \tag{5.4}$$

with $\hat{\boldsymbol{\Phi}}_j$, $j = 1, \ldots, p$, the OLS coefficient matrix estimates. Again, the lag augmentation order in (5.4) can be selected in practice by using a standard information criterion, setting the minimum possible lag length allowed to be one. The multiple predictor residual augmented IVX estimator vector is then defined as

$$\begin{aligned}\hat{\boldsymbol{\beta}}_{h,ivx}^{trf,res} &:= \left(\sum_{t=p}^{T-1} \tilde{\boldsymbol{z}}_t^{trf,(h)} \tilde{\boldsymbol{z}}_t^{trf,(h)\prime}\right)^{-1} \sum_{t=p}^{T-1} \tilde{\boldsymbol{z}}_t^{trf,(h)} \left(\bar{y}_{t+1} - \hat{\boldsymbol{\gamma}}'\hat{\boldsymbol{\nu}}_{t+1}\right) \\ &= \left(\sum_{t=1}^{T-h} \boldsymbol{z}_t \bar{\boldsymbol{x}}_t^{\dagger\prime}\right)^{-1} \sum_{t=p}^{T-1} \boldsymbol{z}_t^{trf,(h)} \left(\bar{y}_{t+1} - \hat{\boldsymbol{\gamma}}'\hat{\boldsymbol{\nu}}_{t+1}\right).\end{aligned} \tag{5.5}$$

where $\boldsymbol{z}_t$ is a $K \times 1$ vector of instruments with elements as defined in (3.6) for each predictor in $\boldsymbol{x}_t^{\dagger}$ and

$$\tilde{\boldsymbol{z}}_t^{trf,(h)} := \left(\sum_{t=p}^{T-1} \boldsymbol{z}_t^{trf,(h)} \boldsymbol{z}_t^{trf,(h)\prime}\right)^{-1} \left(\sum_{t=1}^{T-h} \boldsymbol{z}_t \bar{\boldsymbol{x}}_t^{\dagger\prime}\right) \boldsymbol{z}_t^{trf,(h)} \tag{5.6}$$

in which $\boldsymbol{z}_t^{trf,(h)}$ is a $K \times 1$ vector of instruments, whose elements are obtained by applying the definition in (4.4) to each element of $\boldsymbol{z}_t$.

For inference purposes we need to estimate the covariance matrix of $\hat{\boldsymbol{\beta}}_{h,ivx}^{trf,res}$. This can be done by using the familiar "sandwich" formula,

$$\widehat{\text{Cov}\left(\hat{\boldsymbol{\beta}}_{h,ivx}^{trf,res}\right)} := \mathbf{B}_T^{-1}\mathbf{M}_T\left(\mathbf{B}_T^{-1}\right)' \tag{5.7}$$

where $\mathbf{B}_T := \sum_{t=1}^{T-h} \boldsymbol{z}_t \bar{\boldsymbol{x}}_t^{\dagger\prime}$ and

$$\begin{aligned}\mathbf{M}_T := \; & \textstyle\sum_{t=p}^{T-1} \boldsymbol{z}_t^{trf,h} \boldsymbol{z}_t^{trf,(h)\prime} \hat{\varepsilon}_{t+1}^2 + \left[\boldsymbol{\gamma}' \otimes \left(\frac{1}{T}\sum_{t=p}^{T-1} \boldsymbol{z}_t^{trf,h} \bar{\boldsymbol{\mathcal{X}}}_{t,K}'\right)\left(\sum_{t=p}^{T-1} \bar{\boldsymbol{\mathcal{X}}}_{t,K}\bar{\boldsymbol{\mathcal{X}}}_{t,K}'\right)^{-1}\right] \times \\ & \times \left(\textstyle\sum_{t=p}^{T-1} \hat{\boldsymbol{\nu}}_t \hat{\boldsymbol{\nu}}_t' \otimes \bar{\boldsymbol{\mathcal{X}}}_{t,K}\bar{\boldsymbol{\mathcal{X}}}_{t,K}'\right)\left[\hat{\boldsymbol{\gamma}} \otimes \left(\sum_{t=p}^{T-1} \bar{\boldsymbol{\mathcal{X}}}_{t,K}\bar{\boldsymbol{\mathcal{X}}}_{t,K}'\right)^{-1}\left(\frac{1}{T}\sum_{t=p}^{T-1} \bar{\boldsymbol{\mathcal{X}}}_{t,K} \boldsymbol{z}_t^{trf,(h)\prime}\right)\right]\end{aligned}$$

where $\bar{\boldsymbol{\mathcal{X}}}_{t,K}$ is the vector formed from stacking the p lags of each of the K demeaned regressors; that is, $\bar{\boldsymbol{\mathcal{X}}}_{t,K} := (\bar{x}_{t,1}, \ldots, \bar{x}_{t,K},\ \bar{x}_{t-1,1}, \ldots, \bar{x}_{t-1,K}, \ldots, \bar{x}_{t-p+1,1}, \ldots, \bar{x}_{t-p+1,K})'$.

The limiting distribution of $\hat{\boldsymbol{\beta}}_{h,ivx}^{trf,res}$ is normal in the case where the elements of $\boldsymbol{x}_t$ are weakly persistent and mixed normal in the case where they are strongly persistent; the proofs are simple multivariate extensions of the results from the single-regressor case given in section 4.3 and are therefore omitted. More importantly, the associated individual and joint significance tests on the elements of $\boldsymbol{\beta}_h$ have standard normal (if one linear restriction is being tested using a t-type ratio) and χ^2 (for multiple restrictions) limiting null distributions irrespective of whether the elements of $\boldsymbol{x}_t$ are weakly or strongly persistent, and regardless of any heterogeneity present in the DGP, provided the heteroskedasticity-robust covariance matrix estimator in (5.7) is used.

6 Numerical Results

6.1 Set-up

In this section, we report the results from a Monte Carlo study exploring the finite sample performance of the residual augmented transformed regression based long-horizon predictability test, $t_{h,ivx}^{trf,res}$, from section 4.2. We will compare the finite sample performance of this test with the Bonferroni-based test, t_h^{Bonf}, of Hjalmarsson (2011) outlined in section 3.1, the implied test, t_h^{Xu}, of Xu (2020) outlined in section 3.2, and the reversed predictive regression based test, $t_{h,ivx}^{rev,PL}$, of Phillips and Lee (2013) outlined in section 3.3. We also considered the unaugmented transformed regression test, $t_{h,ivx}^{trf}$ defined in (4.7) but we found that this did not perform as well as the $t_{h,ivx}^{trf,res}$ test (its performance was in fact very similar to that of the $t_{h,ivx}^{rev,PL}$ test), and so we only report results for $t_{h,ivx}^{trf,res}$. Empirical size results are reported in section 6.2 and empirical power properties in section 6.3. A number of additional Monte Carlo results are also presented in the supplementary appendix.

For all of the reported experiments, data is generated from (2.1)-(2.2). All of the tests considered are for the null hypothesis of no long-run predictability $H_0 : \beta_h = 0$ in (2.8). We will consider tests directed against both one-sided (left-tailed tests for $H_1 : \beta_h < 0$, and right-tailed tests for $H_1 : \beta_h > 0$), and two-sided alternatives ($H_1 : \beta_h \neq 0$). All tests are run at the 5% nominal (asymptotic) significance level. The simulations were preformed in MATLAB, version R2020a, using the Mersenne Twister random number generator function using 10000 and 5000 Monte Carlo replications for the empirical size and empirical power simulations, respectively.

In implementing the t_h^{Bonf} test, we follow the steps outlined in Hjalmarsson (2011), however, we use the GLS detrended ADF approach as suggested in Campbell and Yogo (2006a) to compute the confidence interval for c instead of Chen and Deo (2009) because it gave better results. With the exception of the IVX instrument, z_t, all variables entering the estimated predictive regressions are demeaned. As discussed in Kostakis et al. (2015, p. 1514) the IVX instrument z_t, does not need to be demeaned because the slope estimator in the predictive regression is invariant to whether z_t is demeaned or not. For implementation of the residual augmented transformed regression based predictive test statistic $t_{h,ivx}^{trf,res}$ in (4.10) we start by estimating an autoregressive model of order p, where p was chosen applying the AIC over $p \in (1, ..., \lfloor 4(T/100)^{1/4} \rfloor$. The resulting residuals, $\hat{\nu}_{t+1}$ are then used to compute $\bar{y}_{t+1} - \hat{\gamma}\hat{\nu}_{t+1}$, from a regression of $\bar{y}_{t+1}$ on $\hat{\nu}_{t+1}$.

6.2 Empirical Size

In this section we investigate the finite sample size properties of our proposed $t_{h,ivx}^{trf,res}$ test with the t_h^{Bonf}, t_h^{Xu} and $t_{h,ivx}^{rev,PL}$ test.[6] To that end, we generate data from (2.1)-(2.2) with $\beta_1 = 0$. In generating the simulation data we set the intercepts, α_1 and μ_x, in (2.1)-(2.2), respectively, to zero without loss of generality. The autoregressive process for x_t was generated as in (2.2) with $\rho = 1 + c/T$ for $c \in \{0, -5, -10, -20, -50\}$ and was initialized at $\xi_0 = 0$. Results are reported for samples of length $T = 250$ and $T = 500$.

We allow the innovations driving the predictor process in (2.2) to either be serially uncorrelated or to follow an AR(1) process; in particular we set $v_{t+1} = \psi v_t + \nu_{t+1}$, and consider $\psi \in \{-0.5, 0, 0.5\}$. The innovation vector $(u_{t+1}, \nu_{t+1})'$ in (2.1)–(2.2) is drawn from an i.i.d. bivariate Gaussian distribution with mean zero and covariance matrix[7] $\mathbf{\Sigma} := \begin{bmatrix} \sigma_u^2 & \phi\sigma_u\sigma_\nu \\ \phi\sigma_u\sigma_\nu & \sigma_\nu^2 \end{bmatrix}$, where ϕ corresponds to the correlation between the innovations u_{t+1} and v_{t+1}. In our analysis we set $\phi = -0.95$.[8] Tables 1, 2 and 3 report results for $\psi = 0$, $\psi = 0.5$, and $\psi = -0.5$, respectively.

The results in Tables 1-3 clearly show the superiority of the IVX-based tests, $t_{h,ivx}^{trf,res}$ and $t_{h,ivx}^{rev,PL}$, over the non-IVX based t_h^{Bonf}, t_h^{Xu} tests in terms of controlling size across both strongly and weakly persistent predictors. Taking the case where $\psi = 0$ to illustrate, it is seen from the results in Table 1, which are for the case where v_{t+1} is serially uncorrelated, that the empirical rejection frequencies of the two-sided $t_{h,ivx}^{trf,res}$ and $t_{h,ivx}^{rev,PL}$ tests when $h \leq 20$, for $T = 250$ are in the range $[0.032, 0.061]$ and $[0.037, 0.061]$, respectively, and for $T = 500$ in the range $[0.030, 0.058]$ and $[0.047, 0.062]$, respectively, taken across all of the values of c considered. For $h = 50$ these two approaches become slightly conservative with the empirical rejection frequencies of both procedures decreasing to $[0.020, 0.030]$ and $[0.027, 0.041]$, respectively when $T = 250$, and to $[0.028, 0.041]$ and $[0.043, 0.051]$, respectively, for $T = 500$. Interestingly, a comparison of the results in Table 1 with those in Tables 2-3 shows that these results change very little when the innovations v_{t+1} are positively ($\psi = 0.5$) or negatively ($\psi = -0.5$) autocorrelated. While the two-sided $t_{h,ivx}^{trf,res}$ and $t_{h,ivx}^{rev,PL}$ tests both show good finite sample size control it can be seen from the results in Tables 1-3 that when considering one-sided alternatives ($H_1 : \beta_h < 0$ and $H_1 : \beta_h > 0$) the $t_{h,ivx}^{trf,res}$ tests display considerably better finite sample size control than the $t_{h,ivx}^{rev,PL}$ tests. This is particularly evident in the case of the right-sided tests. To illustrate, while the right-sided version of the $t_{h,ivx}^{trf,res}$ test displays empirical rejection frequencies, taken across all of the results in Tables 1-3, in the range $[0.028, 0.071]$ for $T = 250$ and $[0.046, 0.069]$ for $T = 500$, the right-sided version of $t_{h,ivx}^{rev,PL}$ displays significant over-sizing when $c \geq -20$, regardless of T, h or ψ, with empirical size generally close to or in excess of 10%. In contrast the left-sided versions of these tests display conservative behaviour, which is a common characteristic

[6]We are grateful to Kei-Li Xu for making code for computing his test available on his website https://sites.google.com/site/xukeli2015/research.

[7]Additional results are reported in the supplementary appendix for the cases where: (i) $(u_{t+1}, \nu_{t+1})'$ is conditionally heteroskedastic with a $GARCH(1,1)$ formulation characterising the volatility dynamics, and (ii) the unconditional variances of u_{t+1} and ν_{t+1} are allowed to display a one-time break at $T/2$. The results for (i) (see Table S.1) are qualitatively similar to those reported here for i.i.d. innovations for all of the tests reported. For (ii) (see Table **??**), for both $t_{h,ivx}^{trf,res}$ and $t_{h,ivx}^{rev,PL}$ the size results are again very similar to those for the i.i.d. case, while for t_h^{Bonf}, t_h^{Xu} additional size distortions relative to the i.i.d. case result.

[8]Notice that because we report results for both left-sided and right-sided tests we do not need to report results for the case where $\phi = 0.95$ because, as noted in Campbell and Yogo (2006a), flipping the sign of ϕ also flips the sign of β. Consequently, the empirical size and power properties for the left-sided and right-sided implementations of any given test in what follows for $\phi = -0.95$ will be identical to those for the right-sided and left-sided implementations of those tests, respectively, for $\phi = 0.95$.

of IVX-based predictability tests; see, for example, Demetrescu et al. (2021)). In general, however, the degree of undersizing observed in the left-tailed IVX-based tests is less pronounced, often very significantly so, for $t_{h,ivx}^{trf,res}$ than it is for $t_{h,ivx}^{rev,PL}$.

In contrast to the IVX-based tests, the empirical rejection frequencies of the t_h^{Xu} test are very sensitive to the strength of the persistence of the predictor. For example, in Table 1 it can be seen that for $h \leq 5$ the t_h^{Xu} test displays substantial size distortions when $c \geq -10$, regardless of the sample size and for both one-sided and two-sided implementations of the test. The finite sample behaviour of the one-sided and two-sided t_h^{Xu} tests become generally more erratic when the innovations v_{t+1} are autocorrelated, and are particularly unreliable in the case of negatively autocorrelated v_{t+1}; see Table 3. We recall from the discussion in section 3.2 that the t_h^{Xu} test is not valid when v_{t+1} is autocorrelated and these results illustrate this well.

The t_h^{Bonf} tests display empirical rejection frequencies close to the nominal significance level of 5% considered, for both one-sided and two sided implementations, in the case where v_{t+1} is serially uncorrelated (Table 1), for $c \geq -20$ and $h < 20$. As discussed in section 3.1 this test is based on the assumption that the predictor is strongly persistent and so the deterioration in the empirical rejection rates for $c = -50$ is to be expected. Perhaps most striking, however, is the highly erratic behaviour of the t_h^{Bonf} tests when the innovations v_{t+1} are autocorrelated (Tables 2 and 3). Here the t_h^{Bonf} tests can be either massively over-sized, with size sometimes in excess of 50%, or massively under-sized. On the basis of these results this approach would appear to be too unreliable to use in empirical applications.

We can conclude from the results in Tables 1–3 that only the IVX-based long-horizon predictability tests, $t_{h,ivx}^{trf,res}$ and $t_{h,ivx}^{rev,PL}$, display reliable enough finite sample size control across predictors whose degree of persistence is unknown and which are not driven by uncorrelated innovations to be empirically useful. The Bonferroni-based t_h^{Bonf} test and the t_h^{Xu} test of Xu (2020) are too unreliable to be used in practical applications. Of the $t_{h,ivx}^{trf,res}$ and $t_{h,ivx}^{rev,PL}$ tests our results suggest that the former delivers significantly better finite sample size control.

6.3 Empirical Power

In this section we will compare the finite sample power properties of the left- and right-sided $t_{h,ivx}^{trf,res}$ and $t_{h,ivx}^{rev,PL}$ tests.[9] Because of the highly unreliable size properties of the t_h^{Bonf} and t_h^{Xu} tests reported in section 6.2 we will not include these tests in our comparison. However results for these tests can be found in the supplementary appendix. In order to investigate the finite sample power properties of the $t_{h,ivx}^{trf,res}$ and $t_{h,ivx}^{rev,PL}$ tests we simulate data from (2.2)- (2.1) under the alternative hypothesis $H_1 := b/T$, across the following values of the drift parameter, $b \in \{-15, -14.5, -14, ..., 14, 14.5, 15\}$. The innovations $(u_{t+1}, \nu_{t+1})'$ were generated as described in section 6.2 with results reported only for $\psi = 0.5$; results for $\psi \in \{-0.5, 0\}$ are qualitatively very similar and can be found in the supplementary appendix. We again report results for $\phi = -0.95$ (cf. footnote 8), for prediction horizons $h = \{1, 5, 10, 20, 50\}$ and for five values of the persistence parameter, c, associated with x_t; specifically, $c = \{0, -5, -10, -20, -50\}$. Figures 1-4 plot simulated finite sample local power curves for each of the prediction horizons considered $h = 1, 5, 10, 20$ and 50. These provide the simulated finite sample power curves for the left-sided (Figures 1 and 3) and the right-sided (Figures 2 and 4)

[9]To save space we do not report power curves for the corresponding two-sided tests as these can essentially be inferred from the power curves of the left- and right-sided tests.

versions of the $t^{trf,res}_{h,ivx}$ and $t^{rev,PL}_{h,ivx}$ tests.

Consider first Figures 1 and 3 which plot the power curves of the left-sided $t^{trf,res}_{h,ivx}$ and $t^{rev,PL}_{h,ivx}$ tests against $H_1 : \beta_h < 0$. It is clearly seen from these figures that the left-sided $t^{trf,res}_{h,ivx}$ test displays significantly superior power performance than the left-sided $t^{rev,PL}_{h,ivx}$ test and that this holds regardless of the prediction horizon or the strength of persistence of the predictor. It can also be seen from Figures 1 and 3 that for both tests power decreases as c decreases (i.e. as the persistence of the predictor weakens), other things being equal. This pattern is to be expected as the signal from the predictor becomes stronger the more persistent is the predictor, x_t. Finally, we observe that the power superiority of $t^{trf,res}_{h,ivx}$ over $t^{rev,PL}_{h,ivx}$ generally becomes more pronounced as h becomes larger, other things equal.

Turning to the right-sided tests in Figures 2 and 4 we observe that the $t^{rev,PL}_{h,ivx}$ test displays somewhat higher empirical rejection frequencies than $t^{trf,res}_{h,ivx}$ for $c = 0, -5, -10$. However, this is an artifact of the significant over-sizing seen with the $t^{rev,PL}_{h,ivx}$ test in these scenarios; see Tables 1-3. Indeed, when we compare the power properties of the two tests for $c \leq -10$ and $h > 10$ where their empirical sizes are broadly comparable, we observe that $t^{trf,res}_{h,ivx}$ tends to display superior power to $t^{rev,PL}_{h,ivx}$. Again, as h becomes larger $t^{trf,res}_{h,ivx}$ tends to perform better than $t^{rev,PL}_{h,ivx}$; for example for $h = 50$ we see that $t^{trf,res}_{h,ivx}$ is generally more powerful than $t^{rev,PL}_{h,ivx}$ for $c \leq -5$ even though the latter is rather oversized for $c = -5$, $c = -10$ and $c = -20$.

7 Empirical Application

Exchange rate predictability has been a topic of considerable interest in the international finance and macroeconomics literature. Following the seminal work of Meese and Rogoff (1983), a long held view is that forecasts based on macroeconomic fundamentals cannot outperform a random walk benchmark; see Rossi (2013) for a survey of the literature. To overcome this exchange rate puzzle, several alternative approaches have been considered which include: analysis of the behavior of exchange rates in present-value models (Engel and West, 2005); use of nonlinear methods, such as for example the exponential smooth transition autoregressive model (Kilian and Taylor, 2003), and the use of time-varying parameter models (e.g., Rossi (2007) and Byrne et al., 2016).

Engel and West (2005) and Engel et al. (2007) illustrate that models that can be cast in a standard present-value asset pricing framework imply that exchange rates are approximately random walks. Engel et al. (2007), Molodtsova and Papell (2009), and Rossi (2013) find that empirical exchange rate models conditioned on information sets from Taylor rules can outperform the random walk benchmark in out-of-sample forecasting, particularly at short-horizons. However, Rossi and Sekhposyan (2011), detect significant instabilities in models that employ classic and Taylor rule fundamentals. Although, there have been attempts to account for time-variation in parameters when forecasting exchange rates, Rossi (2013) and Rogoff and Stavrakeva (2008) argue that the problem has not been fully resolved. Analysing exchange rate dynamics in the period before and after the 2008 turmoil, Mumtaz and Sunder-Plassmann (2013) observe high volatility in exchange rates in recent years. Similarly, Taylor (2009) argues that prior to the Global Financial crisis the US Federal Reserve conduct of monetary policy was characterized by a non-linear Taylor rule and after the crisis central banks around the world adopted unconventional monetary policy when confronted with the zero lower bound constraint on nominal interest rates.

The exchange rate literature provides, at least two reasons for the often poor behaviour of many of the models used. First, the poor forecasting performance of exchange rate models is, to some extent, explained by estimation error and not just misspecification error (Engel et al., 2007). The significant role of estimation error is confirmed, among other things, by the relative good forecasting performance of economic models estimated with large panel datasets (Mark and Sul, 2001; Engel et al., 2007; Ince, 2014) or long time series (Lothian and Taylor, 1996). The second reason for reservations about the usefulness of exchange rate models comes from the evidence in favor of the PPP model. According to Taylor and Taylor (2004), the exchange rate literature has turned full circle to the pre-1970s view that PPP holds in the long run. The mean reverting nature of real exchange rates has found support from panel unit root tests (Sarno and Taylor, 2002), however only a small number of studies have tested whether the mean-reverting properties of the real exchange rate can be exploited in a forecasting context.

A number of studies have been more skeptical about what is typically dubbed the "Rogoff consensus". For example, Kilian and Zha (2002) propose a prior probability distribution based on a survey of professional international economists and derived a posterior probability distribution of the half-life PPP deviation on the basis of a Bayesian autoregressive model. In similar vein, Murray and Papell (2002) stress how univariate methods provide virtually no information on the size of half-lives. Finally, a large cross-country heterogeneity in terms of point estimates and confidence intervals has also been found by Murray and Papell (2005) and Rossi (2006).

Notwithstanding the pioneering study of Meese and Rogoff (1983), which shows the superiority of the random walk model in out-of-sample exchange-rate forecasting, there is evidence that exchange rate movement may be predictable at longer time horizons. In this section we apply the transformed residual augmented regression tests developed in section 4 to the problem of testing for long-horizon nominal exchange rate and relative price predictability. In particular, we will revisit the recent study of Eichenbaum et al. (2020) [henceforth EJR] who document that: (i) current real exchange rates (RER) predict nominal exchange rates (NER) in the long-run;[10] (ii) RER is a poor predictor of future inflation rates, and (iii) that these regularities depend on the monetary policy regime in effect. Defining the RER as the price of the foreign-consumption basket in units of the home-consumption basket, and the NER as the price of the foreign currency in units of the home currency, EJR further observe that current RER is strongly negatively correlated with future changes in NER and that this correlation increases with the prediction horizon, and that RER is virtually uncorrelated with future inflation rates at all horizons. These empirical observations suggest that RER adjusts to shocks in the medium and long run overwhelmingly through changes in NER, not through inflation rate differentials. We revist the predictive power of RER for predicting changes in NER and future inflation rates across 45 countries. As indicated by EJR, if monetary policy seeks to limit the volatility of the NER, then RER should converge to its unconditional mean primarily via inflation differentials rather than through sustained predictable movements in the NER. Thus, our contribution to this literature is to provide further evidence on the stylised features of exchange rate predictability using the new long-horizon predictability tests developed in this paper to provide evidence on the usefulness of current RERs as predictors of future changes in NERs and inflation differentials.

[10]Mark (1995) and Engel et al. (2007) have also found evidence of predictability of NER at medium and long horizons; see Rossi (2013) for a survey.

7.1 Data

In our empirical analysis we use a similar data set to that considered in EJR, but our sample datset contains both a larger group of countries and a larger sample size. All of the data used are obtained from the International Financial Statistics of the IMF and covers the period from 1973:Q1 to 2020:Q1. Our analysis will be conducted over four different sample periods: (i) the full sample - 1973:Q1 to 2020:Q1; (ii) from 1973:Q1 to 2008:Q4; (iii) from 1990:Q1 to 2008:Q4; and (iv) from 1999:Q1 to 2020:Q1. Our sample includes 45 countries split into two groups according to the MCSI classification namely developed markets and emerging markets; see https://www.msci.com/market-classification. The developed markets group comprises: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Hong Kong, Ireland, Israel, Italy, Japan, Luxembourg, Netherlands, New Zealand, Norway, Portugal, Singapore, Spain, Sweden, Switzerland, and United Kingdom. The emerging markets group comprises: Brazil, Bulgaria, Chile, China, Colombia, Czech Republic, Egypt, Greece, Hungary, Iceland, India, Indonesia, Korea, Mexico, Peru, Philippines, Poland, Romania, Russian Federation, South Africa, Thailand, and Ukraine.

Notice that although the overall sample period for analysis considered was from 1973:Q1 to 2020:Q1, the samples for some of the countries we consider are slightly smaller due to lack of available data at the beginning and/or end of the sample. Specifically, for Hungary and Iceland the sample starts in 1976:Q1, for Brazil and Poland in 1980:Q1, for Hong-Kong in 1980:Q4, for China in 1986:Q1, for Romania in 1990:Q4, for Bulgaria in 1991:Q1, for the Czech Republic and the Ukraine in 1993:Q1 and finally for the Russian Federation in 1995:Q2. Moreover, for Egypt and the Ukraine the ending dates are also shorter than for the rest of the countries in the sample (2019:Q3 and 2019:Q4, respectively).

7.2 Empirical Results

7.2.1 The Nominal Exchange Rate Long-Horizon Predictive Regression

The NER long-horizon predictive regression consider by EJR is given by,

$$log\left(\frac{NER_{i,t+h}}{NER_{it}}\right) = \alpha_{ih}^{NER} + \beta_{ih}^{NER} log(RER_{it}) + u_{i,t+h}^{NER} \tag{7.1}$$

where i corresponds to the country under analysis and h is the prediction horizon (in quarters), $h = \{1, 4, 8, 12, 20\}$. The predictor is the real exchange rate of country i relative to the U.S,, which we define as RER_{it},[11] which is computed as

$$RER_{it} := NER_{it} \frac{P_{it}}{P_t} \tag{7.2}$$

where NER_{it} is the average quarterly nominal exchange rate (domestic currency per U.S. dollar) and P_t and P_{it} denote the consumer price index (CPI) for all items in the U.S. and in country i, respectively.

In our analysis, to provide an indication of the strength of persistence of the RER_{it} predictors,

[11] The RER between two countries may be defined as the relative price of one country's consumption basket in terms of the consumption basket of the other country.

we estimate the following augmented Dickey-Fuller regression for each country,

$$RER_{it} = \alpha_i^{RER} + \rho_i^{RER} RER_{i,t-1} + \sum_{k=1}^{p} \delta_k \Delta RER_{i,t-k} + \nu_{it}^{RER},\ i = 1, ..., 45 \tag{7.3}$$

where for each series p is determined based using the AIC information criteria with a maximum lag order determined by Schwert's rule, $\lfloor 4(T/100)^{1/4} \rfloor$; for country i we denote this selected lag length as $\hat{p}_i$. We report the resulting OLS estimate of ρ_i^{RER}, denoted $\hat{\rho}_i^{RER}$, for each country under analysis. We also report for each country an estimate of the contemporaneous correlation between the innovations, ϕ_i, (under the assumption that this is constant) based on the OLS residuals from estimating the short-horizon predictive regression, (7.1) for $h = 1$, and the OLS residuals from estimating (7.3); specifically,

$$\hat{\phi}_i := \frac{(T - \hat{p}_i)^{-1} \sum_{t=\hat{p}_i}^{T-1} \hat{u}_{i,t+1}^{NER} \hat{\nu}_{i,t+1}^{RER}}{\sqrt{((T-1)^{-1} \sum_{t=1}^{T-1} (\hat{u}_{i,t+1}^{NER})^2)((T - \hat{p}_i)^{-1} \sum_{t=\hat{p}_i}^{T-1} (\hat{\nu}_{i,t+1}^{NER})^2)}}. \tag{7.4}$$

EJR assume that RER is mean reverting (weakly persistent) and highlight a number of features they observe from the estimation of the predictive regression in (7.1) by OLS. Their analysis is based on testing for long-horizon predictability by comparing the conventional OLS t-statistic from (7.1) computed with Newey-West standard errors, which we will denote by t_h^{NW}, with critical values from the standard normal distribution. As is well known and is discussed in section 2.2 these tests are not theoretically valid and likely to spuriously reject the null hypothesis if RER is strongly persistent. The estimates of $\hat{\rho}_i^{RER}$ reported in Tables 4 - 7 suggest that for most of the countries considered RER is indeed strongly persistent with an estimated autoregressive root very close to unity. We observe, for instance, that in general for all countries $\hat{\rho}^{RER} \geq 0.953$ when considering the sample from 1973:Q1 to 2020:Q1 (except of the Russian Federation, where $\hat{\rho}^{RER} = 0.898$); $\hat{\rho}^{RER} \geq 0.932$ in the sample from 1973:Q1 to 2008:Q4 (except for the Russian Federation and Ukraine, where $\hat{\rho}^{RER} = 0.868$ and $\hat{\rho}^{RER} = 0.853$, respectively); $\hat{\rho}^{RER} \geq 0.910$ from 1990:Q1 to 2008:Q4 (except for Peru, the Russian Federation and Ukraine, where $\hat{\rho}^{RER} = 0.666$, $\hat{\rho}^{RER} = 0.868$ and $\hat{\rho}^{RER} = 0.853$, respectively); and finally $\hat{\rho}^{RER} \geq 0.918$ from 1999:Q1 to 2020:Q1 (except for Korea where $\hat{\rho}^{RER} = 0.887$).

Based on the outcomes of t_h^{NW} tests, EJR strongly support the conclusion that current RER is highly negatively correlated with changes in future NERs at horizons of three or more years. These results are consistent with those obtained by Cheung et al. (2019) using vector error-correction models. Furthermore, EJR also found that RER only predicts the nominal rate in currencies of countries with floating exchange rates, meaning the price of the country's currency in U.S. dollars is allowed to float according to supply and demand; and that the central banks of the two countries must follow an inflation-targeting policy (i.e., country i must be willing to adjust interest rates to keep the inflation rate around a target value). Our analysis and the way the different samples are organised looks to provide further evidence concerning these previous findings. Finally, one result that is mentioned by EJR which requires further analysis relates to the increase in the absolute value of β_{ih}^{NER} as h increases since as observed in section 2.2 this may be an artifact of the aggregation when estimating long-horizon predictive regressions; see Equation (2.8).

EJR base their analysis on a benchmark group of six countries - Australia, Canada, Germany, New Zealand, Sweden, and the UK, which (other than Germany) had adopted inflation targeting before 1997. We consider the 45 countries listed above, most of which adopted inflation targeting, but at a later stage than the benchmark group considered in EJR. Many of these countries adopted this policy in 1999 and a few between 1999 and 2005; see Ilzetzki et al. (2017) for details.

In Tables 4 - 7, for the various sample periods discussed above, we report for each country considered and for each horizon h, the t_h^{NW} test as well as our new IVX-based $t_{h,ivx}^{trf}$ and $t_{h,ivx}^{trf,res}$ tests from section 4. We also report the $t_{h,ivx}^{rev,PL}$ test of Phillips and Lee (2013). The IVX-based tests were implemented exactly as detailed for he simulation study in section 6. Although we report results for the t_h^{NW} test, these should therefore be treated with great caution given the strength of persistence in the predictors used, discussed above. As suggested in EJR, the Newey-West standard errors in the reported t_h^{NW} statistics were computed using the Bartlett kernel and setting the number of lags to $h + 8$.[12]

Consider first the full sample results in Table 4. Here we observe negative outcomes for the IVX-based statistics for almost all of the countries (the exceptions are a small number of the emerging markets nations) for all of the values of h considered. This entails that the IVX estimates of the β_{ih}^{NER} slope coefficeints are negative, albeit many are these tests outcomes are not statistically significant. These findings support EJR's conclusion that current RER and changes in future NERs are negatively correlated. The results in Tables 5 - 7 suggest that this finding also appears robust to the sample period considered. In addition to the observation that the outcomes of the IVX-based statistics are mostly negative, we also observe that the estimated innovation correlations, $\hat{\phi}_i$, are positive for all of the countries and are generally very high. As the Monte Carlo simulation results in section 6.2 demonstrate (recalling footnote 8), this is precisely the case where the left-sided $t_{h,ivx}^{trf}$ and $t_{h,ivx}^{rev,PL}$ tests will be significantly oversized, while our preferred residual-augmented $t_{h,ivx}^{trf,res}$ test is approximately correctly sized. We might therefore expect to see fewer rejection with the $t_{h,ivx}^{trf,res}$ test than with the $t_{h,ivx}^{trf}$ and $t_{h,ivx}^{rev,PL}$ tests, and that should be borne in mind in the discussion which follows.

Overall, the results in Table 4 provide increasing evidence of predictability as h increases. This is particularly, noticeable in the top panel which contains the results for the developed markets nations, where an increase in the number of statistically significant cases is observed for larger h. However, we also note that the number of rejections is largest for t_h^{NW} and smallest for $t_{h,ivx}^{trf,res}$. This is unsurprising given that, as discussed above, the former is likely to be invalid for these data and that the latter is the only one of these tests reported which displays reliable size control in this setting. In the case of the emerging markets nations, a similar situation as for the developed markets nations can be observed from the results for the $t_{h,ivx}^{trf}$, $t_{h,ivx}^{trf,res}$ and $t_{h,ivx}^{rev,PL}$ tests. The t_h^{NW} finds that changes in NER of more than 50% of these countries are predictable by RER when $h = 1$, but as h increases the number of statistically significant results decreases slightly. From the results of $t_{h,ivx}^{trf}$, $t_{h,ivx}^{trf,res}$ and $t_{h,ivx}^{rev,PL}$ tests we observe that for forecast horizons $h \geq 8$ predictability seems to increase ($h = 20$ displays the largest number of significant cases). The results obtained based on $t_{h,ivx}^{trf}$, $t_{h,ivx}^{trf,res}$, $t_{h,ivx}^{rev,PL}$ seem to suggest, for the full sample (1970:Q1 to 2020:Q1), that of the benchmark countries considered by EJR only Canada seems to become significant when $h \geq 12$, whereas from the results of t_h^{NW} all EJR's benchmark countries are statistically significant except

[12] For all but one of the countries considered the fitted lag length, $\hat{p}_i$, from 7.3 was greater than zero in all of the sample periods considered. For that reason, we do not report results for the t_h^{Bonf} test of Hjalmarsson (2011) or the t_h^{Xu} test of Xu (2020) given their likely unreliability in such cases; see section 6.2.

Canada, Germany and Sweden.

Because the results may be affected by the period were short-term US nominal interest rates were at or near their effective lower bound (see Amador et al., 2020, for a discussion) the analysis is also conducted for the period from 1973:Q1 to 2008:Q4 (see Table 5). However, even with the exclusion of the information from 2009:Q1 to 2020:Q1 the conclusions are essentially in line with what we have observed from the results in Table 4 for the full sample (1973:Q1 to 2020:Q1). The smaller number of significant results obtained for $t_{h,ivx}^{trf}$, $t_{h,ivx}^{trf,res}$ and $t_{h,ivx}^{rev,PL}$, suggest that there is less evidence of predictability in this period (particularly for the emerging markets), which potentially highlights the importance of inflation targeting policies suggested by EJR.

If we consider the period where most countries adopted inflation targeting policies for most of the time (recall that after 1999 most countries considered had already adopted inflation targeting) we clearly observe the general conclusion of EJR that the RER's predictive power seems to increase as h increases particularly for $h \geq 8$ (see Table 6). This pattern is most clearly seen for the developed markets group. Finally, if we focus on the period from 1999:Q1 to 2020:Q1 (see Table 7), which roughly corresponds to a period where most countries adopted inflation targeting policies, we observe that the number of significant cases reduces considerably, indicating a reduction in predictability of changes in NER by the RER.

7.2.2 The Relative Price Predictive Regression

In this section we now consider the relative-price long-horizon predictive regression of EJR,

$$log\left(\frac{P_{i,t+h}/P_{t+h}}{P_{it}/P_t}\right) = \alpha_{ih}^{\pi} + \beta_{ih}^{\pi} log(RER_{it}) + u_{i,t+h}^{\pi}. \tag{7.5}$$

According to EJR, in countries with inflation-targeting policies, the way that the RER reverts towards the mean is through changes in the NER. Hence, current RER should predict future nominal exchange rates, but not changes in relative rates of inflation.

Table 8 reports the long-horizon predictability tests computed from (7.5) along with estimates of the contemporaneous correlation $\hat{\phi}_i$ in (7.4) where, in this case, for estimation we replace $\hat{u}_{i,t+1}^{NER}$ by $\hat{u}_{i,t+1}^{\pi}$, for the sample from 1970:Q1 to 2020:Q1. This corresponds to a period during which inflation dynamics changed considerably. Inflation rates around the world have been falling over the last few decades (see Rogoff, 2003). Inflation in industrial economies started to decline in the early 1980s while inflation in emerging economies only began declining in the 1990s. Average inflation was the highest in the seventies, it decreased at the beginning of the eighties and it has been even lower since the beginning of the 1990s.

Given the large number of statistically significant results in Table 8, particularly those obtained from t_h^{NW}, but also for $t_{h,ivx}^{trf}$, $t_{h,ivx}^{trf,res}$ and $t_{h,ivx}^{rev,PL}$, especially when compared with the results for the changes in NER in Table 4, these results seem to suggest that a large number of countries adjust RER through predictable inflation differentials rather than through changes in NER. This finding is also observed by EJR for countries with fixed and quasi-fixed exchange rates (e.g. China and Hong Kong, and France, Ireland, Italy, Portugal, and Spain starting in 1999). The results in Table 9, corresponding to the 1970:Q1 - 2008:Q4 period, are also very similar to those just described in Table 8. Potential justifications for the large number of significant results observed may be related to uncontrolled changes in exchange rate policy, since many countries, particular in the emerging markets group adopted several exchange rate regimes between 1973 and 2020 (Ilzetzki et al., 2017); and to the persistence changes of inflation dynamics observed over this period.

The impact of the changes in exchange rate policy in emerging markets' countries is observable when we compare the results of Tables 8 and 9 with those of Table 11. The results in the latter are computed in the sample from 1999:Q1 onward, a period were most of this countries adopted an inflation targeting policy, show that inflation differentials are less predictive. Note that in the developed markets group $t_{h,ivx}^{trf}$, $t_{h,ivx}^{trf,res}$ and $t_{h,ivx}^{rev,PL}$ suggest rejection of the null hypothesis of no predictability for Ireland and Israel. Similarly, and in contrast to the results in Tables 8 and 9, also for the emerging markets do these statistics suggest a relevant decrease in significant results.

8 Conclusions

In this paper, we have contributed to the long-horizon predictability literature by proposing new tests developed within a transformed regression framework using the IVX estimation approach of Kostakis et al. (2015). We have demonstrated that our proposed tests are (asymptotically) robust to whether the predictors are weakly or strongly persistent and to the induced serial correlation in the errors arising from the temporal aggregation of the dependent variable used in the long-horizon predictive regression. Within a residual augmentation framework we have show that the estimation effect from fitting an autoregression to the predictor to obtain the necessary residuals to augment the predictive regression is asymptotically negligible in the set-up we consider and leads to more efficient estimation of the transformed predictive regression model on which our long-horizon tests are based. Specifically, the residual augmentation approach eliminates endogeneity in the limit, such that the bias of the IVX slope coefficient estimator is reduced compared to the corresponding IVX estimation from the transformed regression without this additional regressor. We have formally established the conditions required for the asymptotic validity of our proposed tests, such that the statistics on which they are based have standard limiting null distributions, free of nuisance parameters arising from the innovations. These conditions allow for quite general patterns of unconditional and conditional time variation in the innovations with no need for the practitioner to specify a parametric model for either the conditional or unconditional time-variation. Our Monte Carlo results contrast the finite size and power properties of our proposed tests with the leading long-horizon predictability tests in the literature. The results obtained suggest that our proposed tests overall display superior finite sample properties to the extant tests displaying robustness against features which are frequently found in time series, making them a useful addition to the literature. We have also provided an empirical application investigating the predictive power of real exchange rates for changes in nominal exchange rates and future inflation rates of a large number of developed and emerging countries, extending the analysis in Eichenbaum et al. (2020) to a wider range of countries and providing conclusions based on the robust statistics developed in this paper. Overall we find somewhat less evidence of predictability than Eichenbaum et al. (2020). This is perhaps to be expected given their analysis is based on standard t-statistics which would appear to be inappropriate for the predictors being considered which appear to be strongly persistent.

References

Amador, M., J. Bianchi, L. Bocola, and F. Perri (2020). Exchange Rate Policies at the Zero Lower Bound. *The Review of Economic Studies 87*(4), 1605–1645.

Amihud, Y. and C. M. Hurvich (2004). Predictive regressions: A reduced-bias estimation method. *Journal of Financial and Quantitative Analysis 39*(4), 813–841.

Amihud, Y., C. M. Hurvich, and Y. Wang (2009). Multiple-predictor regressions: Hypothesis testing. *Review of Financial Studies 22*(1), 413–434.

Amihud, Y., C. M. Hurvich, and Y. Wang (2010). Predictive regression with order-p autoregressive predictors. *Journal of Empirical Finance 17*(3), 513–525.

Ang, A. and G. Bekaert (2007). Return predictability: Is it there? *Review of Financial Studies 20*(3), 651–707.

Bekaert, G. and R. J. Hodrick (1992). Characterizing predictable components in excess returns on equity and foreign exchange markets. *Journal of Finance 47*(2), 467–509.

Boudouk, J. and M. Richardson (1994). The statistics of long-horizon regressions revisited. *Mathematical Finance 4*(2), 103–119.

Boudoukh, J. and R. Matthew (1993). Stock returns and inflation: A long-horizon perspective. *American Economic Review 83*(5), 1346–55.

Boudoukh, J., M. Richardson, and R. Whitelaw (2008). The myth of long horizon predictability. *Review of Financial Studies 21*(4), 1577–1605.

Breitung, J. and M. Demetrescu (2015). Instrumental variable and variable addition based inference in predictive regressions. *Journal of Econometrics 187*(1), 358–375.

Britten-Jones, M., A. Neuberger, and I. Nolte (2011). Improved inference in regression with overlapping observations. *Journal of Business Finance & Accounting 38*(5-6), 657–683.

Byrne, J. P., D. Korobilis, and P. J. Ribeiro (2016). Exchange rate predictability in a changing world. *Journal of International Money and Finance 62*, 1–24.

Cai, Z. and Y. Wang (2014). Corrigendum to "Testing predictive regression models with nonstationary regressors" [J. Econometrics 178 (2014) 4–14]. *Journal of Econometrics 181*(2), 194.

Campbell, J. Y. and J. H. Cochrane (1999). By force of habit: A consumption-based explanation of aggregate stock market behavior. *Journal of Political Economy 107*(1), 205–251.

Campbell, J. Y. and R. J. Shiller (1987). Cointegration and tests of present value models. *Journal of Political Economy 95*(5), 1062–1088.

Campbell, J. Y. and R. J. Shiller (1988). Stock prices, earnings, and expected dividends. *Journal of Finance 43*(3), 661–676.

Campbell, J. Y. and L. M. Viceira (1999). Consumption and portfolio decisions when expected returns are time varying. *The Quarterly Journal of Economics 114*(2), 433–495.

Campbell, J. Y. and M. Yogo (2006a). Efficient tests of stock return predictability. *Journal of Financial Economics 81*(1), 27–60.

Campbell, J. Y. and M. Yogo (2006b). Predicting excess stock returns out of sample: Can anything beat the historical average? *Journal of Financial Economics 81*(1), 27–60.

Cavanagh, C. L., G. Elliott, and J. H. Stock (1995). Inference in models with nearly integrated regressors. *Econometric Theory 11*, 1131–1147.

Chang, C., I. J., H. Laurila, and M. McAleer (2018). Long run returns predictability and volatility with moving averages. *Risk 6*(4), 105.

Cheung, Y.-W., M. D. Chinn, A. G. Pascual, and Y. Zhang (2019). Exchange rate prediction redux: New models, new data, new currencies. *Journal of International Money and Finance 95*, 332–362.

Cochrane, J. H. (1991). Production-based asset pricing and the link between stock returns and economic fluctuations. *Journal of Finance 46*(1), 209–237.

Cochrane, J. H. (2008). The dog that did not bark: A defense of return predictability. *The Review of Financial Studies 21*(4), 1533–1575.

Cochrane, J. H. (2011). Presidential address: Discount rates. *Journal of Finance 66*(4), 1047–1108.

Davidson, J. (1994). *Stochastic Limit Theory: An Introduction for Econometricians*. Oxford university press.

Demetrescu, M., I. Georgiev, P. Rodrigues, and A. M. R. Taylor (2020). Testing for episodic predictability in stock returns. *Journal of Econometrics forthcoming*.

Demetrescu, M., I. Georgiev, P. Rodrigues, and A. M. R. Taylor (2021). Extensions to IVX methods of inference for return predictability. *Working paper*.

Demetrescu, M. and B. Hillmann (2020). Nonlinear predictability of stock returns? Parametric vs. nonparametric inference in predictive regressions. *Journal of Business & Economic Statistics forthcoming*.

Demetrescu, M. and P. M. M. Rodrigues (2020). Residual-augmented IVX predictive regression. *Journal of Econometrics forthcoming*.

Eichenbaum, M. S., B. K. Johannsen, and S. T. Rebelo (2020). Monetary Policy and the Predictability of Nominal Exchange Rates. *The Review of Economic Studies 88*(1), 192–228.

Elliott, G., U. K. Müller, and M. W. Watson (2015). Nearly optimal tests when a nuisance parameter is present under the null hypothesis. *Econometrica 83*(2), 771–811.

Elliott, G. and J. H. Stock (1994). Inference in time series regression when the order of integration of a regressor is unknown. *Econometric Theory 10*(3-4), 672–700.

Engel, C., N. C. Mark, K. D. West, K. Rogoff, and B. Rossi (2007). Exchange rate models are not as bad as you think [with comments and discussion]. *NBER Macroeconomics Annual 22*, 381–473.

Engel, C. and K. West (2005). Exchange rates and fundamentals. *Journal of Political Economy 113*(3), 485–517.

Fama, E. F. (1998). Market efficiency, long-term returns, and behavioral finance. *Journal of Financial Economics 49*(3), 283–306.

Fama, E. F. and K. R. French (1988). Dividend yields and expected stock returns. *Journal of Financial Economics 22*(1), 3–25.

Fama, E. F. and K. R. French (2018). Long-horizon returns. *The Review of Asset Pricing Studies 8*(2), 232–252.

Georgiev, I., D. I. Harvey, S. J. Leybourne, and A. M. R. Taylor (2018). A bootstrap stationarity test for predictive regression invalidity. *Journal of Business & Economic Statistics*.

Gonzalo, J. and J.-Y. Pitarakis (2012). Regime-specific predictability in predictive regressions. *Journal of Business & Economic Statistics 30*(2), 229–241.

Hjalmarsson, E. (2011). New methods for inference in long-horizon regressions. *Journal of Financial and Quantitative Analysis 46*(3), 815–839.

Hjalmarsson, E. (2012). Some curious power properties of long-horizon tests. *Finance Research Letters 9*(2), 81–91.

Hodrick, R. J. (1992). Dividend yields and expected stock returns: Alternative procedures for inference and measurement. *Review of Financial Studies 5*(3), 357–386.

Ilzetzki, E., C. M. Reinhart, and K. S. Rogoff (2017, February). The Country Chronologies to Exchange Rate Arrangements into the 21st Century: Will the Anchor Currency Hold? NBER Working Papers 23135, National Bureau of Economic Research, Inc.

Im, K. S. and P. Schmidt (2008). More efficient estimation under non-normality when higher moments do not depend on the regressors, using residual augmented least squares. *Journal of Econometrics 144*(1), 219–233.

Ince, O. (2014). Forecasting exchange rates out-of-sample with panel methods and real-time data. *Journal of International Money and Finance 43*, 1–18.

Jegadeesh, N. (1991). Seasonality in stock price mean reversion: Evidence from the U.S. and the U.K. *Journal of Finance 46*(4), 1427–1444.

Johannes, M., A. Korteweg, and N. Polson (2014). Sequential learning, predictability, and optimal portfolio returns. *Journal of Finance 69*, 611–644.

Kandel, S. and R. F. Stambaugh (1996). On the predictability of stock returns: An asset-allocation perspective. *Journal of Finance 51*(2), 385–424.

Kilian, L. and M. Taylor (2003). Why is it so difficult to beat the random walk forecast of exchange rates? *Journal of International Economics 60*(1), 85–107.

Kilian, L. and T. Zha (2002). Quantifying the uncertainty about the half-life of deviations from PPP. *Journal of Applied Econometrics 17*(2), 107–125.

Kim, J. H. (2014). Predictive regression: An improved augmented regression method. *Journal of Empirical Finance 26*, 13 – 25.

Kostakis, A., T. Magdalinos, and M. P. Stamatogiannis (2015). Robust econometric inference for stock return predictability. *Review of Financial Studies 28*(5), 1506–1553.

Lee, J. H. (2016). Predictive quantile regression with persistent covariates: IVX-QR approach. *Journal of Econometrics 192*(1), 105–118.

Lettau, M. and S. Van Nieuwerburgh (2008). Reconciling the return predictability evidence. *The Review of Financial Studies 21*(4), 1607–1652.

Lewellen, J. (2004). Predicting returns with financial ratios. *Journal of Financial Economics 74*(2), 209–235.

Lothian, J. and M. Taylor (1996). Real exchange rate behavior: The recent float from the perspective of the past two centuries. *Journal of Political Economy 104*(3), 488–509.

Mankiw, G. N. and M. D. Shapiro (1986). Do we reject too often?: Small sample properties of tests of rational expectations models. *Economics Letters 20*(2), 139–145.

Mark, N. (1995). Exchange rates and fundamentals: Evidence on long-horizon predictability. *American Economic Review 85*(1), 201–18.

Mark, N. and D. Sul (2001). Nominal exchange rates and monetary fundamentals: Evidence from a small post-bretton woods panel. *Journal of International Economics 53*(1), 29–52.

Maynard, A. and D. Ren (2014). *Assessing the Power of Long-Horizon Predictive Tests in Models of Bull and Bear Markets*. Essays in Honor of Peter C. B. Phillips (Advances in Econometrics, Vol. 33), Emerald Group Publishing Limited.

Meese, R. and K. Rogoff (1983). Empirical exchange rate models of the seventies: Do they fit out of sample? *Journal of International Economics 14*(1-2), 3–24.

Menzly, L., T. Santos, and P. Veronesi (2004). Understanding predictability. *Journal of Political Economy 112*(1), 1–47.

Mishkin, F. S. (1990). What does the term structure of interest rates tell us about future inflation? *Journal of Monetary Economics 25*(1), 77–90.

Molodtsova, T. and D. H. Papell (2009). Out-of-sample exchange rate predictability with Taylor rule fundamentals. *Journal of International Economics 77*(2), 167–180.

Mumtaz, H. and L. Sunder-Plassmann (2013). Time-varying dynamics of the real exchange rate: An empirical analysis. *Journal of Applied Econometrics 28*(3), 498–525.

Murray, C. J. and D. Papell (2002). The purchasing power parity persistence paradigm. *Journal of International Economics 56*(1), 1–19.

Murray, C. J. and D. Papell (2005). The purchasing power parity puzzle is worse than you think. *Empirical Economics 30*, 783–790.

Nelson, C. R. and M. J. Kim (1993). Predictable stock returns: The role of small sample bias. *Journal of Finance 48*(2), 641–661.

Newey, W. K. and K. D. West (1987). A simple, positive semi-definite, heteroskedasticity and autocorrelation consistent covariance matrix. *Econometrica 55*(3), 703–708.

Phillips, P. (1991). Optimal inference in cointegrated systems. *Econometrica 59*(2), 283–306.

Phillips, P. C. B. (2014). On confidence intervals for autoregressive roots and predictive regression. *Econometrica 82*(3), 1177–1195.

Phillips, P. C. B. (2015). Pitfalls and possibilities in predictive regression. *Journal of Financial Econometrics 13*(3), 521–555.

Phillips, P. C. B. and J. H. Lee (2013). Predictive regression under various degrees of persistence and robust long-horizon regression. *Journal of Econometrics 177*(2), 250–264.

Phillips, P. C. B. and T. Magdalinos (2009). Econometric inference in the vicinity of unity. Cofie working paper 7, singapore management university.

Rogoff, K. (2003). Globalization and global disinflation. *Economic Review 88*(Q IV), 45–78.

Rogoff, K. and V. Stavrakeva (2008). The continuing puzzle of short-horizon exchange rate forecasting. Download paper, July 2008. Download NBER Working Paper No. 14071, June 2008.

Rossi, B. (2006). Are exchange rates really random walks? some evidence robust to parameter instability. *Macroeconomic Dynamics 10*(1), 20–38.

Rossi, B. (2007). Expectations hypotheses tests at long horizons. *The Econometrics Journal 10*(3), 554–579.

Rossi, B. (2013, December). Exchange rate predictability. *Journal of Economic Literature 51*(4), 1063–1119.

Rossi, B. and T. Sekhposyan (2011). Understanding models' forecasting performance. *Journal of Econometrics 164*(1), 158–172.

Sarno, L. and M. Taylor (2002). Purchasing power parity and the real exchange rate. *IMF Staff Papers 49*(1).

Stambaugh, R. F. (1986). Bias in regressions with lagged stochastic regressors. *CRSP Working Paper No 156, University of Chicago*.

Stambaugh, R. F. (1999). Predictive regressions. *Journal of Financial Economics 54*(3), 375–421.

Taylor, A. M. and M. P. Taylor (2004, Fall). The purchasing power parity debate. *Journal of Economic Perspectives 18*(4), 135–158.

Taylor, M. P. (2009). Editorial: Long-run purchasing power parity and real exchange rates: introduction and overview. *Applied Economics Letters 16*(1), 1–4.

Valkanov, R. (2003). Long-horizon regressions: Theoretical results and applications. *Journal of Financial Economics 68*(2), 201–232.

Wei, M. and J. H. Wright (2013). Reverse regressions and long-horizon forecasting. *Journal of Applied Econometrics 28*(3), 353–371.

Weigand, R. A. and R. Irons (2007). The market p/e ratio, earnings trends, and stock return forecasts. *Journal of Portfolio Management 33*(4), 87–101.

Welch, I. and A. Goyal (2008). A comprehensive look at the empirical performance of equity premium prediction. *Review of Financial Studies 21*(4), 1455–1508.

Xu, K.-L. (2020). Testing for multiple-horizon predictability: Direct regression based versus implication based. *Review of Financial Studies 33*(9), 4403–4443.

Table 1: Empirical rejection frequencies of one-sided (left and right tail) and two-sided long-horizon predictability tests, for sample sizes $T = 250$ and $T = 500$. **DGP (homoskedastic IID innovations):** $y_{t+1} = \beta x_t + u_{t+1}, x_{t+1} = \rho x_t + v_{t+1}$ and $v_{t+1} = \psi v_t + \nu_{t+1}$, where $\beta = 0$, $\rho = 1 - c/T$, $\psi = 0$ and $(u_{t+1}, \nu_{t+1})' \sim NIID(\mathbf{0}, \mathbf{\Sigma})$, with $\mathbf{\Sigma} = [1 \quad -0.95; \quad -0.95 \quad 1]$.

$T = 250$

		t_h^{Xu}			t_h^{Bonf}			$t_{h,ivx}^{trf,res}$			$t_h^{rev,PL}$		
h	c	$\beta_h < 0$	$\beta_h > 0$	$\beta_h \neq 0$	$\beta_h < 0$	$\beta_h > 0$	$\beta_h \neq 0$	$\beta_h < 0$	$\beta_h > 0$	$\beta_h \neq 0$	$\beta_h < 0$	$\beta_h > 0$	$\beta_h \neq 0$
1	0	0.001	0.197	0.122	0.026	0.039	0.050	0.000	0.061	0.032	0.000	0.112	0.055
	-5	0.111	0.092	0.118	0.038	0.043	0.064	0.007	0.062	0.035	0.009	0.111	0.061
	-10	0.104	0.067	0.105	0.048	0.039	0.070	0.017	0.057	0.035	0.017	0.093	0.053
	-20	0.073	0.060	0.074	0.078	0.032	0.091	0.029	0.056	0.044	0.024	0.076	0.052
	-50	0.062	0.054	0.062	0.213	0.031	0.219	0.043	0.051	0.046	0.031	0.058	0.044
5	0	0.000	0.159	0.092	0.018	0.036	0.040	0.001	0.064	0.035	0.001	0.104	0.053
	-5	0.110	0.073	0.101	0.030	0.040	0.053	0.006	0.063	0.037	0.009	0.107	0.059
	-10	0.102	0.061	0.100	0.042	0.033	0.058	0.017	0.068	0.043	0.019	0.098	0.056
	-20	0.069	0.052	0.066	0.065	0.025	0.072	0.031	0.066	0.052	0.024	0.080	0.055
	-50	0.054	0.048	0.052	0.169	0.018	0.159	0.047	0.065	0.061	0.030	0.060	0.043
10	0	0.000	0.124	0.068	0.011	0.036	0.033	0.001	0.065	0.038	0.000	0.100	0.051
	-5	0.104	0.057	0.080	0.023	0.036	0.044	0.005	0.067	0.038	0.009	0.099	0.057
	-10	0.096	0.046	0.084	0.035	0.027	0.045	0.016	0.065	0.040	0.015	0.092	0.055
	-20	0.064	0.046	0.057	0.051	0.018	0.049	0.028	0.070	0.052	0.022	0.075	0.048
	-50	0.046	0.049	0.047	0.119	0.007	0.100	0.048	0.060	0.054	0.023	0.054	0.036
20	0	0.000	0.089	0.054	0.004	0.033	0.026	0.001	0.057	0.033	0.001	0.097	0.049
	-5	0.091	0.057	0.074	0.0153	0.026	0.028	0.005	0.058	0.034	0.007	0.101	0.054
	-10	0.083	0.052	0.079	0.023	0.016	0.024	0.014	0.062	0.038	0.011	0.096	0.053
	-20	0.049	0.050	0.051	0.029	0.007	0.023	0.030	0.053	0.043	0.016	0.077	0.048
	-50	0.052	0.051	0.053	0.045	0.001	0.028	0.051	0.046	0.048	0.022	0.054	0.037
50	0	0.003	0.170	0.157	0.001	0.020	0.010	0.001	0.040	0.020	0.001	0.076	0.037
	-5	0.058	0.115	0.113	0.003	0.011	0.008	0.008	0.040	0.023	0.004	0.080	0.039
	-10	0.068	0.077	0.092	0.006	0.006	0.005	0.020	0.040	0.028	0.007	0.081	0.041
	-20	0.038	0.067	0.059	0.004	0.002	0.002	0.029	0.037	0.030	0.008	0.077	0.040
	-50	0.060	0.060	0.067	0.003	0.000	0.001	0.029	0.028	0.029	0.009	0.051	0.027

$T = 500$

		t_h^{Xu}			t_h^{Bonf}			$t_{h,ivx}^{trf,res}$			$t_h^{rev,PL}$		
h	c	$\beta_h < 0$	$\beta_h > 0$	$\beta_h \neq 0$	$\beta_h < 0$	$\beta_h > 0$	$\beta_h \neq 0$	$\beta_h < 0$	$\beta_h > 0$	$\beta_h \neq 0$	$\beta_h < 0$	$\beta_h > 0$	$\beta_h \neq 0$
1	0	0.002	0.196	0.119	0.032	0.032	0.048	0.000	0.063	0.035	0.000	0.111	0.057
	-5	0.119	0.082	0.121	0.042	0.033	0.058	0.005	0.059	0.036	0.009	0.104	0.061
	-10	0.107	0.065	0.104	0.052	0.032	0.067	0.016	0.058	0.035	0.017	0.091	0.052
	-20	0.072	0.059	0.074	0.073	0.029	0.082	0.028	0.058	0.042	0.024	0.082	0.053
	-50	0.062	0.056	0.064	0.178	0.021	0.177	0.039	0.056	0.046	0.033	0.068	0.050
5	0	0.000	0.180	0.105	0.025	0.032	0.041	0.002	0.059	0.031	0.001	0.107	0.052
	-5	0.112	0.081	0.109	0.038	0.032	0.054	0.005	0.067	0.035	0.009	0.110	0.062
	-10	0.103	0.065	0.099	0.047	0.032	0.062	0.017	0.067	0.041	0.017	0.100	0.061
	-20	0.070	0.058	0.066	0.065	0.026	0.073	0.026	0.065	0.049	0.023	0.084	0.056
	-50	0.055	0.050	0.053	0.156	0.016	0.148	0.035	0.068	0.052	0.029	0.064	0.048
10	0	0.000	0.156	0.091	0.018	0.032	0.035	0.001	0.062	0.035	0.000	0.107	0.054
	-5	0.109	0.072	0.099	0.033	0.031	0.050	0.006	0.061	0.034	0.010	0.106	0.060
	-10	0.100	0.058	0.096	0.041	0.030	0.054	0.017	0.065	0.041	0.017	0.096	0.058
	-20	0.067	0.054	0.064	0.057	0.023	0.063	0.029	0.068	0.051	0.022	0.087	0.054
	-50	0.059	0.049	0.056	0.129	0.010	0.116	0.043	0.068	0.058	0.033	0.068	0.053
20	0	0.000	0.127	0.067	0.011	0.030	0.029	0.001	0.056	0.030	0.001	0.099	0.048
	-5	0.105	0.054	0.081	0.028	0.029	0.042	0.005	0.060	0.033	0.009	0.105	0.058
	-10	0.094	0.049	0.080	0.034	0.026	0.045	0.015	0.062	0.040	0.014	0.096	0.055
	-20	0.062	0.045	0.054	0.045	0.019	0.043	0.026	0.061	0.045	0.022	0.076	0.053
	-50	0.051	0.051	0.053	0.082	0.004	0.061	0.041	0.065	0.053	0.026	0.067	0.047
50	0	0.000	0.086	0.064	0.004	0.028	0.020	0.001	0.055	0.029	0.001	0.091	0.045
	-5	0.083	0.064	0.078	0.017	0.018	0.022	0.006	0.053	0.028	0.007	0.094	0.051
	-10	0.088	0.052	0.087	0.019	0.013	0.020	0.017	0.049	0.032	0.012	0.086	0.051
	-20	0.047	0.056	0.055	0.018	0.006	0.013	0.026	0.047	0.036	0.017	0.077	0.046
	-50	0.052	0.060	0.062	0.014	0.001	0.007	0.038	0.046	0.041	0.020	0.067	0.043

Notes: t_h^{Xu} denotes the implied statistic of Xu (2020), t_h^{Bonf} is the Bonferroni based statistic of Hjalmarsson (2011), $t_{h,ivx}^{trf,res}$ is the residual augmented transformed regression based statistic in (4.10) proposed in section 4.2; and $t_{h,ivx}^{trf,PL}$ is the Phillips and Lee (2013) statistic. h is the forecast horizon considered and c is the local to unity parameter that characterises the persistence of the predictor. The columns labeled $\beta_h < 0$, $\beta_h > 0$, and $\beta_h \neq 0$ refer to left-, right- and two-sided tests, respectively.

Table 2: Empirical rejection frequencies of one-sided (left and right tail) and two-sided predictability tests, for sample sizes $T = 250$ and $T = 500$. **DGP (Positive Autocorrelation):** $y_{t+1} = \beta x_t + u_{t+1}, x_{t+1} = \rho x_t + v_{t+1}$ and $v_{t+1} = \psi v_t + \nu_{t+1}$, where $\beta = 0$, $\rho = 1 + c/T$, $\psi = 0.5$ and $(u_{t+1}, \nu_{t+1})' \sim NIID(\mathbf{0}, \mathbf{\Sigma})$, with $\mathbf{\Sigma} = [1 \quad -0.95; \quad -0.95 \quad 1]$.

T = 250

		t_h^{Xu}			t_h^{Bonf}			$t_{h,ivx}^{trf,res}$			$t_h^{rev,PL}$		
h	c	$\beta_h < 0$	$\beta_h > 0$	$\beta_h \neq 0$	$\beta_h < 0$	$\beta_h > 0$	$\beta_h \neq 0$	$\beta_h < 0$	$\beta_h > 0$	$\beta_h \neq 0$	$\beta_h < 0$	$\beta_h > 0$	$\beta_h \neq 0$
1	0	0.001	0.240	0.145	0.030	0.036	0.045	0.001	0.060	0.033	0.000	0.111	0.056
	-5	0.032	0.057	0.046	0.006	0.031	0.024	0.007	0.062	0.035	0.009	0.112	0.063
	-10	0.127	0.043	0.092	0.014	0.029	0.030	0.015	0.060	0.037	0.017	0.099	0.059
	-20	0.175	0.041	0.138	0.027	0.027	0.036	0.027	0.061	0.040	0.024	0.084	0.052
	-50	0.091	0.046	0.075	0.042	0.020	0.046	0.037	0.059	0.048	0.030	0.069	0.050
5	0	0.000	0.167	0.095	0.034	0.005	0.025	0.001	0.062	0.032	0.001	0.104	0.051
	-5	0.016	0.050	0.032	0.074	0.002	0.051	0.005	0.059	0.033	0.009	0.107	0.059
	-10	0.116	0.043	0.081	0.188	0.000	0.147	0.016	0.062	0.040	0.019	0.100	0.059
	-20	0.172	0.038	0.125	0.358	0.000	0.299	0.027	0.062	0.045	0.024	0.085	0.057
	-50	0.086	0.041	0.068	0.656	0.000	0.590	0.040	0.059	0.049	0.032	0.068	0.050
10	0	0.001	0.110	0.064	0.042	0.002	0.028	0.001	0.063	0.033	0.000	0.097	0.049
	-5	0.010	0.044	0.025	0.0136	0.001	0.096	0.005	0.062	0.036	0.008	0.099	0.055
	-10	0.099	0.035	0.059	0.306	0.000	0.247	0.012	0.062	0.039	0.015	0.094	0.056
	-20	0.165	0.036	0.120	0.509	0.000	0.439	0.027	0.067	0.047	0.022	0.081	0.051
	-50	0.072	0.037	0.058	0.716	0.000	0.650	0.036	0.064	0.052	0.024	0.065	0.043
20	0	0.004	0.065	0.041	0.030	0.001	0.020	0.001	0.056	0.031	0.001	0.094	0.047
	-5	0.002	0.045	0.032	0.151	0.000	0.108	0.005	0.056	0.031	0.006	0.100	0.052
	-10	0.077	0.041	0.047	0.318	0.000	0.255	0.012	0.058	0.034	0.011	0.097	0.055
	-20	0.150	0.041	0.114	0.453	0.000	0.380	0.024	0.057	0.042	0.017	0.083	0.049
	-50	0.069	0.045	0.062	0.448	0.000	0.3431	0.040	0.055	0.049	0.024	0.068	0.044
50	0	0.056	0.059	0.064	0.003	0.003	0.001	0.002	0.039	0.020	0.001	0.074	0.037
	-5	0.003	0.093	0.077	0.040	0.000	0.020	0.007	0.041	0.023	0.004	0.081	0.039
	-10	0.022	0.105	0.091	0.082	0.000	0.047	0.017	0.041	0.026	0.007	0.082	0.042
	-20	0.113	0.104	0.135	0.066	0.000	0.034	0.027	0.042	0.030	0.008	0.082	0.044
	-50	0.055	0.067	0.070	0.032	0.002	0.012	0.031	0.036	0.032	0.010	0.069	0.039

T = 500

		t_h^{Xu}			t_h^{Bonf}			$t_{h,ivx}^{trf,res}$			$t_h^{rev,PL}$		
h	c	$\beta_h < 0$	$\beta_h > 0$	$\beta_h \neq 0$	$\beta_h < 0$	$\beta_h > 0$	$\beta_h \neq 0$	$\beta_h < 0$	$\beta_h > 0$	$\beta_h \neq 0$	$\beta_h < 0$	$\beta_h > 0$	$\beta_h \neq 0$
1	0	0.001	0.240	0.144	0.036	0.029	0.044	0.001	0.065	0.035	0.000	0.115	0.057
	-5	0.029	0.056	0.044	0.008	0.026	0.020	0.005	0.060	0.035	0.009	0.104	0.060
	-10	0.128	0.037	0.090	0.016	0.025	0.027	0.015	0.059	0.033	0.017	0.094	0.053
	-20	0.173	0.041	0.136	0.026	0.024	0.033	0.028	0.058	0.042	0.024	0.085	0.055
	-50	0.088	0.041	0.072	0.038	0.021	0.039	0.037	0.056	0.044	0.031	0.072	0.051
5	0	0.000	0.196	0.113	0.042	0.004	0.030	0.001	0.062	0.032	0.001	0.105	0.053
	-5	0.022	0.053	0.036	0.079	0.001	0.056	0.006	0.068	0.037	0.009	0.112	0.065
	-10	0.120	0.041	0.082	0.200	0.001	0.154	0.013	0.062	0.039	0.016	0.099	0.059
	-20	0.172	0.041	0.136	0.381	0.000	0.319	0.029	0.062	0.042	0.026	0.090	0.060
	-50	0.084	0.042	0.070	0.735	0.000	0.679	0.037	0.060	0.047	0.030	0.072	0.051
10	0	0.001	0.163	0.094	0.059	0.002	0.041	0.001	0.061	0.033	0.001	0.109	0.053
	-5	0.017	0.045	0.031	0.151	0.000	0.112	0.005	0.063	0.035	0.009	0.105	0.058
	-10	0.112	0.036	0.069	0.340	0.000	0.279	0.014	0.060	0.037	0.016	0.094	0.057
	-20	0.167	0.039	0.132	0.587	0.000	0.521	0.027	0.066	0.047	0.025	0.090	0.060
	-50	0.083	0.035	0.063	0.894	0.000	0.856	0.039	0.063	0.053	0.032	0.071	0.052
20	0	0.001	0.107	0.062	0.062	0.001	0.042	0.001	0.057	0.029	0.001	0.096	0.044
	-5	0.010	0.041	0.023	0.197	0.000	0.149	0.004	0.062	0.034	0.008	0.104	0.059
	-10	0.097	0.034	0.054	0.408	0.000	0.345	0.016	0.064	0.038	0.016	0.093	0.059
	-20	0.161	0.031	0.121	0.641	0.000	0.568	0.027	0.062	0.046	0.024	0.083	0.054
	-50	0.077	0.040	0.061	0.852	0.000	0.794	0.036	0.061	0.048	0.026	0.068	0.047
50	0	0.007	0.062	0.044	0.028	0.002	0.016	0.001	0.047	0.023	0.001	0.083	0.041
	-5	0.001	0.049	0.037	0.161	0.000	0.117	0.003	0.048	0.025	0.005	0.090	0.048
	-10	0.061	0.046	0.042	0.331	0.000	0.265	0.015	0.047	0.030	0.014	0.087	0.051
	-20	0.147	0.051	0.115	0.431	0.000	0.344	0.024	0.050	0.033	0.016	0.081	0.045
	-50	0.067	0.055	0.067	0.372	0.000	0.260	0.039	0.048	0.045	0.019	0.075	0.043

Notes: See Notes to Table 1.

Table 3: Empirical rejection frequencies of one-sided (left and right tail) and two-sided predictability tests, for sample sizes $T = 250$ and $T = 500$. **DGP (Negative Autocorrelation):** $y_{t+1} = \beta x_t + u_{t+1}, x_{t+1} = \rho x_t + v_{t+1}$ and $v_{t+1} = \psi v_t + \nu_{t+1}$, where $\beta = 0$, $\rho = 1 + c/T$, $\psi = -0.5$ and $(u_{t+1}, \nu_{t+1})' \sim NIID(\mathbf{0}, \mathbf{\Sigma})$, with $\mathbf{\Sigma} = [1 \quad -0.95; \quad -0.95 \quad 1]$.

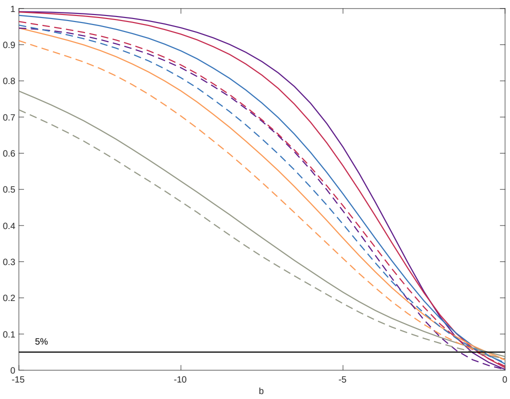
$T = 250$

		t_h^{Xu}			t_h^{Bonf}			$t_{h,ivx}^{trf,res}$			$t_h^{rev,PL}$		
h	c	$\beta_h < 0$	$\beta_h > 0$	$\beta_h \neq 0$	$\beta_h < 0$	$\beta_h > 0$	$\beta_h \neq 0$	$\beta_h < 0$	$\beta_h > 0$	$\beta_h \neq 0$	$\beta_h < 0$	$\beta_h > 0$	$\beta_h \neq 0$
1	0	0.008	0.250	0.160	0.014	0.023	0.024	0.001	0.062	0.033	0.000	0.110	0.054
	-5	0.032	0.129	0.093	0.031	0.028	0.037	0.008	0.064	0.036	0.009	0.103	0.056
	-10	0.033	0.095	0.071	0.060	0.028	0.063	0.019	0.059	0.039	0.018	0.083	0.047
	-20	0.039	0.074	0.062	0.141	0.032	0.142	0.031	0.056	0.045	0.023	0.066	0.047
	-50	0.045	0.066	0.052	0.335	0.075	0.378	0.042	0.055	0.048	0.027	0.052	0.034
5	0	0.005	0.221	0.136	0.001	0.226	0.199	0.001	0.063	0.035	0.001	0.112	0.055
	-5	0.028	0.110	0.077	0.001	0.419	0.379	0.005	0.064	0.035	0.009	0.095	0.051
	-10	0.031	0.095	0.066	0.000	0.521	0.475	0.018	0.068	0.046	0.015	0.088	0.054
	-20	0.036	0.077	0.059	0.002	0.611	0.563	0.031	0.071	0.056	0.023	0.073	0.049
	-50	0.041	0.064	0.053	0.038	0.584	0.576	0.040	0.070	0.051	0.025	0.051	0.035
	-50	0.041	0.064	0.053	0.038	0.584	0.576	0.040	0.070	0.051	0.025	0.051	0.035
10	0	0.004	0.213	0.135	0.000	0.257	0.230	0.001	0.065	0.038	0.001	0.103	0.051
	-5	0.028	0.115	0.082	0.001	0.466	0.417	0.005	0.063	0.037	0.009	0.094	0.051
	-10	0.028	0.094	0.068	0.000	0.558	0.507	0.016	0.070	0.045	0.012	0.086	0.049
	-20	0.032	0.079	0.059	0.001	0.616	0.546	0.029	0.071	0.053	0.020	0.068	0.043
	-50	0.045	0.065	0.055	0.018	0.478	0.603	0.042	0.065	0.058	0.023	0.049	0.037
20	0	0.003	0.235	0.174	0.000	0.262	0.228	0.001	0.063	0.034	0.001	0.098	0.051
	-5	0.024	0.131	0.097	0.000	0.429	0.358	0.005	0.067	0.037	0.007	0.094	0.050
	-10	0.023	0.098	0.067	0.000	0.469	0.375	0.017	0.058	0.037	0.013	0.078	0.045
	-20	0.034	0.079	0.061	0.000	0.412	0.290	0.032	0.059	0.046	0.017	0.064	0.038
	-50	0.046	0.065	0.056	0.006	0.109	0.043	0.039	0.045	0.040	0.019	0.047	0.032
50	0	0.001	0.327	0.277	0.000	0.200	0.144	0.002	0.045	0.023	0.001	0.075	0.038
	-5	0.016	0.146	0.099	0.000	0.209	0.120	0.008	0.043	0.025	0.003	0.084	0.043
	-10	0.023	0.101	0.069	0.000	0.130	0.059	0.020	0.040	0.029	0.005	0.071	0.034
	-20	0.042	0.074	0.065	0.000	0.037	0.009	0.029	0.032	0.030	0.008	0.056	0.029
	-50	0.045	0.067	0.059	0.000	0.001	0.000	0.023	0.021	0.021	0.009	0.049	0.030

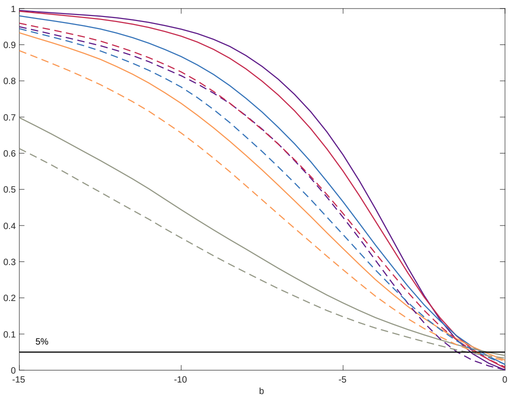
$T = 500$

		t_h^{Xu}			t_h^{Bonf}			$t_{h,ivx}^{trf,res}$			$t_h^{rev,PL}$		
h	c	$\beta_h < 0$	$\beta_h > 0$	$\beta_h \neq 0$	$\beta_h < 0$	$\beta_h > 0$	$\beta_h \neq 0$	$\beta_h < 0$	$\beta_h > 0$	$\beta_h \neq 0$	$\beta_h < 0$	$\beta_h > 0$	$\beta_h \neq 0$
1	0	0.009	0.251	0.162	0.015	0.015	0.019	0.001	0.063	0.036	0.000	0.109	0.055
	-5	0.030	0.122	0.090	0.038	0.018	0.034	0.005	0.061	0.034	0.009	0.103	0.058
	-10	0.034	0.094	0.067	0.058	0.016	0.052	0.017	0.058	0.035	0.017	0.085	0.049
	-20	0.036	0.080	0.062	0.120	0.013	0.103	0.028	0.059	0.044	0.022	0.073	0.048
	-50	0.045	0.067	0.060	0.346	0.009	0.322	0.044	0.056	0.051	0.032	0.059	0.044
5	0	0.009	0.238	0.150	0.001	0.217	0.189	0.002	0.060	0.033	0.001	0.104	0.052
	-5	0.029	0.123	0.087	0.001	0.432	0.397	0.005	0.068	0.038	0.009	0.109	0.059
	-10	0.031	0.095	0.069	0.000	0.575	0.533	0.018	0.069	0.042	0.018	0.096	0.057
	-20	0.032	0.081	0.058	0.001	0.700	0.668	0.026	0.065	0.047	0.022	0.078	0.051
	-50	0.043	0.065	0.058	0.023	0.724	0.706	0.043	0.062	0.055	0.031	0.058	0.043
	-50	0.043	0.065	0.057	0.013	0.724	0.706	0.043	0.062	0.055	0.031	0.058	0.043
10	0	0.005	0.229	0.136	0.001	0.251	0.227	0.001	0.063	0.035	0.000	0.107	0.055
	-5	0.028	0.120	0.086	0.000	0.500	0.462	0.006	0.061	0.034	0.010	0.105	0.059
	-10	0.031	0.094	0.070	0.000	0.648	0.609	0.017	0.069	0.042	0.017	0.091	0.054
	-20	0.034	0.081	0.060	0.000	0.765	0.732	0.029	0.067	0.051	0.021	0.078	0.049
	-50	0.042	0.066	0.057	0.007	0.759	0.729	0.043	0.064	0.055	0.029	0.058	0.045
20	0	0.005	0.222	0.143	0.001	0.268	0.241	0.001	0.057	0.029	0.001	0.099	0.048
	-5	0.028	0.117	0.088	0.000	0.517	0.468	0.005	0.064	0.034	0.009	0.102	0.056
	-10	0.027	0.101	0.072	0.000	0.648	0.597	0.015	0.063	0.043	0.014	0.091	0.051
	-20	0.031	0.077	0.057	0.000	0.736	0.682	0.028	0.062	0.047	0.021	0.070	0.046
	-50	0.043	0.067	0.060	0.003	0.646	0.559	0.044	0.057	0.051	0.028	0.059	0.042
50	0	0.002	0.260	0.199	0.001	0.255	0.221	0.001	0.054	0.030	0.001	0.094	0.044
	-5	0.021	0.144	0.109	0.000	0.431	0.353	0.007	0.057	0.029	0.007	0.094	0.049
	-10	0.025	0.097	0.067	0.000	0.470	0.361	0.016	0.052	0.034	0.012	0.081	0.046
	-20	0.032	0.083	0.064	0.000	0.389	0.256	0.028	0.047	0.035	0.016	0.067	0.039
	-50	0.047	0.067	0.060	0.000	0.089	0.030	0.038	0.038	0.037	0.021	0.063	0.042

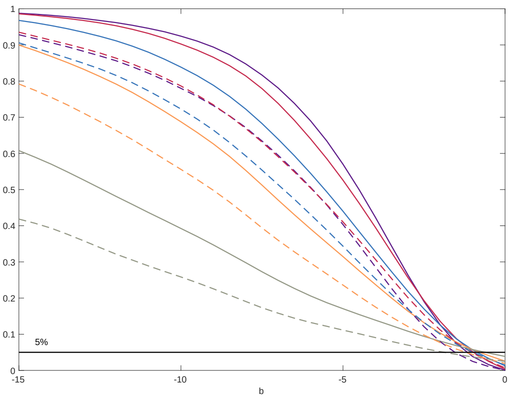
Notes: See Notes to Table 1.



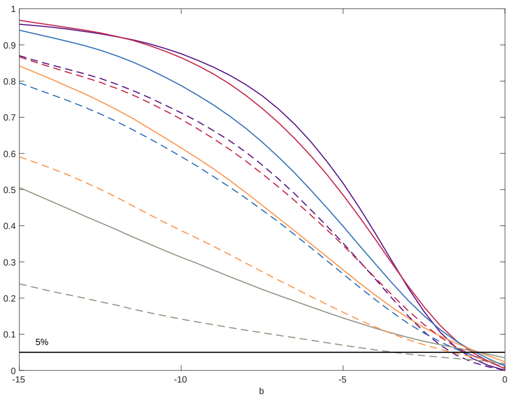
(a) $h = 1$



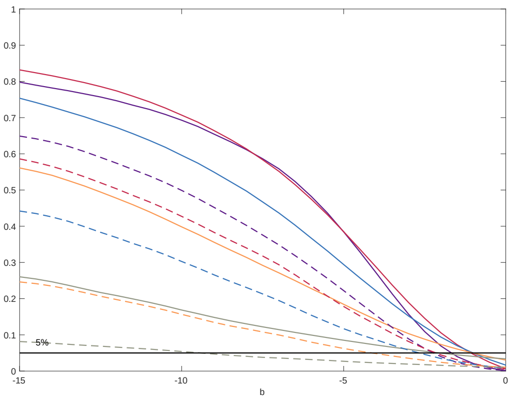
(b) $h = 5$



(c) $h = 10$



(d) $h = 20$



(e) $h = 50$

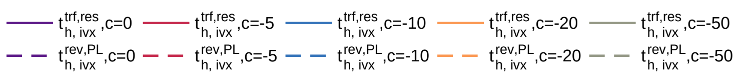
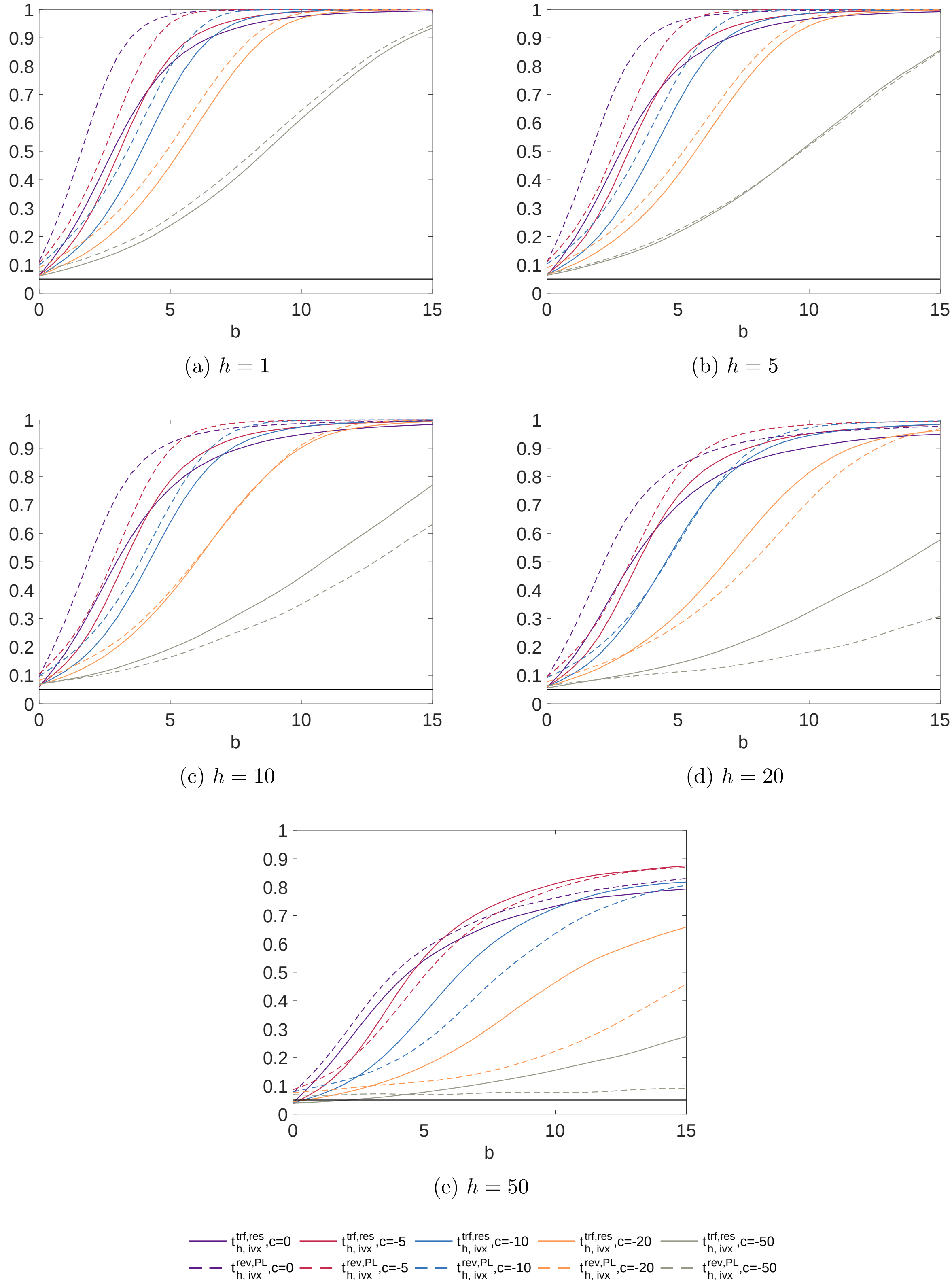


Figure 1: Power curves of the **LEFT**-sided tests $t_{h,ivx}^{trf,res}$ and $t_{h,ivx}^{rev,PL}$ for prediction horizon $h = \{1, 5, 10, 20, 50\}$ and $T = 250$. **DGP:** $y_{t+1} = \beta x_t + u_{t+1}, x_{t+1} = \rho x_t + v_{t+1}$ and $v_{t+1} = \psi v_t + \nu_{t+1}$, where $\beta = b/T$, $\rho = 1 + c/T$, with $c = \{0, -5, -10, -20, -50\}$, $\psi = 0.5$ and $(u_{t+1}, \nu_{t+1})' \sim NIID(\mathbf{0}, \mathbf{\Sigma})$, with $\mathbf{\Sigma} = [1 \quad -0.95; \quad -0.95 \quad 1]$.



(a) $h = 1$

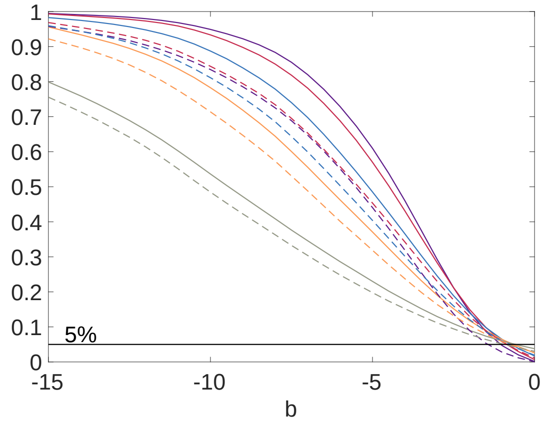
(b) $h = 5$

(c) $h = 10$

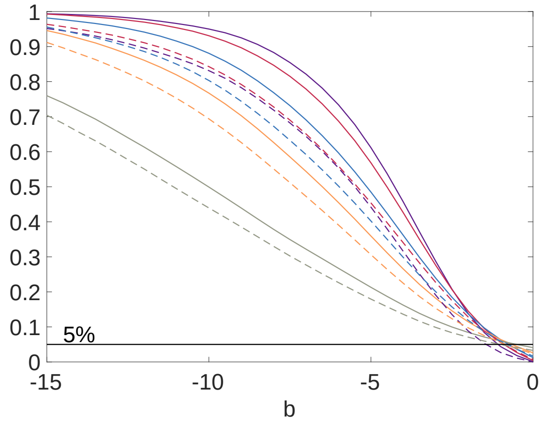
(d) $h = 20$

(e) $h = 50$

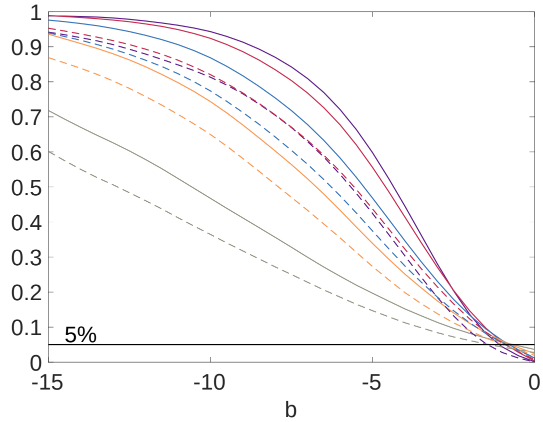
Figure 2: Power curves of the **RIGHT**-sided tests $t_{h,ivx}^{trf,res}$ and $t_{h,ivx}^{rev,PL}$ for prediction horizon $h = \{1, 5, 10, 20, 50\}$ and $T = 250$. **DGP:** $y_{t+1} = \beta x_t + u_{t+1}, x_{t+1} = \rho x_t + v_{t+1}$ and $v_{t+1} = \psi v_t + \nu_{t+1}$, where $\beta = b/T$, $\rho = 1 + c/T$, with $c = \{0, -5, -10, -20, -50\}$, $\psi = 0.5$ and $(u_{t+1}, \nu_{t+1})' \sim NIID(\mathbf{0}, \mathbf{\Sigma})$, with $\mathbf{\Sigma} = [1 \quad -0.95; \quad -0.95 \quad 1]$.



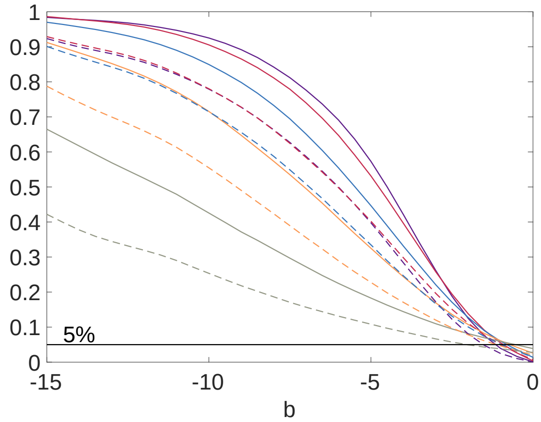
(a) $h = 1$



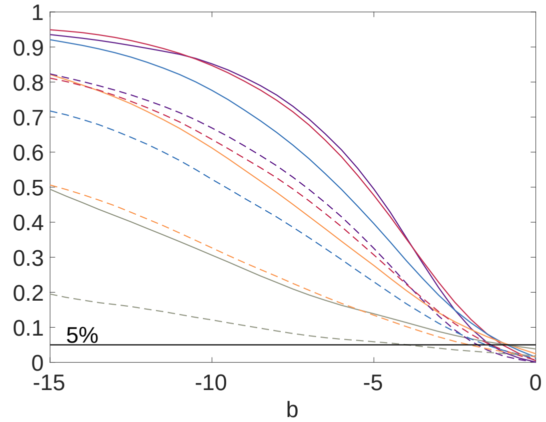
(b) $h = 5$



(c) $h = 10$



(d) $h = 20$



(e) $h = 50$

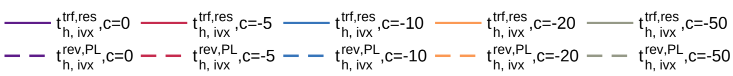
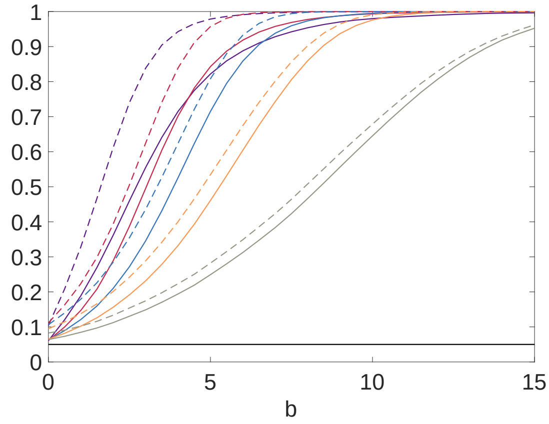
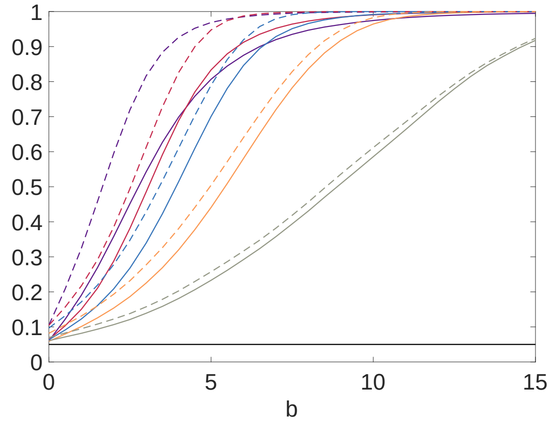


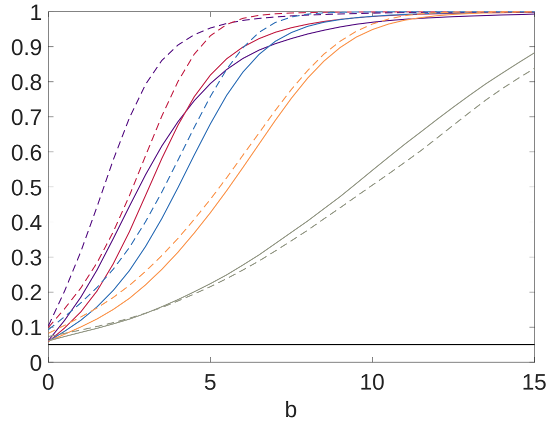
Figure 3: Power curves of the **LEFT**-sided tests $t^{trf,res}_{h,ivx}$ and $t^{rev,PL}_{h,ivx}$ for prediction horizon $h = \{1, 5, 10, 20, 50\}$ and $T = 500$. **DGP:** $y_{t+1} = \beta x_t + u_{t+1}, x_{t+1} = \rho x_t + v_{t+1}$ and $v_{t+1} = \psi v_t + \nu_{t+1}$, where $\beta = b/T$, $\rho = 1 + c/T$, with $c = \{0, -5, -10, -20, -50\}$, $\psi = 0.5$ and $(u_{t+1}, \nu_{t+1})' \sim NIID(\mathbf{0}, \mathbf{\Sigma})$, with $\mathbf{\Sigma} = [1 \quad -0.95; \quad -0.95 \quad 1]$.



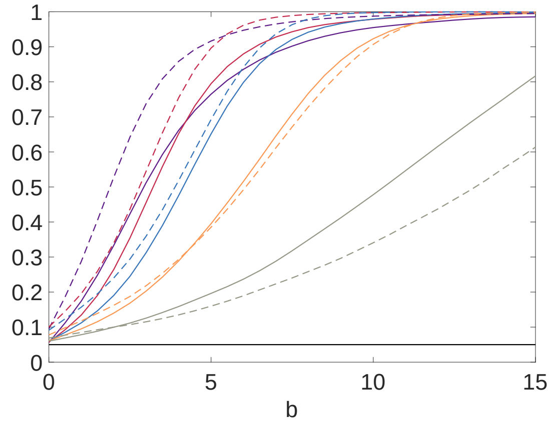
(a) $h = 1$



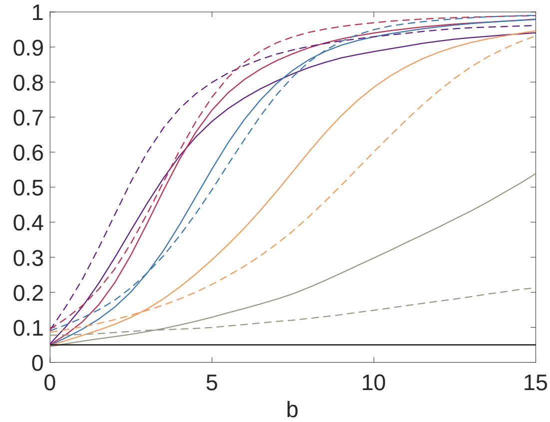
(b) $h = 5$



(c) $h = 10$



(d) $h = 20$



(e) $h = 50$

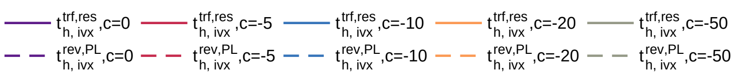


Figure 4: Power curves of the **RIGHT**-sided tests $t^{trf,res}_{h,ivx}$ and $t^{rev,PL}_{h,ivx}$ for prediction horizon $h = \{1, 5, 10, 20, 50\}$ and $T = 500$. **DGP:** $y_{t+1} = \beta x_t + u_{t+1}, x_{t+1} = \rho x_t + v_{t+1}$ and $v_{t+1} = \psi v_t + \nu_{t+1}$, where $\beta = b/T$, $\rho = 1 + c/T$, with $c = \{0, -5, -10, -20, -50\}$, $\psi = 0.5$ and $(u_{t+1}, \nu_{t+1})' \sim NIID(\mathbf{0}, \mathbf{\Sigma})$, with $\mathbf{\Sigma} = [1 \quad -0.95; \quad -0.95 \quad 1]$.

Table 4: Nominal exchange rate regression results for sample from 1973:Q1 to 2020:Q2.

			$h=1$				$h=4$				$h=8$				$h=12$				$h=20$				
	$\hat{\phi}$	$\hat{\rho}^{RER}$	$t_{h,NW}$	$t^{trf}_{h,ivx}$	$t^{trf,res}_{h,ivx}$	$t^{rev,PL}_{h,ivx}$	$t_{h,NW}$	$t^{trf}_{h,ivx}$	$t^{trf,res}_{h,ivx}$	$t^{rev,PL}_{h,ivx}$	$t_{h,NW}$	$t^{trf}_{h,ivx}$	$t^{trf,res}_{h,ivx}$	$t^{rev,PL}_{h,ivx}$	$t_{h,NW}$	$t^{trf}_{h,ivx}$	$t^{trf,res}_{h,ivx}$	$t^{rev,PL}_{h,ivx}$	$t_{h,NW}$	$t^{trf}_{h,ivx}$	$t^{trf,res}_{h,ivx}$	$t^{rev,PL}_{h,ivx}$	T
Australia	0.933	0.973	-1.653*	-0.739	-1.301	-0.750	-2.436**	-0.996	-1.390	-1.053	-2.736***	-1.028	-1.325	-0.912	-2.943***	-1.045	-1.273	-0.801	-2.986***	-1.041	-1.129	-0.705	187
Austria	0.958	0.975	-0.575	-0.791	-0.256	-0.795	-0.758	-1.133	-0.326	-1.191	-0.794	-1.193	-0.280	-1.145	-0.888	-1.134	-0.200	-1.258	-0.989	-1.042	-0.103	-0.859	187
Belgium	0.970	0.968	-0.081	-1.523	-1.528	-1.523	-0.199	-2.342**	-1.915*	-2.375**	-0.217	-2.770***	-1.832*	-2.767***	-0.268	-2.920***	-1.637	-3.089***	-0.228	-2.807***	-1.268	-2.642***	187
Canada	0.951	0.973	-0.178	-0.913	-0.544	-0.920	-0.400	-1.329	-0.644	-1.434	-0.465	-1.480	-0.675	-1.456	-0.572	-1.709*	-0.739	-1.808*	-0.715	-1.949*	-0.783	-1.790*	187
Denmark	0.952	0.965	0.044	-1.096	-0.936	-1.103	-0.086	-1.640	-1.161	-1.609	-0.111	-1.883*	-1.180	-1.926*	-0.147	-1.966*	-1.139	-1.964**	-0.128	-1.943*	-1.038	-1.952*	187
Finland	0.934	0.953	0.288	-0.790	-0.304	-0.798	0.201	-1.249	-0.444	-1.274	0.180	-1.460	-0.462	-1.573	0.186	-1.427	-0.415	-1.436	0.297	-1.168	-0.319	-1.094	187
France	0.962	0.970	0.197	-0.799	-0.718	-0.805	0.036	-1.308	-0.952	-1.195	0.006	-1.529	-0.985	-1.552	-0.012	-1.636	-0.963	-1.576	0.006	-1.592	-0.847	-1.516	187
Germany	0.974	0.978	-0.714	-0.586	-0.358	-0.590	-1.031	-0.884	-0.432	-0.963	-1.148	-0.945	-0.383	-0.968	-1.340	-0.908	-0.306	-1.089	-1.587	-0.855	-0.220	-0.720	187
Hong Kong	0.334	0.969	1.222	-2.049**	-0.875	-1.920*	1.145	-3.031***	-1.345	-1.849*	1.120	-4.793***	-1.765*	-1.575	1.100	-7.138***	-1.914*	0.544	1.060	-10.572***	-1.789*	0.196	157
Ireland	0.910	0.964	-1.975**	-0.679	-1.032	-0.690	-2.410**	-0.977	-1.145	-0.968	-2.887***	-1.112	-1.142	-1.084	-2.988***	-1.178	-1.108	-0.843	-2.991***	-1.065	-0.937	-0.723	187
Israel	0.862	0.996	-3.068***	-0.839	-1.205	-0.882	-2.897***	-1.216	-1.567	-1.480	-2.773***	-1.699*	-1.990**	-2.012**	-2.862***	-2.159**	-2.350**	-2.621***	-3.755***	-3.087***	-2.937***	-3.563***	187
Italy	0.862	0.977	1.134	-0.657	-0.235	-0.667	1.105	-0.849	-0.267	-0.735	1.095	-0.959	-0.266	-0.893	1.079	-1.012	-0.242	-0.557	1.110	-0.993	-0.170	-0.596	187
Japan	0.961	0.992	-1.195	-0.745	-0.495	-0.748	-1.377	-0.939	-0.579	-1.051	-1.654*	-1.012	-0.620	-1.221	-1.932*	-1.005	-0.596	-1.297	-2.189**	-0.835	-0.498	-0.645	187
Luxembourg	0.966	0.967	-0.081	-1.461	-1.206	-1.461	-0.201	-2.256**	-1.505	-2.299**	-0.219	-2.656***	-1.442	-2.661***	-0.271	-2.791***	-1.292	-2.962***	-0.230	-2.691***	-1.004	-2.532**	187
Netherlands	0.972	0.979	-0.666	-0.814	-0.669	-0.819	-0.889	-1.180	-0.770	-1.213	-0.951	-1.256	-0.690	-1.189	-1.082	-1.207	-0.572	-1.334	-1.237	-1.142	-0.426	-0.959	187
New Zealand	0.896	0.974	-1.492	-0.672	-0.977	-0.683	-2.071**	-0.908	-1.069	-1.013	-2.316**	-0.942	-1.051	-0.918	-2.624***	-1.001	-1.033	-0.585	-2.702***	-1.020	-0.950	-0.632	187
Norway	0.952	0.975	0.623	-1.082	-0.477	-1.086	0.504	-1.651*	-0.610	-1.551	0.517	-1.749*	-0.574	-1.722*	0.500	-1.869*	-0.553	-1.649*	0.590	-1.762*	-0.429	-1.604	187
Portugal	0.650	0.987	1.430	-0.808	-0.536	-0.832	1.387	-0.975	-0.629	-0.949	1.335	-1.129	-0.679	-1.197	1.280	-1.233	-0.687	-1.064	1.229	-1.337	-0.636	-0.809	187
Singapore	0.866	0.992	-1.541	-0.033	-0.302	-0.033	-1.958*	-0.018	-0.293	-0.215	-1.963**	-0.025	-0.278	-0.084	-2.271**	-0.059	-0.266	-0.466	-2.661***	-0.169	-0.255	-0.469	187
Spain	0.864	0.980	0.883	-0.577	-0.099	-0.585	0.864	-0.804	-0.144	-0.827	0.863	-0.929	-0.143	-1.027	0.861	-0.978	-0.121	-0.822	0.911	-0.960	-0.071	-0.644	187
Sweden	0.941	0.977	0.782	-0.872	-0.523	-0.880	0.721	-1.255	-0.488	-1.210	0.751	-1.335	-0.435	-1.424	0.777	-1.330	-0.371	-1.252	0.916	-1.194	-0.287	-1.091	187
Switzerland	0.977	0.985	-1.767*	-0.768	-1.089	-0.766	-2.279**	-0.975	-1.059	-0.986	-2.441**	-0.987	-0.879	-0.791	-2.768***	-0.914	-0.680	-0.774	-3.519***	-0.797	-0.386	-0.397	187
United Kingdom	0.928	0.955	-1.522	-0.678	-0.407	-0.688	-1.724*	-0.875	-0.383	-0.864	-2.098**	-0.977	-0.311	-0.917	-2.419**	-0.994	-0.236	-0.644	-2.494**	-0.842	-0.109	-0.445	187
Brazil	0.888	0.997	-3.492***	-0.840	-1.276	-0.884	-3.300***	-1.148	-1.566	-1.323	-3.169***	-1.517	-1.871*	-1.863*	-3.173***	-1.887*	-2.138**	-2.371**	-3.626***	-2.656***	-2.575**	-3.198***	160
Bulgaria	0.947	0.973	-2.069**	-0.640	-0.902	-0.664	-1.971**	-1.190	-1.459	-1.202	-2.563**	-1.571	-1.925*	-1.679*	-3.839***	-1.936*	-2.197**	-1.628	-19.090***	-3.019***	-2.743***	-2.340**	115
Chile	0.277	0.977	-0.798	-2.229**	1.088	-0.520	-0.665	-3.081***	1.559	-0.542	-0.593	-4.259***	1.811*	-0.145	-0.487	-5.092***	1.147	0.229	-0.297	-5.872***	7.092***	0.346	187
China	0.741	0.966	0.890	-0.576	-0.165	-0.589	0.790	-0.647	-0.095	-0.377	0.652	-0.838	-0.041	-0.584	0.559	-0.990	-0.007	-0.774	0.379	-1.429	-0.005	-0.541	136
Colombia	0.468	0.996	2.616***	-0.829	-0.432	-0.850	2.380**	-0.938	-0.500	-0.972	2.136**	-1.049	-0.567	-0.988	1.978**	-1.116	-0.587	-1.072	1.764*	-1.184	-0.573	-1.402	187
Czech Rep.	0.946	0.973	-0.473	-0.747	-1.398	-0.751	-0.527	-1.030	-0.813	-1.032	-0.495	-1.197	-0.752	-1.239	-0.491	-1.463	-0.752	-1.511	-0.589	-2.363**	-0.964	-2.404**	107
Egypt	0.782	1.000	-0.069	0.000	-0.017	0.000	-0.168	0.021	-0.125	-0.165	-0.311	0.015	-0.158	-0.365	-0.436	0.035	-0.205	-0.497	-1.010	-0.459	-0.320	-0.695	185
Greece	0.646	0.991	1.902*	-0.647	-0.028	-0.657	1.856*	-0.741	-0.036	-0.827	1.740*	-0.853	-0.059	-1.019	1.638	-0.941	-0.071	-0.908	1.514	-1.032	-0.059	-1.212	187
Hungary	0.803	0.998	1.855*	-0.015	0.485	-0.015	1.742*	-0.144	0.473	-0.172	1.578	-0.280	0.418	-0.398	1.481	-0.393	0.368	-0.699	1.451	-0.508	0.323	-1.125	176
Iceland	0.522	0.988	0.729	-0.969	-0.575	-1.024	0.472	-1.233	-0.755	-1.352	0.174	-1.501	-0.887	-1.474	-0.010	-1.739*	-0.957	-1.442	-0.137	-2.007**	-0.903	-1.216	176
India	0.754	1.000	3.394***	-0.391	0.225	-0.392	3.218***	-0.502	0.184	-0.495	3.040***	-0.489	0.177	-0.600	2.833***	-0.605	0.115	-0.556	2.637***	-0.636	0.049	-1.046	187
Indonesia	0.919	0.997	2.319**	-0.481	0.037	-0.483	2.507**	-0.602	0.063	-0.671	2.558**	-0.512	0.145	-0.694	2.579***	-0.464	0.140	-0.732	2.597***	-0.427	0.104	-0.912	187
Korea	0.888	0.978	1.322	-0.423	0.076	-0.428	1.438	-0.589	0.064	-0.621	1.449	-0.659	0.037	-0.426	1.473	-0.637	0.037	-0.422	1.623	-0.592	0.028	-0.466	187
Mexico	0.719	0.997	-2.184**	-0.932	-1.392	-0.962	-2.254**	-1.161	-1.611	-1.362	-2.307**	-1.354	-1.780*	-1.821*	-2.384**	-1.497	-1.877*	-2.224**	-2.586***	-1.757*	-1.983**	-2.332**	187
Peru	0.933	0.997	-2.704***	-0.339	-0.724	-0.342	-2.526**	-0.656	-1.025	-0.776	-2.414**	-1.063	-1.347	-1.397	-2.359**	-1.495	-1.659*	-2.070**	-2.417**	-2.104**	-2.027**	-2.893***	187
Philippines	0.726	0.996	1.880*	-0.541	0.010	-0.546	1.814*	-0.689	-0.062	-0.817	1.749*	-0.719	-0.068	-0.925	1.647*	-0.774	-0.102	-0.982	1.495	-0.860	-0.140	-1.264	187
Poland	0.900	0.992	-2.463**	-0.871	-1.275	-0.899	-2.327**	-1.412	-1.629	-1.586	-2.340**	-1.766*	-1.864*	-1.682*	-2.453**	-2.086**	-2.029**	-2.189**	-3.408***	-2.595***	-2.205**	-2.773***	160
Romania	0.633	0.961	-8.930***	-1.842*	-2.761***	-2.216**	-11.361***	-2.243**	-2.895***	-1.211	-14.737***	-2.658***	-2.763***	-0.685	-14.191***	-3.005***	-2.489**	-0.343	-12.696***	-3.629***	-1.586	-0.409	117
Russian Federation	0.048	0.898	0.113	-0.477	-0.454	-0.493	0.531	-0.615	-0.270	-0.117	0.669	-0.619	-0.060	-0.146	0.605	-0.620	-0.102	-0.167	0.121	-1.221	-0.797	-0.256	102
South Africa	0.808	0.997	0.350	-0.351	-0.014	-0.352	0.175	-0.438	-0.080	-0.473	0.037	-0.467	-0.119	-0.553	0.017	-0.453	-0.103	-0.350	-0.043	-0.345	-0.045	-0.479	187
Thailand	0.921	0.988	0.651	-0.317	-0.316	-0.318	0.654	-0.653	-0.241	-0.662	0.707	-0.614	-0.112	-0.668	0.743	-0.605	-0.109	-0.705	0.844	-0.762	-0.196	-1.019	187
Ukraine	0.414	0.980	-1.123	-1.224	1.651*	-0.801	-1.091	-1.799*	1.942*	-0.622	-0.983	-2.745***	1.067	-0.209	-0.815	-2.943***	8.111***	-0.075	-0.732	-3.063***	34.635***	-0.208	106

Notes: Shaded cells indicate statistically significant two-sided test statistics and *, **, *** refer to statistically significant at the 10%, 5% and 1% nominal levels. h corresponds to the prediction horizon, $t_{h,NW}$ is the OLS t-statistic with Newey-West standard errors, $t^{trf}_{h,ivx}$ is the t-statistic computed from the transformed regression, $t^{trf,res}_{h,ivx}$ is the t-statistic computed from a residual augmented transformed regression and $t^{rev,PL}_{h,ivx}$ is the t-statistic computed from a reversed regression as suggested by Phillips and Lee (2013). $\hat{\phi}$ corresponds to the estimates of the contemporaneous correlation computed as indicated in (7.4) and $\hat{\rho}^{RER}$ denotes the estimates of ρ_i^{RER} computed as indicated in (7.3).

Table 5: Nominal exchange rate regression results for sub-period from 1973:Q1 to 2008:Q4

			$h=1$				$h=4$				$h=8$				$h=12$				$h=20$				
	$\hat{\phi}$	$\hat{\rho}^{RER}$	$t_{h,NW}$	$t^{trf}_{h,ivx}$	$t^{trf,res}_{h,ivx}$	$t^{rev,PL}_{h,ivx}$	$t_{h,NW}$	$t^{trf}_{h,ivx}$	$t^{trf,res}_{h,ivx}$	$t^{rev,PL}_{h,ivx}$	$t_{h,NW}$	$t^{trf}_{h,ivx}$	$t^{trf,res}_{h,ivx}$	$t^{rev,PL}_{h,ivx}$	$t_{h,NW}$	$t^{trf}_{h,ivx}$	$t^{trf,res}_{h,ivx}$	$t^{rev,PL}_{h,ivx}$	$t_{h,NW}$	$t^{trf}_{h,ivx}$	$t^{trf,res}_{h,ivx}$	$t^{rev,PL}_{h,ivx}$	T
Australia	0.919	0.974	-1.715*	-0.745	-1.082	-0.757	-2.542**	-1.117	-1.275	-1.101	-2.774***	-1.168	-1.296	-0.960	-2.857***	-1.152	-1.222	-0.830	-2.795***	-1.141	-1.084	-0.712	142
Austria	0.965	0.982	-0.795	-0.778	-0.233	-0.781	-1.106	-0.908	-0.321	-1.224	-1.118	-1.167	-0.323	-1.244	-1.196	-1.228	-0.268	-1.383	-1.395	-1.133	-0.150	-0.948	142
Belgium	0.974	0.979	-0.270	-1.292	-0.384	-1.292	-0.501	-1.990**	-1.589	-2.222**	-0.486	-2.965***	-2.671***	-2.908***	-0.507	-3.590***	-2.961***	-3.414***	-0.509	-3.682***	-2.828***	-3.070***	142
Canada	0.933	0.972	-0.182	-0.564	0.261	-0.570	-0.370	-0.591	0.093	-1.120	-0.440	-0.835	-0.205	-1.179	-0.528	-1.160	-0.473	-1.596	-0.593	-1.414	-0.667	-1.321	142
Denmark	0.950	0.969	-0.136	-0.766	0.085	-0.771	-0.377	-1.158	-0.361	-1.263	-0.373	-1.535	-0.650	-1.691*	-0.375	-1.697*	-0.724	-1.740*	-0.388	-1.705*	-0.674	-1.743*	142
Finland	0.931	0.957	0.098	-0.571	0.359	-0.577	-0.118	-0.972	0.045	-1.065	-0.113	-1.286	-0.186	-1.466	-0.068	-1.289	-0.228	-1.335	0.024	-1.029	-0.115	-0.932	142
France	0.959	0.970	0.031	-0.528	-0.343	-0.533	-0.237	-0.942	-0.589	-0.889	-0.243	-1.258	-0.702	-1.356	-0.229	-1.419	-0.720	-1.368	-0.234	-1.397	-0.625	-1.308	142
Germany	0.978	0.984	-0.791	-0.542	-0.298	-0.545	-1.139	-0.620	-0.380	-0.962	-1.265	-0.867	-0.358	-1.027	-1.460	-0.951	-0.293	-1.174	-1.729*	-0.905	-0.159	-0.780	142
Hong Kong	0.396	0.952	1.215	-2.069**	-1.680*	-1.959*	1.124	-3.111***	-2.140**	-1.921*	1.089	-4.996***	-2.648***	-1.677*	1.059	-7.588***	-2.998***	0.772	1.041	-11.643***	-3.084***	0.373	112
Ireland	0.918	0.959	-1.734*	-0.518	-0.779	-0.526	-1.907*	-0.876	-0.931	-0.811	-2.493**	-1.042	-0.959	-0.974	-2.702***	-1.104	-0.911	-0.708	-2.753***	-0.983	-0.738	-0.587	142
Israel	0.844	0.987	-3.049***	-0.806	-1.175	-0.843	-2.881***	-1.189	-1.506	-1.464	-2.757***	-1.588	-1.820*	-2.033**	-2.845***	-1.940*	-2.039**	-2.690***	-3.736***	-2.675***	-2.349**	-3.750***	142
Italy	0.855	0.972	0.948	-0.597	0.053	-0.607	0.765	-0.892	-0.095	-0.683	0.810	-0.997	-0.161	-0.888	0.859	-1.014	-0.158	-0.521	0.937	-0.973	-0.108	-0.568	142
Japan	0.962	0.998	-1.441	-0.500	-0.451	-0.501	-1.598	-0.868	-0.593	-0.806	-1.890*	-1.026	-0.658	-0.966	-2.222**	-0.999	-0.605	-1.020	-2.643***	-0.686	-0.419	-0.315	142
Luxembourg	0.970	0.979	-0.271	-1.289	-0.400	-1.288	-0.504	-1.968**	-1.490	-2.212**	-0.489	-2.860***	-2.343**	-2.844***	-0.511	-3.394***	-2.558**	-3.311***	-0.513	-3.505***	-2.439**	-2.981***	142
Netherlands	0.977	0.985	-0.772	-0.773	-0.695	-0.777	-1.039	-0.934	-0.809	-1.222	-1.105	-1.225	-0.760	-1.276	-1.233	-1.307	-0.640	-1.451	-1.419	-1.257	-0.424	-1.057	142
New Zealand	0.870	0.975	-1.291	-0.701	-0.791	-0.714	-1.974**	-1.066	-0.933	-1.092	-2.281**	-1.155	-0.960	-1.040	-2.547**	-1.214	-0.928	-0.672	-2.573**	-1.255	-0.842	-0.756	142
Norway	0.938	0.969	0.160	-1.091	-0.331	-1.101	-0.192	-1.307	-0.461	-1.530	-0.126	-1.378	-0.457	-1.721	-0.089	-1.513	-0.447	-1.638	0.041	-1.400	-0.299	-1.529	142
Portugal	0.633	0.980	1.241	-0.773	-0.404	-0.797	1.047	-1.068	-0.524	-0.933	1.052	-1.188	-0.545	-1.234	1.068	-1.235	-0.519	-1.101	1.082	-1.295	-0.459	-0.825	142
Singapore	0.858	0.992	-1.456	0.013	-0.313	0.013	-1.914*	0.273	-0.342	-0.190	-1.889*	0.083	-0.421	-0.051	-2.177**	-0.114	-0.474	-0.498	-2.549**	-0.398	-0.464	-0.502	142
Spain	0.868	0.976	0.696	-0.497	0.057	-0.503	0.538	-0.845	-0.091	-0.775	0.588	-0.967	-0.114	-1.032	0.645	-0.978	-0.090	-0.818	0.723	-0.939	-0.046	-0.622	142
Sweden	0.937	0.976	0.561	-0.778	0.182	-0.785	0.271	-1.192	0.004	-1.105	0.317	-1.326	-0.082	-1.410	0.397	-1.340	-0.083	-1.266	0.588	-1.176	-0.014	-1.080	142
Switzerland	0.976	0.979	-1.703*	-0.745	-0.520	-0.746	-2.209**	-0.814	-0.513	-1.017	-2.352**	-0.958	-0.415	-0.835	-2.651***	-0.956	-0.294	-0.815	-3.407***	-0.850	-0.115	-0.418	142
United Kingdom	0.911	0.944	-1.327	-0.605	-0.374	-0.615	-1.225	-0.896	-0.428	-0.817	-1.405	-1.035	-0.454	-0.918	-1.592	-1.081	-0.448	-0.636	-1.385	-0.914	-0.332	-0.413	142
Brazil	0.857	0.989	-3.491***	-0.609	-1.045	-0.631	-3.299***	-0.891	-1.280	-1.073	-3.168***	-1.137	-1.449	-1.619	-3.172***	-1.347	-1.541	-2.138**	-3.626***	-1.723*	-1.581	-3.018***	115
Bulgaria	0.947	0.966	-2.063**	-0.549	-0.813	-0.565	-1.968**	-1.110	-1.326	-1.126	-2.560**	-1.380	-1.643	-1.646*	-3.837***	-1.595	-1.711*	-1.627	-19.267***	-2.341**	-1.787*	-2.501	70
Chile	0.266	0.932	-0.862	-2.386**	3.681***	-0.954	-0.766	-3.404***	6.976***	-0.943	-0.684	-4.727***	15.517***	-0.283	-0.579	-5.658***	-161.630***	0.165	-0.396	-6.437***	-4.197***	0.297	142
China	0.741	0.964	0.879	-0.558	0.273	-0.571	0.837	-0.590	0.375	-0.338	0.888	-0.648	0.467	-0.562	0.916	-0.673	0.534	-0.768	0.932	-0.808	0.526	-0.436	91
Colombia	0.382	0.995	2.391**	-0.571	-0.310	-0.583	1.966**	-0.936	-0.427	-0.731	1.849*	-0.888	-0.343	-0.764	1.882*	-0.757	-0.202	-0.875	2.077**	-0.487	0.016	-1.290	142
Czech Rep.	0.945	0.980	-0.861	0.201	0.693	0.200	-1.080	0.501	0.753	-0.019	-0.905	0.051	0.582	-0.287	-0.808	-0.356	0.426	-0.442	-0.990	-1.166	0.094	-0.642	62
Egypt	0.825	0.997	-1.392	-0.112	-0.705	-0.112	-1.584	-0.209	-0.861	-0.295	-1.765*	-0.311	-0.876	-0.616	-1.950*	-0.314	-0.847	-0.852	-2.287**	-0.136	-0.605	-1.210	142
Greece	0.643	0.990	1.700*	-0.563	-0.084	-0.571	1.468	-0.814	-0.159	-0.764	1.436	-0.857	-0.161	-0.998	1.433	-0.854	-0.135	-0.893	1.441	-0.863	-0.104	-1.249	142
Hungary	0.797	1.001	1.282	0.235	0.648	0.234	0.968	-0.206	0.476	0.021	0.920	-0.286	0.423	-0.280	0.946	-0.274	0.419	-0.663	0.974	-0.409	0.322	-1.205	131
Iceland	0.508	0.977	0.722	-0.937	-0.531	-0.990	0.095	-1.684*	-0.749	-1.382	-0.327	-2.123**	-0.925	-1.570	-0.466	-2.427**	-0.985	-1.595	-0.597	-2.937***	-1.020	-1.415	131
India	0.749	1.001	2.500**	-0.105	0.288	-0.105	2.083**	-0.586	0.219	-0.248	1.869*	-0.691	0.164	-0.402	1.812*	-0.695	0.185	-0.404	1.750*	-0.684	0.201	-0.974	142
Indonesia	0.937	0.999	2.221**	-0.261	0.034	-0.262	2.280**	-0.495	0.072	-0.430	2.315**	-0.408	0.157	-0.422	2.420**	-0.313	0.148	-0.441	2.911***	-0.200	0.114	-0.596	142
Korea	0.904	0.978	1.500	-0.376	0.109	-0.379	1.392	-0.891	0.029	-0.617	1.274	-1.082	-0.046	-0.461	1.232	-1.096	-0.056	-0.492	1.453	-1.000	-0.028	-0.548	142
Mexico	0.724	0.994	-2.241**	-0.641	-1.158	-0.655	-2.320**	-1.009	-1.355	-1.077	-2.368**	-1.178	-1.457	-1.568	-2.444**	-1.265	-1.484	-2.005**	-2.691***	-1.370	-1.422	-2.163**	142
Peru	0.912	0.995	-2.704***	-0.099	-0.497	-0.099	-2.526**	-0.374	-0.690	-0.510	-2.413**	-0.681	-0.869	-1.094	-2.358**	-0.986	-1.021	-1.725*	-2.416	-1.321	-1.117	-2.501**	142
Philippines	0.735	0.995	1.891*	-0.317	0.222	-0.318	1.603	-0.670	0.094	-0.603	1.638	-0.675	0.106	-0.722	1.835*	-0.573	0.149	-0.785	2.652***	-0.272	0.246	-1.057	142
Poland	0.902	0.987	-2.462**	-0.740	-1.201	-0.759	-2.328**	-1.392	-1.579	-1.484	-2.339**	-1.678*	-1.732*	-1.623	-2.453**	-1.912*	-1.786*	-2.206**	-3.415***	-2.255**	-1.735*	-2.948***	115
Romania	0.724	0.949	-10.616***	-1.961**	-3.130***	-2.328**	-13.655***	-2.445**	-3.234***	-1.283	-16.132***	-2.751***	-2.906***	-0.762	-14.351***	-2.895***	-2.230**	-0.407	-14.006***	-3.029***	-0.007	-0.496	72
Russian Federation	0.016	0.868	-0.839	-0.351	-0.610	-0.369	-1.158	-0.548	-0.503	0.064	-1.444	-0.487	-0.289	0.031	-1.652*	-0.375	-0.129	0.005	-1.814*	-1.013	-0.305	-0.133	57
South Africa	0.779	0.998	-0.234	-0.188	-0.164	-0.188	-0.639	-0.536	-0.273	-0.333	-0.880	-0.646	-0.310	-0.440	-0.962	-0.646	-0.294	-0.237	-1.183	-0.637	-0.233	-0.403	142
Thailand	0.923	0.988	0.925	-0.260	-0.297	-0.261	0.878	-0.648	-0.237	-0.606	0.942	-0.587	-0.095	-0.605	1.087	-0.465	-0.062	-0.628	1.508	-0.461	-0.072	-0.939	142
Ukraine	0.493	0.853	-4.168***	-0.951	-17.346***	-1.179	-6.633***	-1.761*	-6.725***	-1.111	-9.569***	-3.191***	-2.749***	0.071	-11.958***	-3.578***	-0.929	0.225	-16.613***	-3.917***	-0.904	0.002	62

Notes: See Notes to Table 4.

Table 6: Nominal exchange rate regression results for sub-period from 1990:Q1 to 2008:Q4.

			$h=1$				$h=4$				$h=8$				$h=12$				$h=20$				
	$\hat{\phi}$	$\hat{\rho}^{RER}$	$t_{h,NW}$	$t_{h,ivx}^{trf}$	$t_{h,ivx}^{trf,res}$	$t_{h,ivx}^{rev,PL}$	$t_{h,NW}$	$t_{h,ivx}^{trf}$	$t_{h,ivx}^{trf,res}$	$t_{h,ivx}^{rev,PL}$	$t_{h,NW}$	$t_{h,ivx}^{trf}$	$t_{h,ivx}^{trf,res}$	$t_{h,ivx}^{rev,PL}$	$t_{h,NW}$	$t_{h,ivx}^{trf}$	$t_{h,ivx}^{trf,res}$	$t_{h,ivx}^{rev,PL}$	$t_{h,NW}$	$t_{h,ivx}^{trf}$	$t_{h,ivx}^{trf,res}$	$t_{h,ivx}^{rev,PL}$	T
Australia	0.953	0.932	-0.354	-1.051	-0.386	-1.058	-0.688	-1.672*	-1.098	-1.701*	-0.789	-2.188**	-1.501	-2.048**	-0.851	-2.225**	-1.467	-2.003**	-1.036	-2.657***	-1.542	-2.401**	75
Austria	0.962	0.954	-0.334	-0.827	-0.223	-0.829	-0.442	-0.790	-0.882	-1.183	-0.482	-1.471	-1.598	-1.442	-0.372	-1.863*	-1.828*	-1.455	-0.357	-2.841***	-1.390	-1.984**	75
Belgium	0.970	0.949	-0.315	-0.852	-0.202	-0.854	-0.432	-0.966	-0.666	-1.291	-0.466	-1.667*	-1.245	-1.571	-0.354	-2.075**	-1.561	-1.584	-0.343	-3.126***	-2.009**	-2.089**	75
Canada	0.974	0.974	-0.214	-0.281	0.254	-0.281	-0.319	0.293	0.218	-0.567	-0.322	0.182	0.016	-0.694	-0.412	-0.462	-0.280	-0.811	-0.579	-1.484	-1.211	-1.115	75
Denmark	0.968	0.952	-0.357	-0.929	-0.466	-0.932	-0.478	-1.009	-1.057	-1.344	-0.548	-1.673*	-1.563	-1.605	-0.446	-2.117**	-1.683*	-1.654*	-0.481	-3.056***	-1.681*	-2.174**	75
Finland	0.967	0.953	0.131	-0.627	0.164	-0.630	0.023	-1.023	-0.207	-1.426	-0.059	-1.783*	-0.431	-1.673*	-0.088	-1.734*	-0.329	-1.164	-0.227	-1.583	-0.233	-0.830	75
France	0.976	0.958	-0.365	-0.758	-0.259	-0.762	-0.465	-0.650	-0.665	-1.046	-0.527	-1.274	-1.030	-1.243	-0.422	-1.764*	-1.240	-1.253	-0.439	-3.159***	-1.871*	-1.562	75
Germany	0.974	0.959	-0.483	-0.749	-0.425	-0.751	-0.518	-0.622	-1.210	-1.060	-0.609	-1.221	-1.441	-1.257	-0.529	-1.613	-1.308	-1.260	-0.596	-2.730***	-0.975	-1.722*	75
Hong Kong	0.058	0.988	-0.299	1.528	0.806	1.542	0.110	1.524	0.892	1.765*	0.420	1.965**	1.016	1.829*	0.577	2.471**	1.109	1.840*	1.224	2.414**	1.151	1.722*	75
Ireland	0.959	0.937	-0.546	-1.458	-1.942*	-1.481	-0.452	-1.632	-2.887***	-1.996**	-0.523	-1.990**	-2.690***	-2.180**	-0.796	-2.304**	-2.170**	-2.256**	-0.924	-2.322**	-1.360	-2.230**	75
Israel	0.690	0.950	0.925	-0.334	0.298	-0.340	0.667	-0.531	0.289	-0.491	0.810	-0.504	0.336	-0.283	0.977	-0.406	0.356	0.021	1.149	-0.365	0.257	-0.094	75
Italy	0.948	0.946	0.319	-0.583	0.133	-0.589	0.209	-0.664	0.087	-1.037	0.219	-0.851	0.066	-1.035	0.273	-0.955	0.089	-0.503	0.153	-1.019	0.149	-0.386	75
Japan	0.948	0.951	-1.079	-0.760	-0.623	-0.753	-0.902	-1.147	-0.255	-0.569	-0.846	-1.468	-0.116	-0.675	-0.815	-1.416	-0.047	-0.520	-0.821	-1.094	-0.133	-0.034	75
Luxembourg	0.965	0.947	-0.320	-0.986	-0.435	-0.989	-0.437	-1.141	-1.054	-1.455	-0.469	-1.841*	-1.582	-1.733*	-0.356	-2.212**	-1.821*	-1.735*	-0.343	-3.092***	-1.943*	-2.240**	75
Netherlands	0.975	0.955	-0.512	-1.015	-1.105	-1.018	-0.588	-1.103	-2.300**	-1.448	-0.681	-1.782*	-2.366**	-1.741*	-0.598	-2.158**	-1.957*	-1.811*	-0.658	-2.884***	-1.276	-2.530**	75
New Zealand	0.970	0.953	-0.267	-0.741	-0.467	-0.746	-0.579	-1.173	-1.469	-1.584	-0.812	-1.819*	-2.060**	-2.154**	-1.047	-2.711***	-2.447**	-2.444**	-1.301	-4.178***	-2.616***	-2.685***	75
Norway	0.948	0.933	-0.015	-0.932	-0.143	-0.935	-0.293	-0.826	-0.628	-1.258	-0.356	-1.180	-0.814	-1.335	-0.280	-1.594	-0.773	-1.319	-0.345	-2.372**	-0.730	-1.666*	75
Portugal	0.931	0.948	0.016	-0.463	0.248	-0.467	-0.095	-0.675	0.173	-1.058	-0.073	-0.747	0.220	-0.998	0.057	-0.716	0.269	-0.812	0.000	-0.825	0.269	-0.714	75
Singapore	0.908	0.952	-0.960	-0.363	0.016	-0.364	-0.928	-0.113	-0.198	-0.321	-0.838	-0.629	-0.182	-0.408	-0.741	-0.982	-0.144	-0.542	-0.569	-1.550	-0.522	-0.338	75
Spain	0.936	0.950	0.311	-0.582	0.177	-0.590	0.219	-0.830	0.111	-1.182	0.229	-0.984	0.087	-1.173	0.308	-0.942	0.118	-0.798	0.212	-1.070	0.100	-0.588	75
Sweden	0.939	0.928	0.359	-1.076	0.093	-1.091	0.145	-1.514	-0.198	-1.945*	0.141	-1.646*	-0.215	-2.001**	0.229	-1.460	-0.097	-1.284	0.100	-1.333	0.014	-1.037	75
Switzerland	0.980	0.976	-0.743	-0.745	-1.328	-0.747	-0.654	-0.551	-1.398	-0.847	-0.873	-1.193	-1.151	-1.060	-0.893	-1.507	-0.870	-1.056	-1.004	-2.379**	-0.287	-0.627	75
United Kingdom	0.945	0.922	-0.496	-1.309	-0.788	-1.317	-0.317	-1.384	-1.057	-1.524	0.023	-1.189	-1.430	-1.507	0.078	-1.447	-1.386	-1.345	0.481	-2.017**	0.036	-1.398	75
Brazil	0.882	0.940	-3.750***	-0.959	-1.799*	-1.123	-4.488***	-1.703*	-2.509**	-1.582	-8.074***	-2.709***	-3.243***	-2.027**	-22.284***	-3.804***	-3.743***	-1.663*	-387.600***	-5.334***	-3.484***	0.221	75
Bulgaria	0.947	0.966	-2.063**	-0.549	-0.813	-0.565	-1.968*	-1.110	-1.326	-1.126	-2.560**	-1.380	-1.643	-1.646*	-3.837***	-1.595	-1.711*	-1.627	-19.267***	-2.341**	-1.787*	-2.501**	70
Chile	0.826	0.950	1.430	-0.432	-0.187	-0.436	0.983	-0.916	-0.107	-0.334	0.922	-0.994	-0.046	-0.363	0.936	-1.068	-0.014	-0.354	1.265	-0.932	0.040	-0.316	75
China	0.800	0.945	0.547	-0.844	0.004	-0.865	0.525	-1.105	0.206	-0.802	0.591	-1.545	0.425	-1.143	0.664	-1.943*	0.597	-1.367	0.692	-3.165***	0.724	1.801*	75
Colombia	0.574	0.970	1.919*	-0.549	-0.116	-0.563	1.485	-0.991	-0.192	-0.443	1.403	-1.008	-0.134	-0.278	1.474	-0.926	-0.050	-0.250	1.822*	-0.722	0.016	-0.578	75
Czech Rep.	0.945	0.980	-0.861	0.201	0.693	0.200	-1.080	0.501	0.753	-0.019	-0.905	0.051	0.582	-0.287	-0.808	-0.356	0.426	-0.442	-0.990	-1.166	0.094	-0.642	62
Egypt	0.593	0.910	-0.409	-2.573**	-1.837*	-0.997	-0.075	-3.627***	-1.324	-0.520	0.365	-4.000	-0.940	-0.284	0.654	-3.873***	-3.154***	-0.307	1.482	-2.991***	-11.346***	-0.232	75
Greece	0.758	0.944	0.726	-0.301	0.273	-0.305	0.552	-0.571	0.356	-0.553	0.502	-0.689	0.408	-0.403	0.525	-0.732	0.430	-0.230	0.477	-0.836	0.410	-0.257	75
Hungary	0.628	0.966	1.116	-0.461	0.242	-0.478	0.816	-0.937	0.112	-0.596	0.743	-1.074	0.062	-0.693	0.753	-1.148	0.042	-0.853	0.717	-1.486	-0.053	-0.887	75
Iceland	0.981	1.046	0.953	0.089	-0.018	0.089	0.747	-2.424**	-0.789	-1.016	0.548	-2.267**	-0.917	-1.278	0.583	-2.416**	-0.949	-1.262	0.427	-1.966**	-0.753	-1.105	75
India	0.671	0.958	2.027**	-1.327	-0.414	-1.348	1.659*	-2.171**	-0.455	-1.504	1.463	-2.358**	-0.402	-0.416	1.448	-2.537**	-0.283	-0.476	1.432	-2.741***	-0.084	-0.405	75
Indonesia	0.942	0.988	1.480	-0.590	0.007	-0.595	1.453	-1.074	-0.147	-0.962	1.459	-1.096	-0.125	-1.032	1.521	-1.035	-0.100	-1.112	1.793*	-1.104	-0.090	-1.546	75
Korea	0.963	0.960	1.001	-0.522	-0.084	-0.535	0.808	-1.238	-0.198	-1.044	0.663	-1.481	-0.222	-1.185	0.599	-1.426	-0.183	-1.076	0.756	-1.382	-0.155	-1.121	75
Mexico	0.713	0.978	1.976*	-0.866	0.219	-0.887	1.740*	-1.527	0.217	-1.047	1.946*	-1.663*	0.245	-1.281	2.046**	-1.648*	0.249	-1.486	1.162	-1.723*	0.179	-0.572	75
Peru	0.440	0.666	-2.314**	-2.806***	1.019	-0.275	-2.432**	-4.968	0.997	0.244	-2.419**	-6.669***	0.790	0.449	-2.421**	-9.224***	4.392***	0.599	-2.691***	-12.428***	22.023***	0.641	75
Philippines	0.862	0.975	1.340	-0.419	-0.035	-0.424	0.938	-0.920	-0.157	-0.248	0.972	-0.769	-0.037	-0.395	1.184	-0.594	0.032	-0.614	1.900*	-0.196	0.060	-0.960	75
Poland	0.399	0.945	-1.180	-0.224	0.030	-0.229	-1.557	-0.677	-0.234	-0.609	-1.607	-0.893	-0.382	-0.484	-1.720*	-1.070	-0.454	-0.290	-2.261**	-1.317	-0.421	-0.110	75
Romania	0.724	0.949	-10.616***	-1.961**	-3.130***	-2.328**	-13.655***	-2.445**	-3.234***	-1.283	-16.132***	-2.751***	-2.906***	-0.762	-14.351***	-2.895***	-2.230**	-0.407	-14.006***	-3.029***	-0.007	-0.496	72
Russian Federation	0.016	0.868	-0.839	-0.351	-0.610	-0.369	-1.158	-0.548	-0.503	0.064	-1.444	-0.487	-0.289	0.031	-1.652*	-0.375	-0.129	0.005	-1.814*	-1.013	-0.305	-0.133	57
South Africa	0.887	0.984	1.087	-0.270	0.310	-0.273	0.826	-0.692	0.287	-0.522	0.733	-0.838	0.258	-0.571	0.728	-0.891	0.245	-0.575	0.751	-0.934	0.188	-0.589	75
Thailand	0.955	0.970	0.539	-0.272	0.093	-0.274	0.489	-0.812	0.004	-0.859	0.544	-0.846	0.053	-0.955	0.678	-0.784	0.076	-1.062	1.025	-1.112	-0.020	-1.921*	75
Ukraine	0.493	0.853	-4.168***	-0.951	-17.346***	-1.179	-6.633***	-1.761*	-6.725***	-1.111	-9.569***	-3.191***	-2.749***	0.071	-11.958***	-3.578***	-0.929	0.225	-16.613***	-3.917***	-0.904	0.002	62

Notes: See Notes to Table 4.

Table 7: Nominal exchange rate regression results for sub-period from 1999:Q1 to 2020:Q1.

			$h=1$				$h=4$				$h=8$				$h=12$				$h=20$				
	$\hat{\phi}$	$\hat{\rho}^{RER}$	$t_{h,NW}$	$t^{trf}_{h,ivx}$	$t^{trf,res}_{h,ivx}$	$t^{rev,PL}_{h,ivx}$	$t_{h,NW}$	$t^{trf}_{h,ivx}$	$t^{trf,res}_{h,ivx}$	$t^{rev,PL}_{h,ivx}$	$t_{h,NW}$	$t^{trf}_{h,ivx}$	$t^{trf,res}_{h,ivx}$	$t^{rev,PL}_{h,ivx}$	$t_{h,NW}$	$t^{trf}_{h,ivx}$	$t^{trf,res}_{h,ivx}$	$t^{rev,PL}_{h,ivx}$	$t_{h,NW}$	$t^{trf}_{h,ivx}$	$t^{trf,res}_{h,ivx}$	$t^{rev,PL}_{h,ivx}$	T
Australia	0.973	0.958	-0.609	-0.692	-1.775*	-0.692	-1.049	-1.102	-2.550**	-1.086	-1.946*	-1.219	-2.316**	-1.388	-3.393***	-1.190	-2.126**	-1.437	-8.985***	-1.208	-1.991**	-0.373	84
Austria	0.960	0.959	-0.169	-0.528	-0.355	-0.532	-0.308	-0.876	-0.543	-0.986	-0.603	-0.929	-0.539	-1.155	-0.713	-0.871	-0.447	-1.288	-0.533	-0.930	-0.403	-0.267	84
Belgium	0.963	0.956	-0.148	-0.550	-0.774	-0.554	-0.276	-0.917	-0.495	-1.026	-0.563	-0.951	-0.401	-1.175	-0.671	-0.876	-0.268	-1.292	-0.491	-0.928	-0.239	-0.257	84
Canada	0.984	0.964	-1.080	-0.454	-2.216**	-0.457	-1.344	-0.637	-2.551**	-0.623	-1.812*	-0.634	-2.331**	-0.726	-2.913***	-0.738	-2.303**	-0.959	-8.884***	-0.961	-2.227**	-0.684	84
Denmark	0.961	0.956	-0.185	-0.630	-0.815	-0.636	-0.339	-0.983	-0.515	-1.108	-0.646	-0.997	-0.423	-1.263	-0.755	-0.914	-0.280	-1.371	-0.572	-0.967	-0.274	-0.282	84
Finland	0.959	0.960	-0.211	-0.533	-0.322	-0.537	-0.371	-0.856	-0.474	-0.986	-0.682	-0.881	-0.422	-1.155	-0.796	-0.819	-0.317	-1.293	-0.618	-0.900	-0.328	-0.261	84
France	0.963	0.957	-0.212	-0.595	-0.723	-0.601	-0.368	-0.903	-0.481	-1.044	-0.678	-0.916	-0.427	-1.209	-0.790	-0.847	-0.326	-1.345	-0.606	-0.916	-0.299	-0.299	84
Germany	0.967	0.959	-0.421	-0.540	-1.028	-0.545	-0.699	-0.841	-0.740	-0.978	-1.199	-0.865	-0.659	-1.151	-1.417	-0.805	-0.519	-1.291	-1.278	-0.880	-0.469	-0.274	84
Hong Kong	0.001	0.943	0.210	0.422	0.268	0.423	0.665	0.359	0.421	0.176	0.710	0.406	0.477	0.130	0.607	0.398	0.465	0.110	0.256	0.306	0.359	0.042	84
Ireland	0.951	0.952	-0.584	-1.316	-1.217	-1.325	-0.830	-1.786*	-1.313	-1.905*	-0.583	-1.768*	-1.235	-2.054**	-0.295	-1.611	-1.111	-2.178**	-0.457	-1.589	-1.132	-0.608	84
Israel	0.947	0.967	-0.691	-0.674	-0.993	-0.676	-0.928	-1.100	-0.221	-1.051	-0.811	-0.849	-0.020	-0.795	-1.019	-0.332	0.156	-0.622	-1.876*	-0.853	-0.178	-0.964	84
Italy	0.961	0.954	-0.125	-0.688	-0.837	-0.694	-0.239	-1.038	-0.523	-1.162	-0.518	-1.048	-0.449	-1.304	-0.628	-0.960	-0.316	-1.422	-0.448	-1.020	-0.302	-0.319	84
Japan	0.939	0.962	-0.284	-0.589	-0.559	-0.595	-0.113	-0.818	-0.337	-0.575	-0.096	-0.997	-0.250	-0.899	-0.235	-1.028	-0.179	-1.189	-0.251	-0.903	-0.090	-0.685	84
Luxembourg	0.959	0.955	-0.149	-0.627	-0.396	-0.632	-0.276	-0.988	-0.571	-1.096	-0.560	-1.018	-0.564	-1.241	-0.668	-0.938	-0.481	-1.359	-0.488	-0.986	-0.472	-0.279	84
Netherlands	0.966	0.959	-0.388	-0.710	-1.341	-0.717	-0.634	-1.056	-1.002	-1.173	-1.030	-1.071	-0.876	-1.312	-1.174	-0.991	-0.682	-1.429	-1.011	-1.054	-0.553	-0.346	84
New Zealand	0.972	0.953	-0.697	-1.015	-1.566	-1.020	-1.176	-1.437	-1.853*	-1.583	-2.082**	-1.293	-1.488	-1.738*	-3.178***	-1.159	-1.240	-1.746*	-4.637***	-0.945	-0.960	-0.259	84
Norway	0.969	0.977	0.317	-0.124	-0.288	-0.124	0.148	-0.522	-0.538	-0.470	-0.094	-0.581	-0.461	-0.548	-0.192	-0.573	-0.380	-0.522	-0.046	-0.683	-0.397	-0.088	84
Portugal	0.941	0.951	-0.142	-0.949	-0.908	-0.959	-0.260	-1.315	-1.021	-1.433	-0.539	-1.300	-0.933	-1.543	-0.645	-1.165	-0.771	-1.621	-0.464	-1.195	-0.755	-0.407	84
Singapore	0.880	0.970	-1.132	0.027	0.149	0.027	-1.135	0.058	0.404	0.013	-1.387	0.068	0.365	-0.190	-1.847*	0.087	0.387	-0.460	-2.638***	-0.104	0.290	-0.443	84
Spain	0.916	0.945	-0.137	-0.878	-0.821	-0.884	-0.255	-1.299	-0.884	-1.401	-0.533	-1.306	-0.889	-1.520	-0.640	-1.187	-0.755	-1.623	-0.459	-1.204	-0.759	-0.367	84
Sweden	0.958	0.953	0.172	-0.640	-1.205	-0.643	0.133	-1.042	-0.753	-1.068	-0.088	-1.031	-0.593	-1.216	-0.319	-0.778	-0.329	-1.122	-0.263	-0.792	-0.295	-0.232	84
Switzerland	0.938	0.985	-1.118	-0.501	-3.237***	-0.505	-1.692*	-0.639	-3.033***	-0.856	-3.822***	-0.643	-2.779***	-1.033	-8.865***	-0.575	-2.462**	-1.168	-7.524***	-0.551	-1.855*	-0.325	84
United Kingdom	0.975	0.969	-0.722	-0.617	-2.481**	-0.619	-0.935	-0.834	-1.111	-0.868	-0.933	-0.774	-0.632	-0.769	-0.922	-0.650	-0.427	-0.649	-1.306	-0.679	-0.226	-0.433	84
Brazil	0.962	0.994	1.124	-0.342	-0.065	-0.340	0.853	-0.779	0.130	-0.546	0.736	-0.932	0.080	-0.482	0.504	-1.227	0.013	-0.336	0.353	-1.405	-0.271	-0.128	84
Bulgaria	0.832	0.918	-0.370	-0.886	-0.443	-0.891	-0.626	-1.478	-0.128	-1.277	-0.723	-1.511	-0.117	-1.025	-0.674	-1.297	-0.001	-0.733	-0.469	-1.032	-0.052	0.010	84
Chile	0.977	0.979	0.934	-0.462	-0.441	-0.459	0.757	-1.229	-0.215	-0.698	0.641	-1.736*	-0.232	-0.784	0.462	-2.090**	-0.225	-0.655	0.292	-2.152**	-0.196	-0.119	84
China	0.530	0.962	-0.958	0.500	0.439	0.490	-0.959	0.362	0.585	0.286	-1.071	0.380	0.610	0.086	-1.179	0.344	0.550	-0.099	-1.603	0.164	0.436	-0.559	84
Colombia	0.965	0.977	1.224	-0.637	-0.443	-0.622	1.028	-1.099	-0.431	-0.621	0.847	-1.386	-0.305	-0.511	0.754	-1.500	-0.072	-0.506	0.577	-1.511	-1.630	-0.288	84
Czech Rep.	0.946	0.960	-0.943	-0.426	-0.695	-0.431	-1.024	-0.578	-0.308	-0.641	-1.188	-0.588	-0.284	-0.773	-1.186	-0.605	-0.241	-0.792	-0.965	-0.869	-0.311	-0.458	84
Egypt	0.943	1.014	1.681*	0.152	0.397	0.152	2.299**	0.226	0.324	-0.065	2.935***	0.396	0.208	-0.063	2.821***	0.561	0.016	0.068	2.878***	0.105	-0.009	0.337	82
Greece	0.865	0.932	-0.021	-1.066	-1.219	-1.072	-0.144	-1.593	-1.158	-1.690*	-0.463	-1.601	-1.090	-1.854*	-0.597	-1.426	-0.897	-1.941*	-0.426	-1.373	-0.958	-0.418	84
Hungary	0.968	0.974	0.516	-0.724	-0.661	-0.719	0.429	-1.056	0.082	-0.727	0.169	-1.010	0.157	-0.322	0.063	-0.746	0.185	0.058	0.251	-0.696	0.097	0.117	84
Iceland	0.971	0.981	0.733	-0.747	0.233	-0.750	0.732	-1.750*	-0.192	-1.746*	0.638	-2.031**	-0.394	-1.552	0.478	-1.985**	-0.374	-1.049	0.891	-1.792*	-0.284	-1.643	84
India	0.850	1.007	2.069**	0.426	0.217	0.425	2.121**	0.249	0.193	0.319	2.055**	0.445	0.288	0.593	1.847*	0.419	0.243	0.725	1.877*	0.523	0.081	0.293	84
Indonesia	0.912	0.988	1.410	-0.491	0.415	-0.486	1.779*	-0.576	0.373	-0.831	1.989**	-0.382	0.338	0.033	1.936*	-0.190	0.234	0.319	2.285**	0.066	0.092	-0.163	84
Korea	0.930	0.887	-0.027	-2.159**	-1.324	-2.170**	-0.018	-2.506**	-2.807***	-2.519**	-0.120	-1.971**	-1.856*	-1.910*	-0.390	-1.493	-1.201	-1.440	-0.498	-1.139	-0.698	-1.010	84
Mexico	0.951	0.997	2.124**	-0.174	0.038	-0.175	2.484**	-0.177	0.212	-0.213	3.264***	-0.021	0.165	-0.150	4.315***	0.091	0.093	-0.290	4.764***	0.345	0.120	0.079	84
Peru	0.931	0.984	0.073	0.418	0.077	0.418	-0.121	-0.084	0.491	-0.044	-0.230	-0.270	0.593	-0.229	-0.312	-0.576	0.454	-0.421	-0.377	-0.954	0.173	-0.864	84
Philippines	0.852	0.937	0.737	-1.019	-0.491	-1.004	0.682	-1.428	-0.546	-1.162	0.576	-1.421	-0.204	-0.652	0.414	-1.474	-0.203	-0.539	0.084	-1.660*	-0.829	-0.360	84
Poland	0.950	0.931	-0.302	-0.713	-1.846*	-0.716	-0.466	-0.832	-0.394	-0.834	-0.534	-0.684	-0.176	-0.639	-0.501	-0.590	-0.062	-0.573	-0.432	-0.917	-0.144	-0.795	84
Romania	0.496	0.919	0.524	-0.955	-1.725*	-0.917	0.312	-1.248	-1.276	-0.801	0.246	-1.470	-0.701	-0.436	0.347	-1.514	-0.173	-0.243	0.822	-1.321	-0.759	-0.124	84
Russian Federation	0.942	0.990	1.479	-0.062	0.585	-0.062	1.547	-0.113	0.508	0.044	1.745*	-0.090	0.369	-0.035	1.813*	-0.048	0.206	0.074	1.962**	0.268	0.425	0.051	84
South Africa	0.981	0.997	1.214	-0.432	-0.018	-0.430	1.212	-0.756	-0.033	-0.688	1.260	-0.896	-0.033	-0.512	1.290	-0.891	-0.009	0.040	1.620	-0.335	0.011	0.002	84
Thailand	0.954	0.980	-0.601	-0.335	-0.425	-0.335	-0.799	-0.702	-0.054	-0.815	-1.052	-1.085	-0.059	-1.527	-1.348	-1.124	-0.078	-1.602	-1.406	-1.424	-0.502	-1.362	84
Ukraine	0.848	1.002	1.443	0.000	0.434	0.000	1.649	0.044	0.360	0.130	2.128**	0.155	0.299	-0.114	3.090***	0.385	0.207	-0.216	6.550***	0.886	0.858	-0.181	83

Notes: See Notes to Table 4.

Table 8: Relative price regression results for sample from 1973:Q1 to 2020:Q2.

		$h=1$				$h=4$				$h=8$				$h=12$				$h=20$				
	$\hat{\phi}$	$t_{h,NW}$	$t^{trf}_{h,ivx}$	$t^{trf,res}_{h,ivx}$	$t^{rev,PL}_{h,ivx}$	$t_{h,NW}$	$t^{trf}_{h,ivx}$	$t^{trf,res}_{h,ivx}$	$t^{rev,PL}_{h,ivx}$	$t_{h,NW}$	$t^{trf}_{h,ivx}$	$t^{trf,res}_{h,ivx}$	$t^{rev,PL}_{h,ivx}$	$t_{h,NW}$	$t^{trf}_{h,ivx}$	$t^{trf,res}_{h,ivx}$	$t^{rev,PL}_{h,ivx}$	$t_{h,NW}$	$t^{trf}_{h,ivx}$	$t^{trf,res}_{h,ivx}$	$t^{rev,PL}_{h,ivx}$	T
Australia	0.124	-0.910	-0.334	-0.564	-0.336	-1.100	-0.689	-0.814	-0.583	-1.256	-0.985	-0.996	-0.448	-1.454	-1.181	-1.090	-0.314	-1.677*	-1.406	-1.142	-0.140	187
Austria	0.088	-2.351**	0.007	0.173	0.007	-2.358**	-0.201	-0.093	-0.295	-2.388**	-0.583	-0.470	-0.684	-2.578***	-0.972	-0.833	-1.327	-2.871***	-1.570	-1.288	-1.691*	187
Belgium	0.139	-0.912	2.760***	2.952***	2.750***	-0.977	1.966**	2.120**	1.826*	-1.250	0.688	0.685	0.261	-1.753*	-0.715	-0.705	-1.471	-3.008***	-2.324**	-2.158**	-3.074***	187
Canada	0.143	-0.355	0.381	0.197	0.381	-0.559	-0.056	-0.018	-0.103	-0.661	-0.231	-0.062	-0.194	-0.704	-0.324	-0.051	-0.039	-0.809	-0.492	-0.195	0.060	187
Denmark	0.096	0.460	1.928*	1.938*	1.892*	0.319	1.299	1.465	1.412	0.235	0.904	1.056	1.086	0.205	0.550	0.698	0.800	0.052	-0.014	0.142	0.434	187
Finland	0.093	0.415	-0.129	-0.153	-0.130	0.337	-0.675	-0.567	-0.431	0.224	-1.273	-1.087	-0.420	0.092	-1.713*	-1.412	-0.219	-0.084	-1.730*	-1.344	0.267	187
France	0.151	0.144	2.187**	2.077**	2.154**	0.061	1.642	1.617	1.761*	-0.031	1.032	0.996	1.315	-0.126	0.301	0.287	0.850	-0.276	-0.883	-0.812	0.084	187
Germany	0.084	-4.688***	-1.211	-1.362	-1.253	-4.753***	-1.605	-1.655*	-1.555	-4.868***	-2.032**	-2.019**	-1.813*	-5.246***	-2.456**	-2.316**	-2.211**	-5.608***	-3.030***	-2.623***	-2.257**	187
Hong Kong	0.403	1.297	0.101	0.229	0.100	1.144	-0.106	0.125	0.002	0.972	-0.419	-0.052	-0.113	0.855	-0.645	-0.185	-0.160	0.740	-0.991	-0.357	-0.605	156
Ireland	0.122	-2.356**	0.451	0.265	0.443	-2.428**	0.000	-0.196	0.216	-2.468**	-0.602	-0.715	0.360	-2.452**	-1.066	-1.106	0.377	-2.158**	-1.505	-1.401	0.206	187
Israel	0.851	-3.040***	-0.734	-1.022	-0.769	-2.749***	-1.122	-1.409	-1.209	-2.612***	-1.654*	-1.881*	-1.885*	-2.700***	-2.173**	-2.290**	-2.687***	-3.502***	-3.299***	-3.004***	-4.400***	187
Italy	0.119	2.486**	-0.512	-0.457	-0.539	2.236**	-0.870	-0.772	-0.715	2.004**	-1.304	-1.048	-0.689	1.844*	-1.742*	-1.290	-0.616	1.610	-2.397**	-1.474	-0.494	187
Japan	0.032	-2.941***	2.172**	1.157	2.120**	-3.552***	1.875*	0.805	0.782	-4.460***	1.473	0.409	-0.167	-5.816***	1.147	0.149	-1.086	-10.475***	0.843	-0.027	-1.799*	187
Luxembourg	0.133	-1.203	2.412**	2.144**	2.387**	-1.244	1.241	1.050	1.196	-1.439	-0.276	-0.454	-0.425	-1.923*	-1.678*	-1.676*	-2.256**	-3.006***	-2.910***	-2.617***	-3.288***	187
Netherlands	0.076	-2.245**	0.348	0.076	0.345	-2.329**	-0.252	-0.393	-0.297	-2.656***	-0.867	-1.012	-1.173	-3.467***	-1.485	-1.555	-2.409**	-5.320***	-2.482**	-2.327**	-3.286***	187
New Zealand	0.110	-0.548	0.387	0.178	0.382	-0.861	0.072	-0.081	-0.023	-1.054	-0.166	-0.301	-0.137	-1.248	-0.444	-0.567	-0.005	-1.542	-0.955	-1.033	-0.129	187
Norway	0.159	1.025	0.834	0.979	0.825	1.011	0.568	0.759	0.424	0.986	0.144	0.391	0.061	0.904	-0.240	0.111	-0.065	0.765	-0.881	-0.377	-0.554	187
Portugal	0.202	2.558**	-0.625	-0.342	-0.659	2.300**	-0.844	-0.519	-0.592	2.069**	-1.171	-0.677	-1.017	1.898*	-1.478	-0.794	-1.442	1.643	-1.980**	-0.908	-0.485	187
Singapore	0.257	-4.348***	-2.019**	-1.498	-2.109**	-9.081***	-2.105**	-1.455	-2.906***	-9.297***	-2.161**	-1.385	-2.483**	-9.040***	-2.209**	-1.301	-1.928*	-8.512***	-2.281**	-1.137	-1.640	187
Spain	0.130	2.650**	-0.835	-0.625	-0.876	2.445**	-1.071	-0.838	-1.141	2.248**	-1.477	-1.070	-1.340	2.101**	-1.850*	-1.241	-1.130	1.879*	-2.313**	-1.307	-0.130	187
Sweden	0.143	0.418	0.224	0.340	0.223	0.411	0.096	0.282	-0.043	0.376	-0.154	0.039	-0.321	0.302	-0.492	-0.234	-0.328	0.179	-0.765	-0.422	-0.097	187
Switzerland	0.102	-3.416***	-0.500	-0.538	-0.502	-3.747***	-0.700	-0.684	-0.779	-3.947***	-1.005	-0.918	-1.159	-4.270***	-1.359	-1.070	-1.251	-4.627***	-1.797*	-0.959	-0.996	187
United Kingdom	0.068	-2.548**	-0.854	-0.886	-0.885	-2.458**	-1.233	-1.108	-0.976	-2.405**	-1.657*	-1.220	-0.519	-2.448**	-1.939*	-1.166	-0.191	-2.562**	-2.278**	-0.993	0.088	187
Brazil	0.889	-3.245***	-0.798	-1.185	-0.840	-3.025***	-1.112	-1.481	-1.263	-2.920***	-1.503	-1.794*	-1.862*	-2.959***	-1.892*	-2.062**	-2.480**	-3.530***	-2.749***	-2.525**	-3.694***	159
Bulgaria	0.942	-3.094***	-0.681	-1.078	-0.709	-2.748***	-1.411	-1.796*	-1.366	-3.270***	-1.941*	-2.351**	-1.706*	-4.784***	-2.376**	-2.629***	-1.779*	-26.919***	-3.702***	-3.189***	-2.478**	115
Chile	0.226	-0.738	-1.823*	137.640***	-0.559	-0.627	-2.787***	-11.940***	-0.625	-0.560	-4.382***	-1.928*	-0.266	-0.474	-6.111***	-2.529**	0.312	-0.282	-8.682***	-8.628***	0.641	187
China	0.288	1.393	-0.564	-0.246	-0.573	1.240	-0.940	-0.453	-0.765	1.141	-1.366	-0.638	-0.885	1.063	-1.661*	-0.733	-0.470	1.020	-2.115**	-0.839	-1.301	135
Colombia	0.189	3.542***	-1.440	-1.056	-1.582	3.113***	-1.516	-1.086	-1.747*	2.731***	-1.612	-1.086	-1.894*	2.469**	-1.700*	-1.065	-2.260**	2.126**	-1.893*	-1.016	-2.280**	187
Czech Rep.	0.209	1.823*	0.869	0.762	0.864	1.578	0.360	0.427	0.472	1.372	-0.021	0.248	0.186	1.276	-0.298	0.170	0.028	1.230	-0.705	0.089	-0.418	108
Egypt	0.141	0.283	0.539	0.461	0.542	0.125	0.516	0.377	0.334	-0.116	0.356	0.264	-0.028	-0.325	0.141	0.124	-0.183	-0.658	-0.476	-0.227	-0.446	185
Greece	0.231	2.663***	-0.803	-0.295	-0.842	2.357**	-0.852	-0.368	-0.527	2.085**	-0.998	-0.424	-0.852	1.897*	-1.151	-0.472	-1.032	1.665*	-1.413	-0.540	-1.548	187
Hungary	0.436	2.997***	0.781	1.020	0.762	2.644***	0.583	0.817	0.324	2.343**	0.284	0.635	-0.374	2.154**	-0.012	0.480	-1.289	1.952*	-0.537	0.212	-3.532***	175
Iceland	0.320	1.014	-1.603	-1.702*	-1.964**	0.712	-2.057**	-2.010**	-2.598***	0.465	-2.598***	-2.268**	-2.775***	0.284	-3.172***	-2.469**	-3.181***	0.068	-4.400***	-2.584***	-2.788***	175
India	0.386	5.730***	1.527	1.957*	1.510	5.488***	1.492	1.872*	1.697*	5.020***	1.387	1.728*	1.590	4.936***	1.368	1.645*	0.835	5.413***	1.429	1.457	0.382	187
Indonesia	0.377	4.289***	-0.008	0.386	-0.008	4.517***	-0.334	0.156	0.026	4.514***	-0.446	0.152	-0.109	4.434***	-0.364	0.229	0.061	4.286***	-0.194	0.305	-0.075	187
Korea	0.281	2.810***	-2.289**	-1.850*	-2.312**	2.649***	-2.799***	-2.084**	-2.139**	2.535**	-3.129***	-2.078**	-1.811*	2.457**	-3.235***	-1.916*	-1.407	2.448**	-3.232***	-1.447	-1.189	187
Mexico	0.600	-2.586***	-0.685	-1.048	-0.709	-2.406**	-1.013	-1.352	-1.163	-2.304**	-1.403	-1.646*	-1.902*	-2.302**	-1.748*	-1.857*	-2.669***	-2.554**	-2.407**	-2.162**	-3.967***	187
Peru	0.928	-2.573**	-0.307	-0.666	-0.310	-2.339**	-0.670	-0.955	-0.800	-2.186**	-1.152	-1.271	-1.508	-2.130**	-1.631	-1.553	-2.285**	-2.215**	-2.318**	-1.895*	-3.521***	187
Philippines	0.390	3.839***	-0.896	-0.693	-0.924	3.773***	-1.369	-0.966	-0.950	3.568**	-1.679*	-1.063	-1.621	3.299***	-1.797*	-1.030	-1.909*	2.917***	-1.892*	-0.927	-2.582***	187
Poland	0.872	-2.295**	-0.521	-0.954	-0.535	-2.161**	-1.119	-1.337	-1.303	-2.107**	-1.685*	-1.651*	-1.441	-2.164**	-2.200**	-1.869*	-2.092**	-2.888***	-3.099***	-2.131**	-3.886***	159
Romania	0.393	-8.395***	-1.519	-2.305**	-2.162**	-11.743***	-2.087**	-2.657***	-1.617	-14.099***	-2.759***	-2.845***	-1.196	-13.423***	-3.278***	-2.716***	-0.090	-12.107***	-4.306***	-2.029**	-0.426	116
Russian Federation	0.016	-0.700	-2.619***	-1.839*	-1.327	-0.576	-3.916***	-1.329	-0.820	-0.289	-4.330***	-0.465	0.264	-0.081	-4.310***	-0.248	0.271	-0.088	-5.796***	-1.652*	0.360	104
South Africa	0.077	0.972	-0.486	-0.563	-0.495	0.606	-0.665	-0.726	-1.009	0.295	-0.840	-0.818	-1.416	0.098	-0.952	-0.840	-1.675*	-0.165	-1.057	-0.743	-2.296*	187
Thailand	0.139	1.152	-0.979	-0.502	-0.977	0.947	-1.613	-0.778	-0.546	0.920	-1.684*	-0.645	-1.051	1.062	-1.514	-0.425	-1.188	1.434	-0.924	0.140	-0.778	187
Ukraine	0.382	-1.499	-2.406**	1.809*	-0.836	-1.419	-3.987***	1.876*	-0.599	-1.381	-6.713***	0.904	-0.422	-1.355	-9.029***	10.102***	-0.143	-1.425	-10.990***	51.700***	-0.097	107

Notes: See Notes to Table 4.

Table 9: Relative price regression results for sample from 1973:Q1 to 2008:Q4.

		$h=1$				$h=4$				$h=8$				$h=12$				$h=20$				
	$\hat{\phi}$	$t_{h,NW}$	$t^{trf}_{h,ivx}$	$t^{trf,res}_{h,ivx}$	$t^{rev,PL}_{h,ivx}$	$t_{h,NW}$	$t^{trf}_{h,ivx}$	$t^{trf,res}_{h,ivx}$	$t^{rev,PL}_{h,ivx}$	$t_{h,NW}$	$t^{trf}_{h,ivx}$	$t^{trf,res}_{h,ivx}$	$t^{rev,PL}_{h,ivx}$	$t_{h,NW}$	$t^{trf}_{h,ivx}$	$t^{trf,res}_{h,ivx}$	$t^{rev,PL}_{h,ivx}$	$t_{h,NW}$	$t^{trf}_{h,ivx}$	$t^{trf,res}_{h,ivx}$	$t^{rev,PL}_{h,ivx}$	T
Australia	0.130	-0.959	-0.385	-0.581	-0.387	-1.153	-0.834	-0.933	-0.683	-1.321	-1.196	-1.207	-0.539	-1.547	-1.419	-1.336	-0.378	-1.975**	-1.763*	-1.507	-0.234	142
Austria	0.077	-2.473**	0.276	0.461	0.274	-2.587***	0.408	0.363	-0.011	-2.697***	0.003	0.000	-0.414	-3.031***	-0.417	-0.311	-1.082	-3.519***	-1.152	-0.665	-1.496	142
Belgium	0.120	-0.880	3.350***	3.637***	3.307***	-1.010	3.607***	3.348***	2.479**	-1.354	2.297**	1.960**	0.982	-1.963**	0.414	0.187	-0.820	-3.897***	-2.333**	-2.056**	-2.847***	142
Canada	0.136	-0.309	0.494	0.452	0.493	-0.520	0.205	0.191	0.011	-0.609	-0.081	0.078	-0.078	-0.621	-0.180	0.078	0.225	-0.644	-0.342	-0.163	0.474	142
Denmark	0.082	0.881	1.790*	1.832*	1.743*	0.629	1.348	1.471	1.339	0.498	0.816	0.957	0.965	0.445	0.362	0.472	0.657	0.284	-0.453	-0.430	0.287	142
Finland	0.081	0.762	-0.387	-0.522	-0.390	0.581	-0.992	-1.072	-0.769	0.439	-1.770*	-1.863*	-0.789	0.295	-2.313**	-2.371**	-0.505	0.126	-2.502**	-2.566**	0.097	142
France	0.139	0.532	1.906*	1.763*	1.861*	0.359	1.384	1.292	1.436	0.258	0.578	0.477	0.898	0.175	-0.315	-0.465	0.377	0.053	-1.759*	-2.044**	-0.426	142
Germany	0.075	-4.525***	-0.950	-1.051	-0.980	-4.645***	-1.097	-1.141	-1.266	-4.807***	-1.611	-1.436	-1.591	-5.252***	-2.122**	-1.655*	-2.078**	-5.597***	-2.872***	-1.698*	-2.183**	142
Hong Kong	0.304	0.919	0.616	0.796	0.575	0.743	0.233	0.429	0.405	0.608	-0.142	-0.008	0.287	0.517	-0.408	-0.354	0.252	0.444	-0.786	-0.833	-0.210	111
Ireland	0.122	-3.605***	-0.334	-0.633	-0.338	-3.953***	-0.934	-1.237	-0.700	-4.414***	-1.667*	-1.877*	-0.629	-4.603***	-2.227**	-2.310**	-0.642	-4.343***	-2.787***	-2.658***	-0.902	142
Israel	0.835	-3.022***	-0.696	-1.030	-0.726	-2.734***	-1.062	-1.342	-1.181	-2.599***	-1.530	-1.680*	-1.883*	-2.686***	-1.957*	-1.934*	-2.719***	-3.488***	-2.841***	-2.308**	-4.536***	142
Italy	0.119	2.962***	-0.874	-0.804	-0.946	2.600***	-1.358	-1.193	-1.207	2.335**	-1.872*	-1.520	-1.279	2.164**	-2.347**	-1.744*	-1.273	1.938*	-3.041***	-1.862*	-1.221	142
Japan	0.020	-2.465**	2.747***	2.538**	2.730***	-3.090***	3.053***	2.629***	1.319	-3.916***	2.561**	2.121**	0.331	-5.214***	2.169**	1.710**	-0.657	-10.848***	1.645*	1.172	-1.439	142
Luxembourg	0.131	-1.114	3.074***	2.908***	3.016***	-1.221	2.814***	2.218**	1.921*	-1.434	0.861	0.416	0.237	-1.988**	-1.000	-1.111	-1.669*	-3.601***	-2.860***	-2.387**	-3.039***	142
Netherlands	0.063	-2.173**	0.849	0.681	0.828	-2.295**	0.880	0.481	0.251	-2.636***	-0.051	-0.320	-0.736	-3.517***	-1.037	-1.045	-2.192**	-5.700***	-2.831***	-2.008**	-3.328***	142
New Zealand	0.120	-0.554	0.174	-0.016	0.172	-0.874	-0.179	-0.294	-0.284	-1.087	-0.438	-0.526	-0.438	-1.288	-0.705	-0.770	-0.286	-1.634	-1.234	-1.231	-0.442	142
Norway	0.149	0.889	0.926	1.138	0.910	0.739	0.759	0.989	0.503	0.709	0.308	0.557	0.223	0.673	-0.110	0.192	0.121	0.631	-0.786	-0.469	-0.373	142
Portugal	0.207	2.937***	-0.876	-0.525	-0.937	2.605***	-1.146	-0.670	-0.902	2.354**	-1.451	-0.783	-1.441	2.171**	-1.715*	-0.854	-1.986**	1.911*	-2.095**	-0.865	-0.940	142
Singapore	0.277	-4.268***	-1.749*	-1.195	-1.827*	-9.897***	-1.711*	-1.026	-2.905***	-10.953***	-1.495	-0.692	-2.461**	-11.187***	-1.454	-0.417	-1.816*	-11.135***	-1.562	-0.187	-1.536	142
Spain	0.143	3.225***	-1.222	-1.047	-1.304	2.891***	-1.629	-1.313	-1.691*	2.650***	-2.153**	-1.614	-2.022**	2.487**	-2.590***	-1.786*	-1.837*	2.253**	-3.184***	-1.835*	-0.666	142
Sweden	0.157	0.921	0.041	0.075	0.041	0.753	-0.276	-0.022	-0.357	0.696	-0.594	-0.323	-0.729	0.618	-0.946	-0.604	-0.735	0.543	-1.131	-0.732	-0.439	142
Switzerland	0.086	-3.542***	-0.374	-0.360	-0.380	-3.882***	-0.331	-0.373	-0.695	-4.075***	-0.702	-0.601	-1.163	-4.369***	-1.263	-0.826	-1.369	-4.586***	-2.170**	-0.759	-1.207	142
United Kingdom	0.063	-2.583***	-0.656	-0.739	-0.676	-2.415**	-1.176	-1.207	-0.829	-2.427**	-1.718*	-1.776*	-0.411	-2.521**	-2.081**	-2.128**	-0.084	-2.841***	-2.507***	-2.705***	0.202	142
Brazil	0.858	-3.244***	-0.590	-0.904	-0.612	-3.024***	-0.838	-1.106	-1.036	-2.919***	-1.113	-1.268	-1.634	-2.959***	-1.357	-1.372	-2.253**	-3.530***	-1.829*	-1.455	-3.500***	114
Bulgaria	0.943	-3.077***	-0.658	-1.259	-0.683	-2.736***	-1.349	-1.923*	-1.390	-3.259***	-1.771*	-2.247**	-1.786*	-4.775***	-2.045**	-2.273**	-1.900*	-27.689***	-2.856***	-2.218**	-2.683***	70
Chile	0.217	-0.776	-1.984**	71.715*	-1.060	-0.677	-3.005***	-25.174***	-1.139	-0.616	-4.701***	-9.709***	-0.638	-0.539	-6.582***	-3.566***	0.119	-0.369	-9.368***	-6.019***	0.522	142
China	0.289	1.136	-0.591	-0.248	-0.603	1.018	-0.941	-0.462	-0.832	0.901	-1.395	-0.665	-0.968	0.784	-1.734*	-0.809	-0.466	0.709	-2.172**	-0.874	-1.496	90
Colombia	0.178	3.481***	-0.884	-0.568	-0.931	3.010***	-0.938	-0.539	-1.096	2.638***	-0.911	-0.452	-1.242	2.408**	-0.856	-0.380	-1.602	2.225**	-0.627	-0.180	-1.607	142
Czech Rep.	0.189	1.914*	0.215	0.083	0.215	1.649*	-0.478	-0.895	-0.285	1.378	-1.392	-1.873*	-0.830	1.254	-2.190**	-2.782***	-1.124	1.266	-3.275***	-4.427***	-0.903	63
Egypt	0.097	-2.049**	-0.112	-0.489	-0.112	-2.056**	-0.361	-0.608	-0.298	-2.162**	-0.717	-0.884	-0.722	-2.207**	-0.930	-0.992	-0.908	-2.402**	-1.185	-1.069	-1.272	142
Greece	0.234	3.275***	-0.806	-0.220	-0.842	2.842***	-0.890	-0.240	-0.511	2.500**	-0.983	-0.260	-0.891	2.272**	-1.064	-0.285	-1.108	2.006**	-1.173	-0.294	-1.732*	142
Hungary	0.523	3.212***	2.073**	2.068**	1.957*	2.815***	1.944*	2.035**	1.569	2.481**	1.757*	1.836*	0.956	2.256**	1.538	1.587	0.175	2.087**	1.338	1.298	-1.731*	130
Iceland	0.332	0.732	-1.378	-1.423	-1.624	0.353	-1.940*	-1.718*	-2.250**	0.092	-2.512**	-2.006**	-2.475**	-0.090	-3.089***	-2.233**	-2.963***	-0.324	-4.353***	-2.554**	-2.744***	130
India	0.403	4.003***	1.321	1.589	1.301	3.662***	1.046	1.503	1.479	3.181***	0.965	1.469	1.406	2.929***	0.899	1.356	0.663	2.722***	0.815	1.136	0.185	142
Indonesia	0.381	4.399***	0.442	0.676	0.442	4.813***	0.036	0.418	0.467	5.014***	-0.109	0.368	0.353	5.149***	-0.022	0.404	0.532	5.548***	0.033	0.320	0.430	142
Korea	0.312	2.981***	-2.322**	-1.865*	-2.332**	2.755***	-3.010***	-2.105**	-2.202**	2.612***	-3.420***	-2.100**	-1.915*	2.535**	-3.537***	-1.927*	-1.513	2.616***	-3.446***	-1.306	-1.311	142
Mexico	0.612	-2.600***	-0.225	-0.592	-0.227	-2.420**	-0.522	-0.852	-0.662	-2.317**	-0.808	-1.061	-1.365	-2.317**	-1.026	-1.175	-2.097**	-2.590***	-1.352	-1.245	-3.360***	142
Peru	0.908	-2.573***	-0.040	-0.463	-0.040	-2.338**	-0.352	-0.710	-0.504	-2.186**	-0.738	-0.953	-1.169	-2.130**	-1.090	-1.145	-1.896*	-2.215**	-1.482	-1.278	-3.059***	142
Philippines	0.403	3.487***	-0.518	-0.370	-0.526	3.353***	-1.062	-0.655	-0.570	3.107***	-1.422	-0.775	-1.223	2.903***	-1.446	-0.696	-1.504	2.612***	-1.358	-0.523	-2.138**	142
Poland	0.880	-2.285**	-0.401	-0.889	-0.409	-2.150**	-0.953	-1.221	-1.188	-2.096**	-1.417	-1.450	-1.355	-2.153**	-1.799*	-1.574	-2.034**	-2.881***	-2.322**	-1.585	-3.898***	114
Romania	0.468	-9.465***	-1.716*	-2.597***	-2.255**	-14.881***	-2.232**	-2.791***	-1.845*	-17.040***	-2.749***	-2.642***	-1.540	-14.760***	-3.000***	-2.016**	-0.350	-13.882***	-3.243***	-0.110	-0.790	71
Russian Federation	0.001	-1.021	-2.741***	-3.081***	-2.029**	-0.942	-4.199***	-2.942***	-1.022	-0.756	-4.468***	-1.926*	0.139	-0.664	-4.159***	-0.897	0.151	-0.904	-5.128***	-0.921	0.024	59
South Africa	0.073	-0.388	0.157	-0.183	0.156	-0.746	-0.247	-0.518	-0.324	-1.001	-0.552	-0.767	-0.726	-1.136	-0.739	-0.873	-1.004	-1.270	-0.740	-0.755	-1.667*	142
Thailand	0.148	1.451	-1.072	-0.588	-1.067	1.271	-1.774*	-0.857	-0.686	1.142	-2.037**	-0.831	-1.218	1.253	-1.969**	-0.676	-1.358	1.419	-1.754*	-0.365	-0.961	142
Ukraine	0.436	-7.005***	-2.084**	-161.510***	-2.097**	-8.414***	-3.648***	-25.364***	-0.526	-8.896***	-6.625***	-12.599***	-0.328	-9.578***	-9.653***	-5.475***	-0.086	-12.795***	-12.276***	-3.349***	-0.061	63

Notes: See Notes to Table 4.

Table 10: Relative price regression results for sample from 1990:Q1 to 2008:Q4.

		$h=1$				$h=4$				$h=8$				$h=12$				$h=20$				
	$\hat{\phi}$	$t_{h,NW}$	$t^{trf}_{h,ivx}$	$t^{trf,res}_{h,ivx}$	$t^{rev,PL}_{h,ivx}$	$t_{h,NW}$	$t^{trf}_{h,ivx}$	$t^{trf,res}_{h,ivx}$	$t^{rev,PL}_{h,ivx}$	$t_{h,NW}$	$t^{trf}_{h,ivx}$	$t^{trf,res}_{h,ivx}$	$t^{rev,PL}_{h,ivx}$	$t_{h,NW}$	$t^{trf}_{h,ivx}$	$t^{trf,res}_{h,ivx}$	$t^{rev,PL}_{h,ivx}$	$t_{h,NW}$	$t^{trf}_{h,ivx}$	$t^{trf,res}_{h,ivx}$	$t^{rev,PL}_{h,ivx}$	T
Australia	0.195	0.146	0.734	1.483	0.732	0.079	1.574	2.147**	0.528	0.156	1.846*	2.283**	0.636	0.205	1.584	1.848*	0.338	0.166	0.967	1.511	-0.370	75
Austria	0.092	-2.119**	-0.016	0.123	-0.016	-2.447**	0.770	0.824	-0.086	-2.574**	0.719	0.972	-0.125	-3.157***	0.676	1.119	-0.159	-6.326***	-0.315	0.476	-0.326	75
Belgium	0.165	-3.354***	-0.538	-0.213	-0.539	-6.340***	0.141	0.396	-0.464	-8.338***	0.587	0.996	-0.452	-10.138***	0.718	1.238	-0.428	-15.678***	0.292	1.343	-0.651	75
Canada	0.141	-2.156**	0.346	0.512	0.346	-2.230**	1.109	0.935	0.222	-2.386**	1.159	1.329	0.290	-2.558**	1.083	1.771*	0.245	-3.060***	0.762	2.826***	0.084	75
Denmark	0.139	-2.440**	0.352	0.517	0.352	-2.788***	1.513	1.453	0.252	-2.741***	1.421	1.393	0.106	-2.815***	0.723	0.732	-0.191	-3.544***	-0.894	-0.241	-0.693	75
Finland	0.183	-2.995***	-0.324	-0.253	-0.325	-3.647***	-0.009	0.196	-0.535	-4.521***	-0.725	-0.520	-0.975	-5.719***	-1.411	-1.220	-1.400	-9.655***	-1.595	-1.345	-1.663*	75
France	0.163	-5.105***	-0.110	0.077	-0.110	-5.771***	1.030	1.203	0.006	-6.664***	1.696*	1.785*	0.158	-8.402***	1.904*	1.962**	0.152	-14.084***	1.630	2.202**	-0.014	75
Germany	0.198	-1.964**	-0.072	-0.205	-0.072	-1.944*	0.198	0.252	-0.329	-2.277**	-0.105	0.021	-0.378	-2.933***	-0.499	-0.136	-0.485	-5.320***	-1.549	-0.473	-0.544	75
Hong Kong	0.974	0.142	1.172	1.524	1.126	0.031	0.531	0.728	0.634	-0.093	-0.309	-0.149	0.109	-0.196	-1.094	-1.036	-0.387	-0.444	-2.712***	-3.605***	-1.357	75
Ireland	0.187	-0.039	2.459**	3.052***	2.449**	-0.135	2.960***	3.529***	2.011**	-0.331	2.356**	2.756***	1.742*	-0.425	1.683*	1.916*	0.960	-0.654	1.274	1.633	0.203	75
Israel	0.373	1.206	-1.461	-1.410	-1.757	0.933	-1.945*	-1.658*	-1.782*	0.761	-2.278**	-1.721*	-1.190	0.701	-2.471**	-1.584	-1.077	0.660	-2.821***	-1.072	-0.753	75
Italy	0.127	1.010	-0.160	0.011	-0.160	0.753	-0.276	-0.102	-0.369	0.641	-0.754	-0.570	-0.057	0.601	-1.425	-1.291	-0.331	0.641	-2.366**	-2.277**	-0.556	75
Japan	0.008	-6.959***	0.402	0.429	0.400	-7.649***	0.911	0.617	0.289	-8.534***	0.681	0.291	0.008	-8.758***	0.535	0.118	-0.158	-9.238***	0.542	0.380	-0.199	75
Luxembourg	0.100	-2.294**	0.252	0.471	0.252	-2.818**	0.995	1.122	0.171	-2.934***	1.071	1.270	0.176	-3.425***	0.983	1.283	0.152	-4.845***	0.790	1.297	0.023	75
Netherlands	0.182	-0.914	1.395	1.605	1.395	-0.970	3.560***	3.562***	1.056	-1.093	2.728***	2.733***	0.889	-1.304	1.676*	1.719*	0.507	-1.978**	-0.576	-0.281	-0.274	75
New Zealand	0.172	-1.805*	-0.598	-0.574	-0.597	-2.058	-0.172	0.208	-0.690	-2.248**	0.554	0.959	-0.360	-2.537**	0.865	1.283	-0.066	-2.676***	1.551	2.065**	-0.236	75
Norway	0.242	-1.867*	-0.127	0.442	-0.127	-2.484	0.880	1.197	-0.092	-2.470**	0.734	1.008	-0.204	-2.513**	0.275	0.617	-0.375	-3.017***	-0.359	0.206	-0.723	75
Portugal	0.069	1.824*	-1.043	-0.840	-1.024	1.615	-1.370	-1.225	-0.528	1.604	-2.017**	-1.919*	-0.260	1.680*	-2.379**	-2.270**	0.051	2.051**	-2.753***	-2.566**	0.339	75
Singapore	0.176	-3.499***	-1.012	-1.096	-1.085	-3.866***	-0.374	-0.383	-0.970	-4.083***	0.391	0.095	-0.888	-4.093***	1.109	0.198	-0.758	-4.728***	1.898*	1.022	-0.430	75
Spain	0.057	2.990***	-0.557	-0.317	-0.554	2.613***	-0.621	-0.437	-0.603	2.539**	-0.745	-0.571	-0.209	2.706***	-0.864	-0.729	-0.305	3.539***	-0.949	-0.770	-0.031	75
Sweden	0.152	-1.429	-0.641	-0.452	-0.641	-1.998**	-0.681	-0.211	-0.424	-2.841***	-1.156	-0.840	-0.579	-3.721***	-1.842*	-1.624	-0.734	-8.284***	-2.378**	-2.379**	-0.966	75
Switzerland	0.113	-2.801***	0.128	0.284	0.128	-2.884***	1.077	0.926	0.116	-3.633***	1.248	1.066	0.057	-4.832***	1.183	1.025	-0.142	-8.428***	1.408	1.506	-0.358	75
United Kingdom	0.204	0.284	-0.558	-0.081	-0.559	0.846	-0.189	0.010	-0.673	1.466	-0.185	0.147	-0.516	1.842*	-0.365	-0.012	-0.571	1.887*	-1.732	-1.132	-0.828	75
Brazil	0.880	-3.440***	-0.858	-1.670*	-0.990	-4.073***	-1.566	-2.408**	-1.610	-7.031***	-2.684***	-3.268***	-2.458**	-17.106***	-4.102***	-3.986***	-2.304**	-217.360***	-7.361***	-4.187***	-0.259	75
Bulgaria	0.943	-3.077***	-0.658	-1.055	-0.683	-2.736***	-1.349	-1.693*	-1.390	-3.259***	-1.771*	-2.075**	-1.786*	-4.775***	-2.045**	-2.131**	-1.900*	-27.689***	-2.856***	-2.115**	-2.683***	70
Chile	0.250	2.517**	-2.618***	-2.627***	-3.000***	2.182**	-3.594***	-2.856***	-2.677***	1.925*	-4.537***	-2.926***	-1.903*	1.778*	-5.353***	-2.830***	-1.620	1.735*	-6.362***	-1.638	-0.773	75
China	0.367	0.867	0.138	0.253	0.137	0.821	-0.378	-0.146	-0.685	0.722	-1.191	-0.640	-1.659	0.643	-1.939*	-1.069	-2.170**	0.526	-2.887***	-1.432	-0.745	75
Colombia	0.033	3.267***	-2.332**	-2.442**	-3.231***	2.847***	-2.781***	-2.525**	-3.183***	2.536**	-3.150***	-2.365**	-2.766***	2.362**	-3.418***	-2.002**	-2.447**	2.351**	-3.674***	-0.776	-1.839*	75
Czech Rep.	0.189	1.914*	0.215	0.083	0.215	1.649*	-0.478	-0.895	-0.285	1.378	-1.392	-1.873*	-0.830	1.254	-2.190**	-2.782***	-1.124	1.266	-3.275***	-4.427***	-0.903	63
Egypt	0.062	3.822***	0.052	-0.035	0.052	3.471***	-0.493	-0.691	-0.207	4.100***	-1.195	-0.925	0.134	4.518***	-1.649*	-4.314***	0.124	4.416***	-2.462**	-37.086***	0.136	75
Greece	0.156	2.374**	-1.208	-0.999	-1.303	2.078**	-1.449	-1.211	-1.075	1.898*	-1.770*	-1.308	-0.675	1.813*	-2.017**	-1.315	-0.438	1.841*	-2.360**	-1.190	-0.225	75
Hungary	0.302	3.073***	-2.007**	-1.348	-2.523**	2.695***	-2.340**	-1.362	-2.005**	2.376**	-2.778***	-1.351	-1.839*	2.169**	-3.249***	-1.298	-1.734*	2.091**	-4.071***	-1.029	-1.746*	75
Iceland	0.595	2.034**	0.977	0.566	1.002	2.171**	-1.074	-0.166	-0.060	2.124**	-1.116	-0.173	0.017	2.113**	-1.643	-0.369	-0.353	2.085**	-0.841	0.103	-0.354	75
India	0.292	3.840***	-0.603	-0.337	-0.620	3.555***	-0.977	-0.430	-0.623	3.150***	-1.091	-0.364	-0.378	2.870***	-1.174	-0.306	-0.496	2.740***	-1.299	-0.117	-0.402	75
Indonesia	0.472	3.483***	0.573	0.389	0.566	3.802***	-0.137	-0.189	-0.198	3.930***	-0.548	-0.348	-0.754	4.038***	-0.485	-0.219	-0.947	4.366***	-0.568	-0.182	-1.393	75
Korea	0.445	2.718***	-0.611	-0.534	-0.620	2.479**	-1.465	-0.937	-1.213	2.330**	-1.807*	-1.099	-1.179	2.248**	-1.874*	-1.113	-1.207	2.391**	-1.752*	-0.860	-0.775	75
Mexico	0.290	2.010**	-0.643	-0.811	-0.682	1.741*	-1.348	-1.201	-1.010	1.619	-1.863*	-1.394	-1.480	1.590	-2.210**	-1.377	-2.211**	1.703*	-2.782***	-1.050	-3.803***	75
Peru	0.431	-2.370**	-2.308**	0.829	-0.284	-2.461	-4.994***	0.897	0.244	-2.502**	-7.908***	0.873	0.466	-2.572**	-10.893***	5.155***	0.535	-2.911***	-17.296***	30.778***	0.478	75
Philippines	0.126	3.838***	-1.755*	-1.997**	-1.828*	3.611	-2.722***	-2.459**	-1.509	3.330***	-3.158***	-2.434**	-1.379	3.169***	-3.112***	-1.931*	-1.243	3.080***	-3.230***	-0.713	-0.758	75
Poland	0.091	-1.082	-2.231**	-2.639***	-3.772***	-1.069	-2.934***	-3.180***	-2.008**	-1.048	-3.805***	-3.570***	-1.400	-1.057	-4.661***	-3.685***	-0.943	-1.124	-6.306***	-3.064***	-0.440	75
Romania	0.468	-9.465***	-1.716*	-2.597***	-2.255**	-14.881***	-2.232**	-2.791***	-1.845*	-17.040***	-2.749***	-2.642***	-1.540	-14.760***	-3.000***	-2.016**	-0.350	-13.882***	-3.243***	-0.110	-0.790	71
Russian Federation	0.001	-1.021	-2.741***	-3.081***	-2.029**	-0.942	-4.199***	-2.942***	-1.022	-0.756	-4.468***	-1.926*	0.139	-0.664	-4.159***	-0.897	0.151	-0.904	-5.128***	-0.921	0.024	59
South Africa	0.166	2.976***	-1.048	-0.971	-1.014	2.620***	-1.729*	-1.229	-1.169	2.283**	-2.207**	-1.373	-0.921	2.034**	-2.554***	-1.397	-1.037	2.021**	-2.603***	-1.064	-0.697	75
Thailand	0.192	1.392	-1.151	-1.113	-1.156	1.426	-2.375**	-2.182**	-2.617***	1.295	-3.084***	-2.578***	-3.012***	1.311	-3.275***	-2.555**	-3.103***	1.344	-3.388***	-2.257**	-2.198**	75
Ukraine	0.436	-7.005	-2.084**	-161.510***	-2.097**	-8.414***	-3.648***	-25.364***	-0.526	-8.896***	-6.625***	-12.599***	-0.328	-9.578***	-9.653**	-5.475***	-0.086	-12.795***	-12.276***	-3.349***	-0.061	63

Notes: See Notes to Table 4.

Table 11: Relative price regression results for sample from 1999:Q1 to 2020:Q1.

		h = 1				h = 4				h = 8				h = 12				h = 20				
	$\hat{\phi}$	$t_{h,NW}$	$t^{trf}_{h,ivx}$	$t^{trf,res}_{h,ivx}$	$t^{rev,PL}_{h,ivx}$	$t_{h,NW}$	$t^{trf}_{h,ivx}$	$t^{trf,res}_{h,ivx}$	$t^{rev,PL}_{h,ivx}$	$t_{h,NW}$	$t^{trf}_{h,ivx}$	$t^{trf,res}_{h,ivx}$	$t^{rev,PL}_{h,ivx}$	$t_{h,NW}$	$t^{trf}_{h,ivx}$	$t^{trf,res}_{h,ivx}$	$t^{rev,PL}_{h,ivx}$	$t_{h,NW}$	$t^{trf}_{h,ivx}$	$t^{trf,res}_{h,ivx}$	$t^{rev,PL}_{h,ivx}$	T
Australia	0.202	2.016**	0.088	-0.166	0.088	1.754*	-0.207	-0.169	-0.184	1.646*	-0.445	-0.432	-0.630	1.756*	-0.735	-0.673	-1.235	2.335**	-0.566	-0.395	-1.143	84
Austria	0.173	-1.303	-0.178	-0.094	-0.178	-1.276	-0.318	-0.149	-0.266	-1.105	-0.385	-0.212	-0.287	-1.058	-0.559	-0.454	-0.627	-1.011	-0.927	-0.869	-0.748	84
Belgium	0.273	-1.226	-0.496	-0.493	-0.498	-1.302	-0.481	-0.077	-0.420	-1.130	-0.512	-0.179	-0.297	-1.133	-0.627	-0.324	-0.721	-1.142	-0.892	-0.598	-0.663	84
Canada	0.335	-0.945	0.607	0.360	0.604	-1.147	0.202	0.164	0.289	-1.300	0.065	-0.064	0.244	-1.656*	-0.251	-0.392	-0.181	-4.238***	-0.638	-0.742	-0.936	84
Denmark	0.226	-2.232**	-0.250	-0.359	-0.250	-2.345**	-0.454	-0.259	-0.465	-2.339**	-0.539	-0.346	-0.466	-2.632***	-0.721	-0.531	-0.973	-3.541***	-0.854	-0.633	-1.180	84
Finland	0.239	-2.635***	-0.432	-0.294	-0.434	-2.564***	-0.544	-0.377	-0.532	-2.599***	-0.714	-0.637	-0.697	-2.766***	-1.026	-1.002	-1.240	-2.926***	-1.312	-1.229	-1.218	84
France	0.203	-4.645***	-0.254	-0.154	-0.254	-5.228***	-0.247	0.157	-0.153	-5.886***	-0.147	0.236	0.176	-7.381***	-0.152	0.189	-0.151	-16.822***	-0.415	-0.073	-0.626	84
Germany	0.194	-4.906***	-0.291	-0.491	-0.292	-5.647***	-0.471	-0.342	-0.391	-6.223***	-0.484	-0.369	-0.268	-7.568***	-0.587	-0.489	-0.654	-12.625***	-0.890	-0.815	-0.662	84
Hong Kong	0.891	-0.797	-0.028	-0.213	-0.028	-0.584	-0.029	-0.252	0.338	-0.438	-0.401	-0.278	0.227	-0.332	-0.714	-0.295	0.078	-0.114	-1.206	-0.329	-0.093	84
Ireland	0.219	1.882*	2.171**	3.091***	2.241**	2.098**	2.071**	2.927***	2.068**	2.403**	2.197**	2.735***	1.998**	2.594***	2.076**	2.417**	1.127	4.149***	2.508**	3.059***	0.707	84
Israel	0.343	-1.479	-0.846	-1.253	-0.834	-1.606	-1.872*	-1.674*	-1.581	-1.450	-2.079**	-1.568	-1.413	-1.445	-1.867*	-1.274	-1.330	-1.739*	-1.465	-0.655	-0.409	84
Italy	0.212	-2.108**	-0.184	-0.254	-0.184	-2.106**	-0.295	0.056	-0.275	-2.011**	-0.195	0.129	-0.149	-2.211**	-0.291	-0.004	-0.573	-3.513***	-0.474	-0.157	-1.168	84
Japan	0.005	-5.346***	-0.161	-0.420	-0.161	-5.301***	-0.512	-0.697	-0.352	-5.190***	-0.894	-1.034	-0.571	-4.988***	-1.223	-1.244	-0.822	-4.740***	-1.602	-1.479	-1.013	84
Luxembourg	0.179	-1.300	-0.263	-0.128	-0.264	-1.259	-0.472	-0.249	-0.425	-1.193	-0.471	-0.230	-0.338	-1.264	-0.572	-0.344	-0.673	-1.562	-0.660	-0.418	-0.844	84
Netherlands	0.233	-0.853	0.642	0.531	0.641	-0.805	0.283	0.527	0.354	-0.927	0.206	0.449	0.167	-1.422	0.066	0.267	-0.495	-3.300***	-0.486	-0.302	-1.027	84
New Zealand	0.159	0.167	0.255	0.046	0.255	-0.125	-0.169	0.021	-0.074	0.103	-0.054	0.072	0.137	0.003	-0.255	-0.116	-0.390	0.143	-0.292	-0.170	-0.204	84
Norway	0.306	-0.312	0.247	0.389	0.247	-0.251	0.177	0.335	0.146	-0.277	-0.096	0.034	-0.165	-0.355	-0.309	-0.254	-0.348	-0.478	-0.692	-0.774	-0.543	84
Portugal	0.171	-1.163	0.707	1.278	0.711	-0.949	0.625	1.144	0.724	-0.904	0.607	1.026	0.576	-1.053	0.443	0.762	0.032	-1.890*	0.413	0.829	-0.486	84
Singapore	0.256	-2.618***	-1.699*	-2.019**	-1.726*	-2.187**	-1.552	-1.567	-1.458	-1.714*	-1.519	-1.403	-1.256	-1.407	-1.559	-1.296	-1.062	-1.015	-1.591	-1.093	-0.734	84
Spain	0.120	-0.518	-0.108	0.735	-0.108	-0.340	0.084	0.576	0.073	-0.220	0.249	0.719	0.218	-0.219	0.214	0.582	0.017	-0.311	0.341	0.762	-0.159	84
Sweden	0.149	-3.382***	0.251	0.689	0.251	-3.540***	0.348	0.782	0.414	-3.944***	0.169	0.501	0.363	-5.158***	-0.154	-0.006	-0.242	-9.531***	-0.538	-0.339	-0.642	84
Switzerland	0.149	-2.055**	0.095	-0.056	0.095	-2.269**	0.049	-0.090	0.045	-2.373**	0.048	-0.112	0.164	-2.715***	0.050	-0.122	-0.039	-4.089***	-0.003	-0.134	-0.106	84
United Kingdom	0.274	0.519	0.228	0.150	0.228	0.334	0.070	0.186	0.032	0.103	-0.225	-0.166	-0.216	-0.049	-0.506	-0.563	-0.487	-0.303	-1.088	-1.470	-0.819	84
Brazil	0.223	3.304***	0.429	0.018	0.421	3.231***	-0.146	-0.262	-0.187	3.501***	-0.432	-0.323	-0.861	4.143***	-0.711	-0.319	-1.214	7.797***	-0.678	-0.464	-0.990	84
Bulgaria	0.205	1.713*	-1.328	-1.215	-1.355	1.495	-1.572	-0.900	-1.180	1.386	-1.442	-0.630	-0.994	1.284	-1.604	-0.673	-1.351	1.349	-1.116	-0.271	-1.022	84
Chile	0.447	2.311**	-0.846	-1.333	-0.835	2.461**	-1.666*	-0.790	-1.479	2.745***	-2.161**	-0.708	-1.899*	3.200***	-2.347**	-0.517	-1.947*	3.630***	-0.977	0.060	-0.910	84
China	0.401	0.270	-1.006	-1.040	-1.003	0.165	-0.827	-0.669	-0.816	0.261	-0.869	-0.627	-0.669	0.413	-0.817	-0.523	-0.438	0.863	-0.875	-0.407	-0.224	84
Colombia	0.180	5.137***	0.310	-0.023	0.307	5.232***	-0.193	-0.216	-0.021	5.721***	-0.530	-0.253	-0.243	6.633***	-0.629	-0.059	-0.273	8.088***	-0.703	-0.950	-0.210	84
Czech Rep.	0.188	0.485	-0.051	-0.195	-0.051	0.252	-0.117	0.007	-0.245	0.120	-0.032	0.116	-0.276	-0.189	0.032	0.229	-0.542	-0.387	0.010	0.320	-0.183	84
Egypt	0.334	6.238***	1.909*	1.971**	1.853*	8.487***	2.045**	1.690*	1.487	11.541***	2.184**	1.220	1.115	13.180***	2.198**	0.107	0.892	34.599***	1.957*	1.736*	0.649	82
Greece	0.188	-0.780	-0.442	1.128	-0.441	-0.535	-0.094	0.768	-0.047	-0.434	-0.055	0.717	0.042	-0.407	-0.081	0.540	-0.277	-0.423	0.225	0.862	-0.369	84
Hungary	0.094	3.151***	-1.353	-1.909*	-1.402	2.868***	-2.519**	-2.268**	-2.086**	2.656***	-3.080***	-2.282**	-2.167**	2.572**	-3.079***	-1.961**	-1.995**	2.651***	-2.948***	-0.966	-1.916*	84
Iceland	0.502	2.758***	-0.285	-0.251	-0.287	2.678***	-1.521	-0.905	-1.679*	2.621***	-2.488**	-1.428	-3.072***	2.603***	-2.853***	-1.555	-2.836***	3.065***	-3.117***	-1.368	-4.410***	84
India	0.435	4.646***	0.527	0.427	0.521	4.384***	0.532	0.329	0.356	4.077***	0.243	0.072	-0.128	4.026***	0.198	0.043	-0.243	4.889***	0.513	0.070	-0.690	84
Indonesia	0.242	5.040***	-0.565	-0.144	-0.580	5.477***	-0.605	-0.166	-1.223	5.552***	-0.742	-0.301	-1.494	5.855***	-0.677	-0.215	-1.000	6.542***	-0.168	-0.025	-0.974	84
Korea	0.293	0.742	0.963	1.203	0.960	0.822	-0.025	0.828	-0.023	1.143	-0.566	-0.272	-0.523	1.420	-1.151	-1.195	-1.150	2.006**	-1.648*	-1.775*	-2.110**	84
Mexico	0.356	5.977***	-0.496	-0.402	-0.491	7.380***	-0.507	-0.186	-0.109	9.263***	-0.466	-0.167	0.018	12.564***	-0.444	-0.133	0.133	28.579***	-0.455	-0.293	0.281	84
Peru	0.470	1.063	-1.232	-2.185**	-1.232	1.047	-1.649*	-1.956*	-1.644	1.041	-2.052**	-2.299**	-2.051**	1.158	-2.367**	-2.470**	-2.161**	1.310	-2.632***	-2.589***	-2.581***	84
Philippines	0.149	5.120***	-0.123	-0.099	-0.123	5.606***	-0.280	0.013	-0.322	5.816***	-0.002	0.074	-0.135	5.495***	0.182	0.115	0.095	6.083***	0.550	0.622	0.197	84
Poland	0.232	0.826	-0.092	-0.712	-0.092	0.519	-0.789	-0.935	-0.852	0.218	-0.978	-1.108	-1.229	0.009	-1.076	-1.105	-1.591	-0.021	-0.934	-0.620	-1.199	84
Romania	0.173	1.092	-1.288	-2.745***	-1.931*	0.978	-1.966**	-2.568**	-1.156	0.932	-2.861***	-1.700*	-0.315	0.907	-3.650***	-0.507	0.013	0.969	-4.497	-3.271	0.082	84
Russian Federation	0.403	3.987***	-2.006**	-1.876*	-2.101**	3.843***	-2.292**	-1.668	-2.214**	3.957***	-2.337**	-1.253	-1.958*	4.351***	-2.132**	-0.722	-1.527	5.711***	-1.144	-1.705*	-1.041	84
South Africa	0.223	5.009***	2.005**	1.578	1.811*	5.067***	1.203	0.751	0.816	5.477***	0.301	0.112	-0.054	6.204***	-0.191	-0.159	-0.575	6.638***	-0.184	-0.029	-0.659	84
Thailand	0.027	-0.650	-1.286	-1.087	-1.271	-0.464	-1.484	-1.030	-1.117	-0.246	-1.669*	-0.981	-0.817	-0.061	-1.528	-0.800	-0.362	0.464	-0.089	0.470	0.955	84
Ukraine	0.543	3.210***	0.300	0.267	0.299	3.580	0.149	0.148	0.185	4.520***	0.044	0.065	0.112	6.458***	0.111	0.051	-0.049	10.855***	0.343	0.272	-0.171	83

Notes: See Notes to Table 4.

On-Line Supplementary Appendix:

Transformed Regression-based Long-Horizon Predictability Tests

by

M. Demetrescu, P.M.M. Rodrigues and A.M.R. Taylor

Summary of Contents

This supplement contains three sections. Section S.1 contains a technical appendix with proofs of the large sample results given in section 4.3. Section S.2 presents additional Monte Carlo results, such as empirical size for the tests results under conditional and unconditional heteroskedasticity and empirical power plots for the t_h^{Xu}, t_h^{Bonf}, $t_{h,ivx}^{trf,res}$ and $t_{h,ivx}^{rev,PL}$ tests under iid and autocorrelated innovations. Finally, section S.3 presents additional derivations related to the transformed regressions introduced in section 4.

S.1 Technical Appendix

Throughout this appendix, we denote by C a generic constant which may take different values at different occurrences, and by $\|\cdot\|_p$ the L_p norm or a random variable or vector.

S.1.1 Auxiliary results

Lemma S.1 *Under Assumption 4, it holds that*

1. $\frac{1}{\sqrt{T}}\sum_{t=1}^{[sT]}\begin{pmatrix}\varepsilon_t\\ v_t\end{pmatrix}\Rightarrow\begin{pmatrix}\int_0^s\sigma_\varepsilon(r)\,\mathrm{d}W_\varepsilon(r)\\ \int_0^s\sigma_\nu(r)\,\mathrm{d}W_\nu(r)\end{pmatrix}$ *with W_ε and W_ν two independent standard Wiener processes;*

2. $\frac{1}{T}\sum_{t=1}^{T-1}\nu_t^2\xrightarrow{p}\int_0^1\sigma_\nu^2(s)\mathrm{d}s$ *and* $\frac{1}{T}\sum_{t=1}^{T-1}\nu_t^2\sigma_{\varepsilon t}^2\xrightarrow{p}\int_0^1\sigma_\nu^2(s)\sigma_\varepsilon^2(s)\mathrm{d}s$;

Lemma S.2 *Under the assumptions of Theorem 4.1, it holds that*

1. $T^{-\eta/2}z_t$ *is uniformly L_4 bounded,* $t=1,\ldots,T-1$;

2. *Let* $R_{t,T}=\frac{1}{h}z_t^{trf,(h)}-z_t$. *Then,* $\sum_{t=1}^{T-1}|R_{t,T}|=o_p\left(T^{1/2+\eta}\right)$ *and* $\sum_{t=1}^{T-1}R_{t,T}^2=o_p\left(T^{\eta+1}\right)=\sum_{t=1}^{T-1}R_{t,T}^4$;

3. $\frac{1}{\sqrt{T}}\xi_{[sT]}\Rightarrow\omega J_{c,\sigma}(s):=\omega\int_0^s e^{-c(s-r)}\sigma_\nu(r)\mathrm{d}W_\nu(r)$;

4. $\frac{1}{T^{1/2+\eta}}\sum_{t=1}^{[sT]}z_t\Rightarrow\frac{\omega}{a}J_{c,\sigma}(s)$;

5. $\frac{1}{T^{\eta+1}}\sum_{t=1}^{T-1}z_t\bar{x}_t\Rightarrow\frac{\omega^2}{a}\left(J_{c,\sigma}(1)\bar{J}_{c,\sigma}(1)-\int_0^1 J_{c,\sigma}(s)\mathrm{d}J_{c,\sigma}(s)\right)$ *where* $\bar{J}_{c,\sigma}(s)=J_{c,\sigma}(s)-\int_0^1 J_{c,\sigma}(s)\mathrm{d}s$;

6. $\frac{1}{T^{\eta/2+1/2}}\sum_{t=1}^{T-1}z_t\varepsilon_{t+1}\Rightarrow\mathcal{N}\left(0;\frac{\omega^2}{2a}\int_0^1\sigma_\nu^2(s)\,\sigma_\varepsilon^2(s)\,\mathrm{d}s\right)$ *independent of* $J_{c,\sigma}(s)$;

7. $\frac{1}{T^{\eta+1}}\sum_{t=1}^{T-1}z_t^2\varepsilon_{t+1}^2\xrightarrow{d}\frac{\omega^2}{2a}\int_0^1\sigma_\nu^2(s)\,\sigma_\varepsilon^2(s)\,\mathrm{d}s$.

Lemma S.3 *Under the assumptions of Theorem 4.3, it holds that*

1. $z_t=\xi_t-\varrho^{t-2}\xi_1+r_t$ *for* $t=2,\ldots,T-1$, *where* $T^{\eta/2}r_t$ *is uniformly L_4-bounded,* $\left\|T^{\eta/2}r_{t-1}\right\|_4<C\ \forall t$;

2. $\frac{1}{\sqrt{hT}}\sum_{t=1}^{T-1}\xi_t^{(h)}\varepsilon_{t+1}\xrightarrow{d}\mathcal{N}\left(0;\frac{\omega^2}{(1-\rho)^2}\int_0^1\sigma_\nu^2(s)\,\sigma_\varepsilon^2(s)\,\mathrm{d}s\right)$;

3. $\frac{1}{T}\sum_{t=1}^{T-1}\xi_t^{(h)}\xi_t=\sum_{k=0}^{h-1}\theta_k\int_0^1\sigma_\nu^2(s)\,\mathrm{d}s+o_p\left(\sqrt{h/T}\right)$ *and, for each* $j=0,\ldots,h-1$, $\frac{1}{T}\sum_{t=1}^{T-1}\xi_{t-j}\xi_t=\theta_j\int_0^1\sigma_\nu^2(s)\,\mathrm{d}s+o_p\left(\sqrt{h/T}\right)$.

S.1.2 Proofs

Proof of Lemma S.1

1. The proof follows with standard arguments on weak convergence of partial sums of MD sequences; for convergence of the sample quadratic variation to the desired limit see Lemma 3 of Demetrescu et al. (2021, Supplementary Appendix).

2. This is a particular case of Lemma 3 of Demetrescu et al. (2021, Supplementary Appendix).

Proof of Lemma S.2

1. We may decompose, for $t = 1, \ldots, T-1$, $z_t = \omega\zeta_t + r_t$ where $\zeta_t = \sum_{j=0}^{t-2} \varrho^j \nu_{t-j}$ for $t \geq 2$ and 0 for $t = 1$. The decomposition is obtained via the Phillips-Solo decomposition for v_t, $v_t = \omega\nu_t + \Delta\bar{v}_t$ where $\bar{v}_t$ is a linear process in ν_t with exponentially decaying coefficients (given that ξ_t is a finite-order autoregression), such that

$$r_t = \sum_{j=0}^{t-2} \varrho^j \Delta\bar{v}_{t-j} - \frac{c}{T} \sum_{j=0}^{t-2} \varrho^j \xi_{t-1-j},$$

for which we have

$$\sum_{j=0}^{t-2} \varrho^j \Delta\bar{v}_{t-j} = \bar{v}_t - \varrho^{t-2}\bar{v}_1 - (1-\varrho) \sum_{j=0}^{t-3} \varrho^j \bar{v}_{t-1-j}$$

where $\bar{v}_t$ is uniformly L_4-bounded since its coefficients are absolutely summable and ν_t are uniformly L_4-bounded by assumption, implying

$$\sup_{t=1,\ldots,T-1} \|\bar{v}_t\|_4 = C < CT^{\eta/2} \qquad \text{and} \qquad \sup_{t=1,\ldots,T-1} \left\|\varrho^{t-2}\bar{v}_1\right\|_4 = C\varrho^{t-2} < CT^{\eta/2}.$$

Furthermore,

$$\left\|\sum_{j=0}^{t-3} \varrho^j \bar{v}_{t-1-j}\right\|_4 \leq \sum_{j=0}^{t-3} \varrho^j \left\|\bar{v}_{t-1-j}\right\|_4 \leq CT^{\eta}$$

hence $(1-\varrho)\sum_{j=0}^{t-3} \varrho^j \bar{v}_{t-1-j}$ is uniformly L_4 bounded as required. To complete the result, note that

$$\left\|\frac{c}{T}\sum_{j=0}^{t-2} \varrho^j \xi_{t-1-j}\right\|_4 \leq C\frac{1}{T}\sum_{j=0}^{t-2} \varrho^j \left\|\xi_{t-1-j}\right\|_4 \leq CT^{\eta-1/2}$$

since $T^{-1/2}\xi_t$ is uniformly L_4 bounded (which can be shown along the lines of Lemma 2(c) in Demetrescu et al. (2021, Supplementary Appendix)). Therefore $\sup_t \|r_t\|_4 = o\left(T^{\eta/2}\right)$. From Lemma 2(c) in Demetrescu et al. (2021, Supplementary Appendix), we immediately conclude that $\sup_t \|\zeta_t\|_4 = O\left(T^{\eta/2}\right)$, as required for the result.

2. We have for $h \leq t \leq T-h$

$$z_t^{trf,(h)} - hz_t = \sum_{i=1}^{h} (z_{t-h+i} - z_t) = \sum_{i=1}^{h-1} \left(\left(1-\varrho^{h-i}\right) z_{t-h+i} + \sum_{j=i+1}^{h} \varrho^{h-j} \Delta x_{t-h+j}\right).$$

Then,

$$\begin{aligned} \left\|\sum_{i=1}^{h-1} \left(1-\varrho^{h-i}\right) z_{t-h+i}\right\|_4 &= |1-\varrho| \left\|\sum_{i=1}^{h-1} \left(1+\varrho+\ldots+\varrho^{h-i-1}\right) z_{t-h+i}\right\|_4 \\ &\leq Ch^2T^{-\eta} \max_{t=1,\ldots,T-1} \|z_t\|_4, \end{aligned}$$

where we know from item 1 of this Lemma that $T^{-\eta/2}z_{t-h+i}$ is uniformly L_4 bounded. Using the same arguments as in the proof of item 1 of this Lemma it is straightforward to show that $h^{-1/2}\sum_{j=i+1}^{h} \varrho^{h-j}\Delta x_{t-h+j}$ is itself uniformly L_4 bounded under our conditions. Then,

$$\left\|z_t^{trf,(h)} - hz_t\right\|_4 \leq Ch^2T^{-\eta/2} + Ch^{3/2}$$

such that, for $h \leq t \leq T-h$, $\frac{1}{\max\left\{hT^{-\eta/2};h^{1/2}\right\}} R_{t,T}$ is uniformly L_4 (and thus L_2 and L_1)

bounded. The result is trivially extended for $t = 1, ..., h-1$ and $t = T-h+1, ..., T-1$. Therefore,

$$0 \leq \mathrm{E}\left(\sum_{t=1}^{T-1} R_{t,T}^2\right) = \sum_{t=1}^{T-1} \|R_{t,T}\|_2^2 \leq T \max\left\{h^2 T^{-\eta}; h\right\} = o\left(T^{\eta+1}\right)$$

if $h/T^\eta \to 0$, which is fulfilled. Moreover,

$$0 \leq \mathrm{E}\left(\sum_{t=1}^{T-1} R_{t,T}^4\right) = \sum_{t=1}^{T-1} \|R_{t,T}\|_4^4 \leq T \max\left\{h^4 T^{-2\eta}; h^2\right\} = o\left(T^{\eta+1}\right)$$

if $h^2/T^\eta \to 0$. Finally, under the rate restriction $h/T^{3\eta/2-1/2} \to 0$, we obtain with the Lyapunov's and Minkowski's inequalities

$$0 \leq \mathrm{E}\left(\sum_{t=1}^{T-1} |R_{t,T}|\right) \leq \left\|\sum_{t=1}^{T-1} |R_{t,T}|\right\|_2 \leq \sum_{t=1}^{T-1} \|R_{t,T}\|_2$$

which is of order $T \max\left\{hT^{-\eta/2}; h^{1/2}\right\}$ and therefore $o\left(T^{1/2+\eta}\right)$ if $h/\min\left\{T^{3\eta/2-1/2}; T^{2\eta-1}\right\} \to 0$. The result is obtained by an application of Markov's inequality.

3. We first note that the Phillips-Solo decomposition implies the normalized partial sums of v_t to converge weakly to $\omega \int_0^s \sigma_\nu(r)\mathrm{d}r$. The result then follows using standard arguments.

4. See Lemma 5 (a) in Demetrescu et al. (2021, Supplementary Appendix).

5. Since demeaning x_t washes out any nonzero μ_x, write

$$\frac{1}{T^{\eta+1}} \sum_{t=1}^{T-1} z_t \bar{x}_t = \frac{1}{T^{\eta+1}} \sum_{t=1}^{T-1} z_t \xi_t - \left(\frac{1}{T^{3/2}} \sum_{t=1}^{T-1} \xi_t\right)\left(\frac{1}{T^{\eta+1/2}} \sum_{t=1}^{T-1} z_t\right),$$

and the result follows with items 3 and 4 of this Lemma as well as Lemma 5 (b) in Demetrescu et al. (2021, Supplementary Appendix).

6. This is a direct consequence of Lemma 5 (d) in Demetrescu et al. (2021, Supplementary Appendix).

7. Following the proof of item 1 of this Lemma, it is is straightforward to show that $z_t^2 = \omega^2 \zeta_t^2 + q_{t,T}$ where $\zeta_t = \sum_{j=0}^{t-2} \varrho^j \nu_{t-j}$ and $\|q_{t,T}\|_2 = o\left(T^\eta\right)$. Therefore,

$$\frac{1}{T^{\eta+1}} \sum_{t=1}^{T-1} z_t^2 \varepsilon_{t+1}^2 = \frac{\omega^2}{T^{\eta+1}} \sum_{t=1}^{T-1} \zeta_t^2 \varepsilon_{t+1}^2 + \frac{\omega^2}{T^{\eta+1}} \sum_{t=1}^{T-1} q_{t,T} \varepsilon_{t+1}^2$$

and the Cauchy-Schwarz inequality and the moment conditions on ε_t imply that

$$\left|\sum_{t=1}^{T-1} q_{t,T} \varepsilon_{t+1}^2\right| \leq \sqrt{\sum_{t=1}^{T-1} q_{t,T}^2 \sum_{t=1}^{T-1} \varepsilon_{t+1}^4} = o_p\left(T^{1/2+\eta/2} T^{1/2}\right) = o_p\left(T^{\eta+1}\right).$$

We then have from the proof of Lemma 5 (d) of Demetrescu et al. (2021, Supplementary Appendix) that $\frac{1}{T^{\eta+1}} \sum_{t=1}^{T-1} \zeta_t^2 \varepsilon_{t+1}^2 \xrightarrow{d} \frac{1}{2a} \int_0^1 \sigma_\nu^2(s) \sigma_\varepsilon^2(s) \mathrm{d}s$, which leads to the desired result.

Proof of Lemma S.3

1. Follows with arguments similar to those used in the proof of Lemma S.2 item 1.

2. To analyze the asymptotic behaviour of $\frac{1}{\sqrt{hT}}\sum_{t=1}^{T-1}\xi_t^{(h)}\varepsilon_{t+1} = \frac{1}{\sqrt{T}}\sum_{t=1}^{T-1}\frac{\sum_{j=0}^{h-1}\xi_{t-j}}{\sqrt{h}}\varepsilon_{t+1}$ we employ a CLT for MD arrays (e.g. Davidson, 1994, Theorem 24.3), and show that

   (a) $\max_t\left|\frac{\sum_{j=0}^{h-1}\xi_{t-j}}{\sqrt{h}}\varepsilon_{t+1}\right| = o_p\left(\sqrt{T}\right)$, and

   (b) $\frac{1}{T}\sum_{t=1}^{T-1}\frac{\left(\sum_{j=0}^{h-1}\xi_{t-j}\right)^2}{h}\varepsilon_{t+1}^2 \stackrel{p}{\to} \frac{\omega^2}{(1-\rho)^2}\int_0^1\sigma_\nu^2(s)\,\sigma_\varepsilon^2(s)\,\mathrm{d}s.$

   Condition (a) follows from uniform $L_{2+\delta}$ boundedness of $\frac{\sum_{j=0}^{h-1}\xi_{t-j}}{\sqrt{h}}\varepsilon_{t+1}$ which is easily established under our moment conditions, while, for condition (b), a tedious use of martingale approximation arguments allows us to conclude that

$$\frac{1}{T}\sum_{t=1}^{T-1}\frac{\left(\sum_{j=0}^{h-1}\xi_{t-j}\right)^2}{h}\varepsilon_{t+1}^2 = \frac{1}{T}\sum_{t=1}^{T-1}\frac{\left(\sum_{j=0}^{h-1}\xi_{t-j}\right)^2}{h}\sigma_{\varepsilon t}^2 + o_p(1);$$

   we omit the details to save space. Since $\sum_{k\geq 0}b_k = \omega/(1-\rho)$, we obtain from the Phillips-Solo decomposition that $\xi_t = \frac{\omega}{1-\rho}\nu_t + \Delta\bar{\nu}_t$, where $\bar{\nu}_t$ is a linear process driven by ν_t with exponentially decaying coefficients. It then follows

$$\begin{aligned}\frac{1}{T}\sum_{t=1}^{T-1}\frac{\left(\sum_{j=0}^{h-1}\xi_{t-j}\right)^2}{h}\varepsilon_{t+1}^2 &= \frac{1}{T}\sum_{t=1}^{T-1}\frac{1}{h}\frac{\omega^2}{(1-\rho)^2}\left(\sum_{j=0}^{h-1}\nu_{t-j}\right)^2\sigma_{\varepsilon t+1}^2 + \frac{1}{h}\frac{1}{T}\sum_{t=1}^{T-1}\left(\bar{\nu}_t-\bar{\nu}_{t-h}\right)^2\sigma_{\varepsilon t+1}^2\\ &\quad -\frac{1}{\sqrt{h}}\frac{2}{T}\sum_{t=1}^{T-1}\left(\frac{1}{\sqrt{h}}\frac{\omega}{1-\rho}\sum_{j=0}^{h-1}\nu_{t-j}\right)\left(\bar{\nu}_t-\bar{\nu}_{t-h}\right)\sigma_{\varepsilon t+1}^2 + o_p(1)\end{aligned}$$

   where the second and the third summand are easily seen to vanish in probability since $h\to\infty$. Write then

$$\frac{1}{T}\sum_{t=1}^{T-1}\frac{1}{h}\left(\sum_{j=0}^{h-1}\nu_{t-j}\right)^2\sigma_{\varepsilon t+1}^2 = \frac{1}{T}\sum_{t=1}^{T-1}\frac{1}{h}\left(\sum_{j=0}^{h-1}\nu_{t-j}^2\right)\sigma_{\varepsilon t+1}^2 + \frac{1}{h}\sum_{j=0}^{h-1}\sum_{\substack{k=0\\ j\neq k}}^{h-1}\frac{1}{T}\sum_{t=1}^{T-1}\left(\nu_{t-j}\nu_{t-k}\right)\sigma_{\varepsilon t+1}^2,$$

   where we note that the terms $\nu_{t-j}\nu_{t-k}$ for $j\neq k$ are MD sequences (in t) of uniformly bounded variance, such that $\max_{j\neq k}\operatorname{Var}\left(\frac{1}{T}\sum_{t=1}^{T-1}\left(\nu_{t-j}\nu_{t-k}\right)\right) = O\left(T^{-1/2}\right)$. Therefore, the second summand on the r.h.s. is of order $O_p\left(h/\sqrt{T}\right) = o_p(1)$. Then, use the partial summation formula to show that

$$\frac{1}{T}\sum_{t=1}^{T-1}\frac{1}{h}\left(\sum_{j=0}^{h-1}\nu_{t-j}^2\right)\sigma_{\varepsilon t+1}^2 = \frac{1}{T}\sum_{t=1}^{T-h}\nu_t^2\left(\frac{1}{h}\sum_{j=1}^{h}\sigma_{\varepsilon t+j}^2\right) + O_p\left(\frac{h^{3/2}}{T}\right),$$

   where the Lipschitz-by-parts property of $\sigma_\varepsilon^2(\cdot)$ further implies that $\frac{1}{h}\sum_{j=1}^{h}\sigma_{\varepsilon t+j}^2 = \sigma_{\varepsilon t}^2 + O\left(h/T\right)$. Condition (b) and therefore the desired result then follows with Lemma S.1 given our rate restrictions on h.

3. To analyze $\frac{1}{T}\sum_{t=1}^{T-1}\xi_t^{(h)}\xi_t$, we note that this is nothing else than the sum of the sample autocovariances of order $0,\ldots,h-1$ (without demeaning). We examine $T^{-1}\sum_{t=1}^{T-1}\xi_t^2$ first, for which we obtain

$$\frac{1}{T}\sum_{t=1}^{T-1}\xi_t^2 = \frac{1}{T}\sum_{t=1}^{T-1}\sum_{k\geq 0}\sum_{\ell\geq 0}b_k b_\ell\nu_{t-k}\nu_{t-\ell} = \sum_{k\geq 0}\sum_{\ell\geq 0}b_k b_\ell\left(\frac{1}{T}\sum_{t=1}^{T-1}\nu_{t-k}\nu_{t-\ell}\right).$$

We now focus on the cases $k \neq \ell$, for which it follows with the MD property and the moment conditions of ν_t that $\left\| \frac{1}{T} \sum_{t=1}^{T-1} \nu_{t-k}\nu_{t-\ell} \right\|_2 = T^{-1/2}$ uniformly in k, ℓ. Given the weighting with the exponentially decaying $b_k b_\ell$, these terms add up to a negligible term of order $O\left(T^{-1/2}\right)$. Then, for $k = \ell$, we have

$$\begin{aligned}\sum_{k\geq 0} \frac{1}{T} \sum_{t=1}^{T-1} \nu_{t-k}^2 &= \sum_{k=0}^{h} b_k^2 \frac{1}{T} \sum_{t=1}^{T-1} \nu_{t-k}^2 + \sum_{k\geq h+1} b_k^2 \frac{1}{T} \sum_{t=1}^{T-1} \nu_{t-k}^2 \\ &\quad + \sum_{k\geq h+1} b_k^2 \int_0^1 \sigma_\nu^2(s)\mathrm{d}s - \sum_{k\geq h+1} b_k^2 \int_0^1 \sigma_\nu^2(s)\mathrm{d}s,\end{aligned}$$

where the 2nd and the 4th summand vanish at exponential rate in h and may therefore be neglected thanks to the minimum rate condition on h (we note that $\frac{1}{T} \sum_{t=1}^{T-1} \nu_{t-k}^2$ is uniformly L_1 bounded in k), while for the third

$$\sum_{k=0}^{h} b_k^2 \frac{1}{T} \sum_{t=1}^{T-1} \nu_{t-k}^2 = \sum_{k=0}^{h} b_k^2 \frac{1}{T} \sum_{t=1}^{T-1} \nu_{t}^2 + O_p\left(\frac{h}{T}\right).$$

We therefore obtain

$$\frac{1}{T} \sum_{t=1}^{T-1} \xi_t^2 = \left(\frac{1}{T} \sum_{t=1}^{T-1} \nu_t^2 - \int_0^1 \sigma_\nu^2(s)\mathrm{d}s\right) \sum_{k=0}^{h} b_k^2 + \sum_{k\geq 0} b_k^2 \int_0^1 \sigma_\nu^2(s)\mathrm{d}s + O_p\left(\frac{h}{T}\right)$$

where we may deduce from the proof of Lemma 3 in Demetrescu et al. (2021, Supplementary Appendix) that the convergence rate of $\frac{1}{T} \sum_{t=1}^{T-1} \nu_t^2$ is $\sqrt{T}$. The same argument applies for $T^{-1} \sum_{t=1}^{T-1} \xi_t \xi_{t-j}$ for $j = 1, \ldots, h-1$, such that

$$\frac{1}{T} \sum_{t=1}^{T-1} \xi_t \xi_{t-j} = \left(\frac{1}{T} \sum_{t=1}^{T-1} \nu_t^2 - \int_0^1 \sigma_\nu^2(s)\mathrm{d}s\right) \sum_{k=0}^{h} b_k b_{k+j} + \sum_{k\geq 0} b_k b_{k+j} \int_0^1 \sigma_\nu^2(s)\mathrm{d}s + O_p\left(\frac{h}{T}\right)$$

and, since $h^3/T \to 0$,

$$\begin{aligned}\frac{1}{T} \sum_{t=1}^{T-1} \xi_t^{(h)} \xi_{t-j} &= \left(\frac{1}{T} \sum_{t=1}^{T-1} \nu_t^2 - \int_0^1 \sigma_\nu^2(s)\mathrm{d}s\right) \sum_{j=0}^{h-1} \sum_{k=0}^{h} b_k b_{k+j} + \sum_{j=0}^{h-1} \theta_j \int_0^1 \sigma_\nu^2(s)\mathrm{d}s + O_p\left(\frac{h^2}{T}\right) \\ &= \sum_{j=0}^{h-1} \theta_j \int_0^1 \sigma_\nu^2(s)\mathrm{d}s + o_p\left(\sqrt{\frac{h}{T}}\right)\end{aligned}$$

as required.

Proof of Theorem 4.1

Start with

$$\hat{\beta}_{h,ivx}^{trf,res} = \frac{\sum_{t=p}^{T-1} z_t^{trf,(h)} \bar{\varepsilon}_{t+1}}{\sum_{t=1}^{T-h} z_t \bar{x}_t} + \beta_1 \frac{\sum_{t=p}^{T-1} z_t^{trf,(h)} \bar{x}_t}{\sum_{t=1}^{T-h} z_t \bar{x}_t} + \frac{\gamma \sum_{t=p}^{T-1} z_t^{trf,(h)} \bar{\nu}_{t+1}}{\sum_{t=1}^{T-h} z_t \bar{x}_t} - \frac{\hat{\gamma} \sum_{t=p}^{T-1} z_t^{trf,(h)} \hat{\nu}_{t+1}}{\sum_{t=1}^{T-h} z_t \bar{x}_t}.$$

Since $\beta_h = \beta_1 \sum_{j=0}^{h-1} \rho^j$, we first analyze the term depending on β_1 to prove that

$$\frac{T^{\eta/2+1/2}}{h} \left(\frac{\sum_{t=p}^{T-1} z_t^{trf,(h)} \bar{x}_t}{\sum_{t=1}^{T-h} z_t \bar{x}_t} - \sum_{j=0}^{h-1} \rho^j \right) \to 0$$

where, recall, $\rho = 1 - c/T$ under strong persistence. We note that, since $h/T \to 0$, this is equivalent with showing that

$$\frac{T^{\eta/2+1/2}}{h}\left(\frac{\sum_{t=p}^{T-1} z_t^{(h)}\bar{x}_t}{\sum_{t=1}^{T-h} z_t\bar{x}_t} - h\right) \to 0.$$

We have

$$\frac{1}{h}\left(\frac{\sum_{t=p}^{T-1} z_t^{(h)}\bar{x}_t}{\sum_{t=1}^{T-h} z_t\bar{x}_t} - h\right) = \frac{\sum_{t=p}^{T-1}\frac{1}{h}\left(z_t^{(h)} - hz_t\right)\bar{x}_t}{\sum_{t=1}^{T-h} z_t\bar{x}_t},$$

where we plug in

$$z_{t-j} = \frac{1}{\varrho^j} z_t + \sum_{i=0}^{j-1}\frac{1}{\varrho^{j-i}}\Delta x_{t-j-i}$$

to obtain

$$\frac{1}{h}\left(\frac{\sum_{t=p}^{T-1} z_t^{(h)}\bar{x}_t}{\sum_{t=1}^{T-h} z_t\bar{x}_t} - h\right) = \frac{\sum_{t=p}^{T-1}\frac{1}{h}\left(\sum_{j=0}^{h-1}\frac{1}{\varrho^j} - h\right)z_t\bar{x}_t}{\sum_{t=1}^{T-h} z_t\bar{x}_t} + \frac{\sum_{t=p}^{T-1}\frac{1}{h}\left(\sum_{j=0}^{h-1}\sum_{i=0}^{j-1}\frac{1}{\varrho^{j-i}}\Delta x_{t-i}\right)\bar{x}_t}{\sum_{t=1}^{T-h} z_t\bar{x}_t},$$

where $\frac{1}{\varrho^h} \to 1$ for $h/T^\eta \to 0$ and it may be shown using standard techniques for autoregressions of near-integrated variables that $\sum_{i=0}^{j-1}\sum_{t=p}^{T-1}\Delta x_{t-i}\bar{x}_t = O_p(T)$ uniformly in h. Since $T^{1/2-\eta/2}/h \to 0$, the second term may be shown to be of order $o_p\left(T^{\eta/2+1/2}\right)$ as required. Then, for $h/T^\eta \to 0$ we use again the convergence $\frac{1}{\varrho^h} \to 1$ to conclude that

$$\frac{\sum_{t=p}^{T-1}\frac{1}{h}\left(\sum_{j=0}^{h-1}\frac{1}{\varrho^j} - h\right)z_t\bar{x}_t}{\sum_{t=1}^{T-h} z_t\bar{x}_t} = \frac{1}{h}\left(\sum_{j=0}^{h-1}\frac{1}{\varrho^j} - h\right) = \frac{1}{h}\sum_{j=0}^{h-1}\frac{1-\varrho^j}{\varrho^j} = O\left(1-\varrho\right) = O\left(T^\eta\right) = o\left(T^{\eta/2+1/2}\right).$$

Summing up,

$$\begin{aligned}\frac{T^{\eta/2+1/2}}{h}\left(\hat{\beta}_{h,ivx}^{trf,res} - \beta_h\right) &= \frac{\frac{1}{hT^{\eta/2+1/2}}\sum_{t=p}^{T-1} z_t^{trf,(h)}\bar{\varepsilon}_{t+1}}{\frac{1}{T^{\eta+1}}\sum_{t=1}^{T-h} z_t\bar{x}_t} + o_p(1)\\ &\quad + \frac{\gamma\frac{1}{hT^{\eta/2+1/2}}\sum_{t=p}^{T-1} z_t^{trf,(h)}\bar{\nu}_{t+1}}{\frac{1}{T^{\eta+1}}\sum_{t=1}^{T-h} z_t\bar{x}_t} - \frac{\hat{\gamma}\frac{1}{hT^{\eta/2+1/2}}\sum_{t=p}^{T-1} z_t^{trf,(h)}\hat{\nu}_{t+1}}{\frac{1}{T^{\eta+1}}\sum_{t=1}^{T-h} z_t\bar{x}_t}.\end{aligned}$$

The behavior of the denominator of the estimator follows from Lemma S.2 item 5.

We move on to the analysis of the numerator terms. For the first, we have

$$\frac{1}{hT^{\eta/2+1/2}}\sum_{t=1}^{T-1} z_t^{trf,(h)}\bar{\varepsilon}_{t+1} = \frac{1}{hT^{\eta/2+1/2}}\sum_{t=1}^{T-1} z_t^{trf,(h)}\varepsilon_{t+1} - \frac{\bar{\varepsilon}}{hT^{\eta/2+1/2}}\sum_{t=1}^{T-1} z_t^{trf,(h)},$$

where $\bar{\varepsilon} = O_p\left(T^{-1/2}\right)$ thanks to the MD property of ε_t and the boundedness of $\sigma_\varepsilon(\cdot)$ implied by the piecewise Lipschitz continuity, such that $\frac{\bar{\varepsilon}}{hT^{\eta/2+1/2}}\sum_{t=1}^{T-1} z_t^{trf,(h)}$ is seen to vanish thanks to Lemma S.2 items 2 and 4. Moreover,

$$\frac{1}{hT^{\eta/2+1/2}}\sum_{t=1}^{T-1} z_t^{trf,(h)}\varepsilon_{t+1} = \frac{1}{T^{\eta/2+1/2}}\sum_{t=1}^{T-1} z_t\varepsilon_{t+1} + \frac{1}{T^{\eta/2+1/2}}\sum_{t=1}^{T-1}\frac{z_t^{trf,(h)} - hz_t}{h}\varepsilon_{t+1}$$

whose second term on the r.h.s. vanishess too, thanks to the md property of ε_{t+1}, the adaptedness of $\frac{1}{h}\left(z_t^{trf,(h)} - hz_t\right)$ and Lemma S.2 item 2. Item 6 of the Lemma then implies

$$\begin{aligned}\frac{1}{hT^{\eta/2+1/2}}\sum_{t=1}^{T-1} z_t^{trf,(h)}\bar{\varepsilon}_{t+1} &= \frac{1}{T^{\eta/2+1/2}}\sum_{t=1}^{T-1} z_t\varepsilon_{t+1} + o_p(1)\\ &\xrightarrow{d} \mathcal{N}\left(0; \frac{\omega^2}{2a}\int_0^1 \sigma_\nu^2(s)\,\sigma_\varepsilon^2(s)\,\mathrm{d}s\right)\end{aligned}$$

independent of J_c. Moving on to the second, we have

$$\frac{\beta_1}{hT^{\eta/2+1/2}}\sum_{t=1}^{T-1} z_t^{trf,(h)}\bar{x}_t = \frac{T^{\eta/2+1/2}\beta_1}{T^{\eta+1}}\sum_{t=1}^{T-1} z_t\bar{x}_t + \frac{T^{\eta/2+1/2}\beta_1}{T^{\eta+1}}\sum_{t=1}^{T-1}\frac{z_t^{trf,(h)} - hz_t}{h}\bar{x}_t, \tag{S.1}$$

where items 2 and 5 of Lemma S.2 imply together with $\sup_t |\bar{x}_t| = O_p\left(\sqrt{T}\right)$ that the second term on the r.h.s. is dominated with

$$\frac{1}{T^{\eta+1}}\sum_{t=1}^{T-1} z_t^{trf,(h)}\bar{x}_t \xrightarrow{d} \frac{\omega^2}{a}\left(J_{c,\sigma}(1)\bar{J}_{c,\sigma}(1) - \int_0^1 J_{c,\sigma}(s)\mathrm{d}J_{c,\sigma}(s)\right).$$

We finally show $\Delta_T := \frac{\gamma}{hT^{\eta/2+1/2}}\sum_{t=1}^{T-1} z_t^{trf,(h)}\bar{\nu}_{t+1} - \frac{\hat{\gamma}}{hT^{\eta/2+1/2}}\sum_{t=1}^{T-1} z_t^{trf,(h)}\hat{\nu}_{t+1}$ to vanish as follows:

$$\Delta_T = \frac{\gamma}{hT^{\eta/2+1/2}}\sum_{t=1}^{T-1} z_t^{trf,(h)}\left(\nu_{t+1} - \hat{\nu}_{t+1}\right) - \frac{\gamma\bar{\nu}}{hT^{\eta/2+1/2}}\sum_{t=1}^{T-1} z_t^{trf,(h)} - \frac{\hat{\gamma}-\gamma}{hT^{\eta/2+1/2}}\sum_{t=1}^{T-1} z_t^{trf,(h)}\hat{\nu}_{t+1}. \tag{S.2}$$

For the first term on the r.h.s. of (S.2), write

$$\frac{\gamma}{hT^{\eta/2+1/2}}\sum_{t=1}^{T-1} z_t^{trf,(h)}\left(\nu_{t+1} - \hat{\nu}_{t+1}\right) = \frac{\gamma}{T^{\eta/2+1/2}}\sum_{t=1}^{T-1} z_t\left(\nu_{t+1} - \hat{\nu}_{t+1}\right) + \frac{\gamma}{T^{\eta/2+1/2}}\sum_{t=1}^{T-1}\frac{z_t^{trf,(h)} - hz_t}{h}\left(\nu_{t+1} - \hat{\nu}_{t+1}\right).$$

Using analogous arguments as in the proof of Theorem 3.2. in Demetrescu and Rodrigues (2020), the first component can be seen to vanish, while the second may be bounded with the help of the Cauchy-Schwarz inequality,

$$\left|\sum_{t=1}^{T-1}\frac{z_t^{trf,(h)} - hz_t}{h}\left(\nu_{t+1} - \hat{\nu}_{t+1}\right)\right| \leq \sqrt{\sum_{t=1}^{T-1}\left(\frac{z_t^{trf,(h)} - hz_t}{h}\right)^2 \sum_{t=1}^{T-1}\left(\nu_{t+1} - \hat{\nu}_{t+1}\right)^2},$$

where $\sum_{t=1}^{T-1}\left(\frac{z_t^{trf,(h)} - hz_t}{h}\right)^2 = o_p\left(T^{\eta+1}\right)$ from Lemma S.2 item 2 while $\sum_{t=1}^{T-1}\left(\nu_{t+1} - \hat{\nu}_{t+1}\right)^2$ is easily shown to be bounded in probability under our assumptions. For the second term on the r.h.s. of (S.2), write

$$\frac{\gamma\bar{\nu}}{hT^{\eta/2+1/2}}\sum_{t=1}^{T-1} z_t^{trf,(h)} = \frac{\gamma\bar{\nu}}{T^{\eta/2+1/2}}\sum_{t=1}^{T-1} z_t - \frac{\gamma\bar{\nu}}{T^{\eta/2+1/2}}\sum_{t=1}^{T-1}\frac{z_t^{trf,(h)} - hz_t}{h}$$

where both terms vanish given Lemma S.2 and the fact that, just like $\bar{\varepsilon}$, $\bar{\nu} = O_p\left(T^{-1/2}\right)$. For the third term on the r.h.s. of (S.2), we note first that it is not difficult to show that

$$\hat{\gamma} = \frac{\sum_{t=1}^{T-1}\hat{u}_{t+1}\hat{\nu}_{t+1}}{\sum_{t=1}^{T-1}\hat{\nu}_{t+1}^2} = \gamma + o_p(1),$$

so it suffices to show that

$$\frac{1}{hT^{\eta/2+1/2}}\sum_{t=1}^{T-1} z_t^{trf,(h)}\hat{\nu}_{t+1} = \frac{1}{hT^{\eta/2+1/2}}\sum_{t=1}^{T-1} z_t^{trf,(h)}\nu_{t+1} + \frac{1}{hT^{\eta/2+1/2}}\sum_{t=1}^{T-1} z_t^{trf,(h)}\left(\hat{\nu}_{t+1} - \nu_{t+1}\right)$$

is bounded in probability. We have in fact that $h^{-1}T^{-\eta/2-1/2}\sum_{t=1}^{T-1} z_t^{trf,(h)}\nu_{t+1} = O_p(1)$ with the same arguments used to show that $h^{-1}T^{-\eta/2-1/2}\sum_{t=1}^{T-1} z_t^{trf,(h)}\varepsilon_{t+1} = O_p(1)$, while the term $h^{-1}T^{-\eta/2-1/2}\sum_{t=1}^{T-1} z_t^{trf,(h)}\left(\hat{\nu}_{t+1} - \nu_{t+1}\right)$ has already been shown to vanish above.

Summing up, Δ_T vanishes as required for the limiting distribution.

Let us now move on to discuss the standard errors. We have namely that

$$\hat{\hat{\varepsilon}}_{t+1} = \varepsilon_{t+1} + o_p\left(1\right)$$

uniformly in t, so the residual effect in

$$\frac{1}{h^2T^{\eta+1}}\mathcal{H}_{z^{trf,(h)}\hat{\varepsilon}z^{trf,(h)}\hat{\varepsilon}} = \frac{1}{h^2T^{\eta+1}}\sum_{t=1}^{T-h}\left(z_t^{trf,(h)}\right)^2\varepsilon_{t+1}^2 + \frac{1}{h^2T^{\eta+1}}\sum_{t=1}^{T-h}\left(z_t^{trf,(h)}\right)^2\left(\hat{\hat{\varepsilon}}_{t+1}^2 - \varepsilon_{t+1}^2\right)$$

is seen to vanish given that $h^{-2}T^{-\eta}\left(z_t^{trf,(h)}\right)^2$ is uniformly L_1 bounded. Focusing on the first summand on the r.h.s., we have

$$\begin{aligned}\frac{1}{h^2T^{\eta+1}}\sum_{t=1}^{T-h}\left(z_t^{trf,(h)}\right)^2\varepsilon_{t+1}^2 &= \frac{1}{T^{\eta+1}}\sum_{t=1}^{T-h} z_t^2\varepsilon_{t+1}^2 \\ &= -\frac{2}{T^{\eta+1}}\sum_{t=1}^{T-h} z_t\frac{hz_t - z_t^{trf,(h)}}{h}\varepsilon_{t+1}^2 + \frac{1}{T^{\eta+1}}\sum_{t=1}^{T-h}\left(\frac{hz_t - z_t^{trf,(h)}}{h}\right)^2\varepsilon_{t+1}^2\end{aligned}$$

where the third term can be seen to vanish thanks to the Cauchy-Schwarz inequality,

$$\left|\sum_{t=1}^{T-h}\left(\frac{hz_t - z_t^{trf,(h)}}{h}\right)^2\varepsilon_{t+1}^2\right| \leq \sqrt{\sum_{t=1}^{T-h}\left(\frac{hz_t - z_t^{trf,(h)}}{h}\right)^4\sum_{t=1}^{T-h}\varepsilon_{t+1}^4},$$

and the second can be seen to vanish thanks to the Cauchy-Schwarz inequality applied twice,

$$\begin{aligned}\left|\sum_{t=1}^{T-h} z_t\frac{hz_t - z_t^{trf,(h)}}{h}\varepsilon_{t+1}^2\right| &\leq \sqrt{\sum_{t=1}^{T-h}\left(z_t\frac{hz_t - z_t^{trf,(h)}}{h}\right)^2\sum_{t=1}^{T-h}\varepsilon_{t+1}^4} \\ &\leq \sqrt{\sqrt{\sum_{t=1}^{T-h} z_t^4\sum_{t=1}^{T-h}\left(\frac{hz_t - z_t^{trf,(h)}}{h}\right)^4}\left(\sum_{t=1}^{T-h}\varepsilon_{t+1}^4\right)},\end{aligned}$$

where the uniform L_4 boundedness of ε_t and items 1 and 2 of Lemma S.2 have been used again. Item 7 Lemma S.2 then leads to

$$\frac{1}{h^2T^{\eta+1}}\mathcal{H}_{z^{trf,(h)}\hat{\varepsilon}z^{trf,(h)}\hat{\varepsilon}} \stackrel{d}{\to} \frac{\omega^2}{2a}\int_0^1 \sigma_\nu^2\left(s\right)\sigma_\varepsilon^2\left(s\right)\mathrm{d}s,$$

and the result follows if $\hat{Q}_T^{trf,(h)} = o_p\left(h^2T^{\eta+1}\right)$. To show this, we must take into account the fact that x_t is near-integrated, such that the sample covariance matrices involved are singular in the limit. Define therefore the invertible matrix

$$D = \begin{pmatrix} 1 & 0 & \cdots & \cdots & 0 \\ 1 & -1 & \ddots & & \vdots \\ 0 & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & -1 & 0 \\ 0 & \cdots & 0 & 1 & -1 \end{pmatrix},$$

for which $D\boldsymbol{x}_t = (x_t, \Delta x_t, \Delta x_{t-1}, \ldots, \Delta x_{t-p+1})$. Then,

$$\begin{aligned} \hat{Q}_T^{trf,(h)} &= \left(D\mathcal{H}_{z^{trf,(h)}\bar{\boldsymbol{x}}}\right)' \left(D\mathcal{H}_{\bar{\boldsymbol{x}}\bar{\boldsymbol{x}}}D'\right)^{-1} D\mathcal{H}_{\bar{\boldsymbol{x}}\bar{\boldsymbol{x}}v}D' \left(D\mathcal{H}_{\bar{\boldsymbol{x}}\bar{\boldsymbol{x}}}D'\right)^{-1} D\mathcal{H}_{z^{trf,(h)}\bar{\boldsymbol{x}}} \\ &:= \mathcal{D}'_{z^{trf,(h)}\bar{\boldsymbol{x}}}\mathcal{D}^{-1}_{\bar{\boldsymbol{x}}\bar{\boldsymbol{x}}}\mathcal{D}_{\bar{\boldsymbol{x}}\bar{\boldsymbol{x}}v}\mathcal{D}^{-1}_{\bar{\boldsymbol{x}}\bar{\boldsymbol{x}}}\mathcal{D}_{z^{trf,(h)}\bar{\boldsymbol{x}}} \end{aligned}$$

where all vectors and matrices $\mathcal{D}$ are computed just like $\mathcal{H}$ but with $D\bar{\boldsymbol{x}}_t$ rather than $\bar{\boldsymbol{x}}_t$. Let $\mathbf{D}_T = \begin{pmatrix} T & \mathbf{0} \\ \mathbf{0} & \sqrt{T}\mathbf{I}_{p-1} \end{pmatrix}$; it is then not difficult to show using standard arguments for near-integrated variables that

$$\mathbf{D}_T \left(\mathbf{D}_T^{-1}\mathcal{D}_{\bar{\boldsymbol{x}}\bar{\boldsymbol{x}}}\mathbf{D}_T^{-1}\right)^{-1} \mathbf{D}_T^{-1}\mathcal{D}_{\bar{\boldsymbol{x}}\bar{\boldsymbol{x}}v}\mathbf{D}_T^{-1} \left(\mathbf{D}_T^{-1}\mathcal{D}_{\bar{\boldsymbol{x}}\bar{\boldsymbol{x}}}\mathbf{D}_T^{-1}\right)^{-1} \mathbf{D}_T$$

has a nonsingular block diagonal limit, and it then suffices to show that

$$\mathbf{D}_T^{-1}\mathcal{D}_{z^{trf,(h)}\bar{\boldsymbol{x}}} = o_p\left(h^2 T^{\eta+1}\right),$$

i.e. that

$$\left(\frac{1}{T}\sum_{t=1}^{T-1} z_t^{trf,(h)}\bar{x}_t\right)^2 = o_p\left(h^2 T^{\eta+1}\right),$$

which follows from (S.1), and, for $j = 1, \ldots, p-1$, that

$$\left(\frac{1}{\sqrt{T}}\sum_{t=1}^{T-1} z_t^{trf,(h)}\Delta x_{t-j}\right)^2 = o_p\left(h^2 T^{\eta+1}\right),$$

which follows along the lines of the final argument of the proof of Theorem 3.2 in Demetrescu and Rodrigues (2020).

Proof of Theorem 4.2

The result follows by noting that none of the derivations prof the proof of Theorem 4.1 made use of the orthogonality of ε_t and ν_t; see the relevant Lemmata in Demetrescu et al. (2021). Therefore we may apply all derivations with u_t replacing ε_t and the result follows by noting that the variance of u_t is given by $\sigma^2_{\varepsilon t} + \gamma^2\sigma^2_{\nu t}$.

Proof of Theorem 4.3

Like in the strongly persistent case we have

$$\hat{\beta}_{h,ivx}^{trf,res} = \frac{\sum_{t=p}^{T-1} z_t^{trf,(h)}\bar{\varepsilon}_{t+1}}{\sum_{t=1}^{T-h} z_t\bar{x}_t} + \beta_1\frac{\sum_{t=p}^{T-1} z_t^{trf,(h)}\bar{x}_t}{\sum_{t=1}^{T-h} z_t\bar{x}_t} + \frac{\gamma\sum_{t=p}^{T-1} z_t^{trf,(h)}\bar{\nu}_{t+1}}{\sum_{t=1}^{T-h} z_t\bar{x}_t} - \frac{\hat{\gamma}\sum_{t=p}^{T-1} z_t^{trf,(h)}\hat{\nu}_{t+1}}{\sum_{t=1}^{T-h} z_t\bar{x}_t}.$$

It follows immediately with Lemma S.3 item 1 that

$$\frac{1}{\sqrt{hT}}\sum_{t=1}^{T-1} z_t^{trf,(h)}\bar{\varepsilon}_{t+1} = \frac{1}{\sqrt{hT}}\sum_{t=1}^{T-1} \xi_t^{(h)}\varepsilon_{t+1} + o_p(1)$$

and, if $T^{1/2-\eta/2}/h \to 0$,

$$\frac{1}{T}\sum_{t=1}^{T-h} z_t \bar{x}_t = \frac{1}{T}\sum_{t=1}^{T-h} \xi_t^2 + o_p\left(\sqrt{\frac{h}{T}}\right).$$

Therefore, with Lemma S.3, it follows that

$$\sqrt{\frac{T}{h}}\frac{\sum_{t=p}^{T-1} z_t^{trf,(h)}\bar{\varepsilon}_{t+1}}{\sum_{t=1}^{T-h} z_t \bar{x}_t} \xrightarrow{d} \mathcal{N}\left(0; \frac{\frac{\omega^2}{(1-\rho)^2}\int_0^1 \sigma_\nu^2(s)\,\sigma_\varepsilon^2(s)\,\mathrm{d}s}{\left(\theta_0 \int_0^1 \sigma_\nu^2(s)\,\mathrm{d}s\right)^2}\right).$$

Using Lemma S.3 item 1, it is tedious, yet straightforward to show that

$$\frac{\sum_{t=p}^{T-1} z_t^{trf,(h)}\bar{x}_t}{\sum_{t=1}^{T-h} z_t \bar{x}_t} = \sum_{j=0}^{h-1} \frac{\sum_{t=1}^{T-1}\xi_{t-j}\xi_t}{\sum_{t=1}^{T-h}\xi_t^2} + o_p\left(\sqrt{\frac{h}{T}}\right)$$

and we omit the details, such that Lemma S.3 implies

$$\frac{\sum_{t=p}^{T-1} z_t^{trf,(h)}\bar{x}_t}{\sum_{t=1}^{T-h} z_t \bar{x}_t} = \sum_{j=0}^{h-1}\frac{\theta_j}{\theta_0} + o_p\left(\sqrt{\frac{h}{T}}\right)$$

and we may write

$$\begin{aligned}\sqrt{\frac{T}{h}}\left(\hat{\beta}_{h,ivx}^{trf,res} - \beta_h\right) \quad = \quad & \frac{\frac{\gamma}{\sqrt{hT}}\sum_{t=p}^{T-1} z_t^{trf,(h)}\bar{\nu}_{t+1}}{\frac{1}{T}\sum_{t=1}^{T-h} z_t\bar{x}_t} - \frac{\frac{\hat{\gamma}}{\sqrt{hT}}\sum_{t=p}^{T-1} z_t^{trf,(h)}\hat{\nu}_{t+1}}{\frac{1}{T}\sum_{t=1}^{T-h} z_t\bar{x}_t} \\ & + \sqrt{\frac{T}{h}}\frac{\sum_{t=p}^{T-1} z_t^{trf,(h)}\bar{\varepsilon}_{t+1}}{\sum_{t=1}^{T-h} z_t\bar{x}_t} + o_p(1).\end{aligned}$$

We now analyze the two terms involving the errors ν_{t+1} and the residuals $\hat{\nu}_{t+1}$, where we write their difference as

$$\frac{\gamma}{\sqrt{hT}}\sum_{t=1}^{T-1} z_t^{trf,(h)}\left(\nu_{t+1} - \hat{\nu}_{t+1}\right) - \frac{\gamma\bar{\nu}}{\sqrt{hT}}\sum_{t=1}^{T-1} z_t^{trf,(h)} - \frac{\hat{\gamma}-\gamma}{\sqrt{hT}}\sum_{t=1}^{T-1} z_t^{trf,(h)}\hat{\nu}_{t+1}$$

and examine the three summands in turn. In fact, the second and the third are not difficult to be seen to vanish in probability (not unlike the strong persistence case), and we omit the details to save space. For the first, we have

$$\frac{\gamma}{\sqrt{hT}}\sum_{t=1}^{T-1} z_t^{trf,(h)}\left(\nu_{t+1} - \hat{\nu}_{t+1}\right) = \frac{\gamma}{\sqrt{hT}}\sum_{t=1}^{T-1}\xi_t^{(h)}\left(\nu_{t+1} - \hat{\nu}_{t+1}\right) + o_p(1),$$

and

$$\nu_{t+1} - \hat{\nu}_{t+1} = \left(\hat{\boldsymbol{\phi}} - \boldsymbol{\phi}\right)'\bar{\boldsymbol{x}}_t$$

where $\bar{\boldsymbol{x}}_t$ stacks p lags of the demeaned x_t, and $\boldsymbol{\phi}$ stacks the coefficients of $(1-\rho L)\,A(L)$. It follows that

$$\frac{1}{\sqrt{hT}}\sum_{t=1}^{T-1}\xi_t^{(h)}\left(\nu_{t+1} - \hat{\nu}_{t+1}\right) = \frac{1}{\sqrt{hT}}\sum_{t=1}^{T-1}\xi_t^{(h)}\bar{\boldsymbol{x}}_t\sqrt{T}\left(\hat{\boldsymbol{\phi}} - \boldsymbol{\phi}\right),$$

where standard OLS algebra indicates that $\hat{\boldsymbol{\phi}} - \boldsymbol{\phi} = \left(\sum_{t=p+1}^{T-1}\bar{\boldsymbol{x}}_t\bar{\boldsymbol{x}}_t'\right)^{-1}\sum_{t=p+1}^{T-1}\bar{\boldsymbol{x}}_t\nu_{t+1}$ and it is a standard exercise to establish that $\sqrt{T}\left(\hat{\boldsymbol{\phi}} - \boldsymbol{\phi}\right) = O_p(1)$ under our assumptions. Yet Lemma S.3

implies that $\sum_{t=1}^{T-1} \xi_t^{(h)} \bar{\boldsymbol{x}}_t = O_p(T)$ such that

$$\frac{1}{\sqrt{hT}} \sum_{t=1}^{T-1} \xi_t^{(h)} \left(\nu_{t+1} - \hat{\nu}_{t+1}\right) = o_p(1).$$

Analyzing the standard errors, we have from the proof of Lemma S.3 item 2 that

$$\frac{1}{hT} \mathcal{H}_{z^{trf,(h)}\hat{\varepsilon} z^{trf,(h)}\hat{\varepsilon}} \xrightarrow{p} \frac{\omega^2}{(1-\rho)^2} \int_0^1 \sigma_\nu^2(s)\, \sigma_\varepsilon^2(s)\, \mathrm{d}s$$

where the differences between $z_t^{trf,(h)}$ and $\xi_t^{(h)}$ are negligible, and we now show that $\frac{1}{hT}\hat{Q}_T^{trf,(h)} \xrightarrow{p} 0$ as follows.

Since p is finite, the matrices $\mathcal{H}_{\bar{\boldsymbol{x}}\bar{\boldsymbol{x}}}$ and $\mathcal{H}_{\bar{\boldsymbol{x}}\bar{\boldsymbol{x}}v}$ are easily seen to converge to positive definite covariance matrices upon normalization with T^{-1}, while $\mathcal{H}'_{z^{trf,(h)}\bar{\boldsymbol{x}}}$ is $O_p(T)$, cf. S.3 item 3 after accounting for the differences between z_t and ξ_t. We therefore have as required

$$\mathcal{H}'_{z^{trf,(h)}\bar{\boldsymbol{x}}} \mathcal{H}^{-1}_{\bar{\boldsymbol{x}}\bar{\boldsymbol{x}}} \mathcal{H}_{\bar{\boldsymbol{x}}\bar{\boldsymbol{x}}v} \mathcal{H}^{-1}_{\bar{\boldsymbol{x}}\bar{\boldsymbol{x}}} \mathcal{H}_{z^{trf,(h)}\bar{\boldsymbol{x}}} = O_p(T).$$

Proof of Theorem 4.4

See the proof of Theorem 4.2.

Proof of Theorem 4.5

We have

$$t_{h,ivx}^{trf,res} = \frac{\hat{\beta}_{h,ivx}^{trf,res}}{s.e.\left(\hat{\beta}_{h,ivx}^{trf,res}\right)} = \frac{\frac{1}{hT^{\eta/2+1/2}} \sum_{t=1}^{T-1} z_t^{trf,(h)} (\bar{y}_{t+1} - \hat{\gamma}\hat{\nu}_{t+1})}{\frac{1}{hT^{\eta/2+1/2}} \sqrt{\mathcal{H}_{z^{trf,(h)}\hat{\varepsilon} z^{trf,(h)}\hat{\varepsilon}} + \hat{\gamma}^2 \hat{Q}_T^{trf,(h)}}}.$$

Start with the analysis of $\sum_{t=1}^{T-1} z_t^{trf,(h)} (\bar{y}_{t+1} - \hat{\gamma}\hat{\nu}_{t+1})$ and write with $\bar{y}_{t+1} = \bar{\varepsilon}_{t+1} + \gamma \bar{\nu}_{t+1} + \frac{b}{T^{\eta/2+1/2}} \bar{x}_t$

$$\begin{aligned}\frac{1}{hT^{\eta/2+1/2}} \sum_{t=1}^{T-1} z_t^{trf,(h)} (\bar{y}_{t+1} - \hat{\gamma}\hat{\nu}_{t+1}) &= \frac{1}{hT^{\eta/2+1/2}} \sum_{t=1}^{T-1} z_t^{trf,(h)} \bar{\varepsilon}_{t+1} + \frac{b}{hT^{\eta+1}} \sum_{t=1}^{T-1} z_t^{trf,(h)} \bar{x}_t \\ &+ \left(\frac{\gamma}{hT^{\eta/2+1/2}} \sum_{t=1}^{T-1} z_t^{trf,(h)} \bar{\nu}_{t+1} - \frac{\hat{\gamma}}{hT^{\eta/2+1/2}} \sum_{t=1}^{T-1} z_t^{trf,(h)} \hat{\nu}_{t+1} \right).\end{aligned}$$

The first result follows using the same arguments as in the proof of Theorem 4.1, also noting that

$$\hat{\gamma} = \frac{\sum_{t=1}^{T-1} \bar{y}_{t+1} \hat{\nu}_{t+1}}{\sum_{t=1}^{T-1} \hat{\nu}_{t+1}^2} = \gamma + o_p(1)$$

under the null as well as under local alternatives, just as $\hat{\bar{\varepsilon}}_{t+1} = \varepsilon_{t+1} + o_p(1)$ even if $\hat{\bar{\varepsilon}}_{t+1}$ are computed under the null. The second result is obtained entirely analogously and we omit the details.

Proof of Theorem 4.6

See the proof of Theorem 4.5.

S.2 Additional Monte Carlo Results

Table S.1: Empirical rejection frequencies of one-sided (left and right tail) and two-sided predictability tests, for sample sizes $T = 250$ and $T = 500$. **DGP - GARCH(1,1):** $y_t = \beta x_{t-1} + u_t, x_t = \rho x_{t-1} + v_t$ and $v_t = \psi v_{t-1} + \nu_t$, where $\beta = 0$, $\rho = 1 - c/T$, $\psi = 0$ and $(u_t, \nu_t)' = [\sigma_{1t} \quad 0; 0 \quad \sigma_{2t}]\, \boldsymbol{\eta}_t$; $\boldsymbol{\eta}_t := (\eta_{1t}, \eta_{2t})' \sim NIID(\mathbf{0}, \boldsymbol{\Omega})$ with $\boldsymbol{\Omega} = [1 \quad -0.95; -0.95 \quad 1]$ and $\sigma_{it}^2 = 0.05 + 0.1 e_{i,t-1}^2 + 0.85 \sigma_{i,t-1}^2$, $i = 1, 2$.

		t_h^{Xu}			t_h^{Bonf}			$t_{h,ivx}^{trf,res}$			$t_h^{rev,PL}$			t_h^{Xu}			t_h^{Bonf}			$t_{h,ivx}^{trf,res}$			$t_h^{rev,PL}$		
						$T = 250$												$T = 500$							
h	c	$\beta_h < 0$	$\beta_h > 0$	$\beta_h \neq 0$	$\beta_h < 0$	$\beta_h > 0$	$\beta_h \neq 0$	$\beta_h < 0$	$\beta_h > 0$	$\beta_h \neq 0$	$\beta_h < 0$	$\beta_h > 0$	$\beta_h \neq 0$	$\beta_h < 0$	$\beta_h > 0$	$\beta_h \neq 0$	$\beta_h < 0$	$\beta_h > 0$	$\beta_h \neq 0$	$\beta_h < 0$	$\beta_h > 0$	$\beta_h \neq 0$	$\beta_h < 0$	$\beta_h > 0$	$\beta_h \neq 0$
1	0	0.003	0.191	0.118	0.028	0.047	0.057	0.001	0.064	0.036	0.001	0.122	0.064	0.002	0.187	0.115	0.032	0.039	0.055	0.001	0.061	0.032	0.001	0.113	0.056
	-5	0.115	0.078	0.112	0.045	0.044	0.071	0.008	0.059	0.031	0.013	0.116	0.063	0.111	0.079	0.112	0.042	0.040	0.063	0.006	0.059	0.032	0.011	0.117	0.068
	-10	0.108	0.067	0.111	0.060	0.046	0.086	0.019	0.057	0.037	0.024	0.110	0.073	0.112	0.066	0.113	0.060	0.043	0.084	0.018	0.062	0.039	0.024	0.114	0.076
	-20	0.080	0.058	0.079	0.089	0.044	0.111	0.028	0.060	0.043	0.034	0.093	0.068	0.078	0.060	0.078	0.080	0.040	0.098	0.028	0.058	0.042	0.033	0.097	0.074
	-50	0.063	0.060	0.072	0.179	0.052	0.209	0.041	0.054	0.049	0.048	0.081	0.070	0.062	0.057	0.063	0.168	0.035	0.185	0.039	0.057	0.048	0.050	0.088	0.078
5	0	0.001	0.157	0.088	0.016	0.043	0.044	0.001	0.065	0.035	0.001	0.116	0.059	0.000	0.177	0.103	0.026	0.040	0.049	0.001	0.059	0.032	0.001	0.112	0.059
	-5	0.110	0.061	0.094	0.034	0.045	0.062	0.006	0.069	0.039	0.011	0.123	0.067	0.110	0.072	0.104	0.041	0.041	0.068	0.007	0.060	0.036	0.011	0.113	0.066
	-10	0.109	0.057	0.100	0.049	0.045	0.072	0.014	0.067	0.039	0.018	0.105	0.067	0.106	0.060	0.102	0.055	0.041	0.076	0.016	0.064	0.041	0.022	0.108	0.071
	-20	0.071	0.053	0.067	0.077	0.039	0.092	0.028	0.068	0.051	0.032	0.100	0.069	0.077	0.056	0.075	0.071	0.040	0.088	0.025	0.064	0.046	0.031	0.097	0.070
	-50	0.055	0.049	0.051	0.155	0.032	0.161	0.044	0.069	0.060	0.043	0.073	0.064	0.055	0.051	0.053	0.144	0.028	0.147	0.037	0.066	0.051	0.042	0.083	0.071
10	0	0.000	0.120	0.062	0.012	0.041	0.038	0.001	0.062	0.033	0.002	0.109	0.057	0.000	0.150	0.085	0.020	0.034	0.040	0.001	0.058	0.031	0.001	0.104	0.053
	-5	0.100	0.049	0.077	0.029	0.039	0.052	0.006	0.066	0.036	0.009	0.112	0.064	0.111	0.060	0.094	0.039	0.038	0.059	0.006	0.063	0.034	0.012	0.113	0.066
	-10	0.093	0.048	0.081	0.040	0.032	0.054	0.014	0.066	0.043	0.019	0.103	0.063	0.100	0.054	0.089	0.048	0.035	0.065	0.015	0.059	0.039	0.019	0.105	0.064
	-20	0.065	0.048	0.061	0.062	0.028	0.068	0.029	0.074	0.054	0.026	0.089	0.064	0.069	0.045	0.063	0.071	0.030	0.080	0.027	0.069	0.048	0.029	0.096	0.067
	-50	0.054	0.054	0.056	0.110	0.014	0.098	0.044	0.067	0.059	0.035	0.073	0.056	0.060	0.049	0.056	0.128	0.020	0.120	0.039	0.070	0.059	0.037	0.082	0.067
20	0	0.000	0.094	0.057	0.005	0.036	0.030	0.000	0.057	0.030	0.001	0.106	0.057	0.000	0.117	0.063	0.013	0.033	0.034	0.001	0.059	0.029	0.001	0.102	0.049
	-5	0.088	0.060	0.086	0.020	0.035	0.036	0.006	0.064	0.035	0.009	0.110	0.062	0.101	0.049	0.079	0.026	0.036	0.046	0.005	0.062	0.032	0.010	0.111	0.063
	-10	0.091	0.053	0.089	0.030	0.022	0.035	0.015	0.060	0.038	0.016	0.102	0.063	0.098	0.046	0.082	0.037	0.031	0.051	0.013	0.065	0.039	0.016	0.101	0.059
	-20	0.052	0.056	0.057	0.038	0.011	0.032	0.030	0.062	0.045	0.022	0.096	0.064	0.067	0.049	0.059	0.055	0.023	0.056	0.026	0.063	0.047	0.026	0.092	0.064
	-50	0.051	0.059	0.057	0.051	0.002	0.034	0.045	0.051	0.049	0.027	0.076	0.054	0.052	0.050	0.054	0.086	0.007	0.069	0.038	0.061	0.050	0.030	0.079	0.057
50	0	0.003	0.185	0.169	0.001	0.021	0.012	0.002	0.045	0.025	0.001	0.103	0.056	0.000	0.095	0.071	0.004	0.031	0.022	0.001	0.047	0.025	0.001	0.103	0.054
	-5	0.056	0.130	0.132	0.003	0.015	0.007	0.009	0.040	0.023	0.007	0.115	0.066	0.083	0.070	0.087	0.018	0.024	0.027	0.006	0.050	0.026	0.008	0.111	0.061
	-10	0.065	0.092	0.103	0.008	0.006	0.007	0.020	0.041	0.027	0.011	0.114	0.069	0.086	0.062	0.096	0.021	0.015	0.024	0.014	0.052	0.030	0.014	0.097	0.058
	-20	0.044	0.066	0.059	0.006	0.002	0.003	0.031	0.039	0.032	0.014	0.108	0.065	0.049	0.059	0.058	0.020	0.008	0.016	0.025	0.050	0.034	0.019	0.096	0.060
	-50	0.064	0.057	0.074	0.002	0.000	0.000	0.034	0.029	0.029	0.015	0.088	0.056	0.053	0.059	0.062	0.024	0.000	0.014	0.039	0.044	0.042	0.023	0.080	0.052

Notes: t_h^{Xu} denotes the implied statistic of Xu (2020), t_h^{Bonf} is the Bonferroni based statistic of Hjalmarsson (2011), $t_{h,ivx}^{trf,res}$ is the residual augmented transformed regression based statistic in (4.10) proposed in section 4.2; and $t_{h,ivx}^{trf,PL}$ is the Phillips and Lee (2013) statistic. h is the forecast horizon considered and c is the local to unity parameter that characterises the persistence of the predictor. The columns labeled $\beta_h < 0$, $\beta_h > 0$, and $\beta_h \neq 0$ refer to left-, right- and two-sided tests, respectively.

Table S.2: Empirical rejection frequencies of one-sided (left and right tail) and two-sided predictability tests, for sample sizes $T = 250$ and $T = 500$. **DGP (Unconditional Heteroskedasticity):** $y_t = \beta x_{t-1} + u_t, x_t = \rho x_{t-1} + v_t$ and $v_t = \psi v_{t-1} + \nu_t$, where $\beta = 0$, $\rho = 1 - c/T, \psi = 0$ and $(u_t, \nu_t)' \sim NIID(\mathbf{0}, \mathbf{\Sigma_t})$, with $\mathbf{\Sigma}_t = \begin{bmatrix}\sigma_{ut}^2 & -0.95\sigma_{ut}\sigma_{\nu t}; & -0.95\sigma_{ut}\sigma_{\nu t} & \sigma_{\nu t}^2\end{bmatrix}$ and $\sigma_{ut}^2 = \sigma_{vt}^2 = \sigma_1 \mathbb{I}(t \leq \lfloor \lambda T \rfloor) + \sigma_2 \mathbb{I}(t > \lfloor \lambda T \rfloor)$.

		t_h^{Xu}			t_h^{Bonf}			$t_{h,ivx}^{trf,res}$			$t_h^{rev,PL}$			t_h^{Xu}			t_h^{Bonf}			$t_{h,ivx}^{trf,res}$			$t_h^{rev,PL}$		
						$T=250$													$T=500$						
h	c	$\beta_h<0$	$\beta_h>0$	$\beta_h\neq0$	$\beta_h<0$	$\beta_h>0$	$\beta_h\neq0$	$\beta_h<0$	$\beta_h>0$	$\beta_h\neq0$	$\beta_h<0$	$\beta_h>0$	$\beta_h\neq0$	$\beta_h<0$	$\beta_h>0$	$\beta_h\neq0$	$\beta_h<0$	$\beta_h>0$	$\beta_h\neq0$	$\beta_h<0$	$\beta_h>0$	$\beta_h\neq0$	$\beta_h<0$	$\beta_h>0$	$\beta_h\neq0$
						$\lambda=1/4$, and $\sigma_1=1$ and $\sigma_2=4$													$\lambda=1/4$, and $\sigma_1=1$ and $\sigma_2=4$						
1	0	0.003	0.140	0.078	0.035	0.050	0.065	0.001	0.064	0.032	0.001	0.124	0.064	0.003	0.138	0.082	0.040	0.048	0.067	0.002	0.065	0.035	0.002	0.126	0.065
	-5	0.084	0.078	0.083	0.049	0.049	0.080	0.005	0.063	0.034	0.010	0.123	0.070	0.084	0.078	0.083	0.057	0.046	0.083	0.006	0.062	0.033	0.011	0.124	0.070
	-10	0.104	0.068	0.105	0.058	0.051	0.087	0.014	0.064	0.039	0.022	0.114	0.071	0.103	0.067	0.103	0.061	0.044	0.087	0.014	0.062	0.038	0.021	0.117	0.071
	-20	0.075	0.062	0.079	0.062	0.051	0.095	0.028	0.062	0.044	0.031	0.101	0.070	0.077	0.061	0.077	0.070	0.045	0.096	0.029	0.061	0.044	0.033	0.104	0.073
	-50	0.060	0.057	0.062	0.090	0.055	0.124	0.041	0.057	0.049	0.041	0.081	0.065	0.061	0.057	0.065	0.095	0.038	0.112	0.039	0.060	0.051	0.046	0.084	0.070
5	0	0.000	0.114	0.060	0.026	0.047	0.054	0.001	0.064	0.034	0.001	0.126	0.061	0.000	0.124	0.070	0.034	0.048	0.062	0.002	0.065	0.036	0.002	0.129	0.067
	-5	0.077	0.061	0.065	0.042	0.047	0.068	0.004	0.066	0.036	0.008	0.124	0.071	0.080	0.071	0.073	0.051	0.044	0.077	0.006	0.065	0.033	0.010	0.129	0.070
	-10	0.099	0.054	0.089	0.048	0.046	0.071	0.012	0.068	0.040	0.019	0.115	0.069	0.101	0.060	0.096	0.056	0.042	0.080	0.013	0.063	0.038	0.021	0.119	0.073
	-20	0.067	0.049	0.063	0.052	0.041	0.072	0.028	0.070	0.049	0.029	0.101	0.069	0.074	0.053	0.069	0.063	0.040	0.083	0.027	0.066	0.048	0.032	0.104	0.074
	-50	0.052	0.049	0.050	0.064	0.034	0.074	0.044	0.078	0.065	0.037	0.086	0.068	0.058	0.051	0.056	0.083	0.030	0.091	0.041	0.070	0.060	0.041	0.088	0.071
10	0	0.000	0.085	0.042	0.017	0.046	0.045	0.001	0.065	0.032	0.001	0.122	0.065	0.000	0.110	0.058	0.029	0.046	0.057	0.002	0.065	0.035	0.002	0.126	0.064
	-5	0.066	0.048	0.047	0.034	0.042	0.057	0.004	0.065	0.036	0.007	0.120	0.068	0.073	0.062	0.058	0.046	0.043	0.069	0.005	0.066	0.034	0.010	0.125	0.069
	-10	0.094	0.045	0.079	0.040	0.038	0.056	0.012	0.069	0.040	0.018	0.111	0.068	0.099	0.052	0.088	0.051	0.039	0.071	0.013	0.064	0.038	0.020	0.116	0.072
	-20	0.062	0.044	0.057	0.040	0.030	0.049	0.028	0.072	0.050	0.027	0.102	0.065	0.069	0.048	0.062	0.056	0.035	0.071	0.028	0.070	0.049	0.029	0.100	0.072
	-50	0.049	0.053	0.050	0.037	0.014	0.034	0.045	0.080	0.064	0.034	0.083	0.060	0.055	0.049	0.052	0.066	0.020	0.064	0.043	0.073	0.062	0.038	0.084	0.069
20	0	0.000	0.063	0.038	0.008	0.039	0.032	0.001	0.059	0.030	0.001	0.122	0.065	0.000	0.085	0.041	0.020	0.044	0.047	0.002	0.063	0.034	0.002	0.122	0.065
	-5	0.048	0.050	0.047	0.023	0.034	0.036	0.004	0.058	0.031	0.006	0.120	0.066	0.063	0.049	0.042	0.038	0.038	0.056	0.005	0.062	0.035	0.008	0.124	0.068
	-10	0.082	0.050	0.079	0.027	0.025	0.034	0.012	0.060	0.036	0.012	0.115	0.068	0.095	0.045	0.079	0.043	0.033	0.057	0.014	0.062	0.036	0.017	0.114	0.070
	-20	0.053	0.052	0.059	0.024	0.013	0.021	0.025	0.062	0.044	0.022	0.102	0.065	0.063	0.044	0.057	0.044	0.027	0.051	0.027	0.063	0.047	0.027	0.102	0.068
	-50	0.050	0.059	0.056	0.011	0.001	0.006	0.042	0.062	0.054	0.026	0.086	0.058	0.051	0.051	0.051	0.038	0.010	0.030	0.044	0.069	0.061	0.034	0.090	0.065
50	0	0.001	0.160	0.142	0.002	0.020	0.011	0.002	0.042	0.023	0.001	0.120	0.066	0.000	0.070	0.052	0.008	0.038	0.029	0.001	0.053	0.028	0.002	0.115	0.065
	-5	0.015	0.118	0.103	0.005	0.011	0.005	0.007	0.040	0.023	0.004	0.122	0.068	0.040	0.062	0.052	0.022	0.026	0.032	0.005	0.051	0.028	0.007	0.119	0.069
	-10	0.059	0.085	0.083	0.007	0.005	0.005	0.018	0.040	0.026	0.007	0.121	0.070	0.076	0.057	0.079	0.024	0.017	0.026	0.013	0.051	0.030	0.013	0.113	0.070
	-20	0.042	0.065	0.059	0.004	0.001	0.001	0.030	0.040	0.032	0.010	0.118	0.071	0.051	0.056	0.059	0.018	0.007	0.014	0.026	0.051	0.039	0.021	0.108	0.069
	-50	0.059	0.058	0.068	0.000	0.000	0.000	0.031	0.037	0.031	0.013	0.099	0.058	0.053	0.058	0.060	0.007	0.001	0.003	0.040	0.051	0.048	0.025	0.101	0.067
						$\lambda=1/4$, and $\sigma_1=1$ and $\sigma_2=10$													$\lambda=1/4$, and $\sigma_1=1$ and $\sigma_2=10$						
1	0	0.003	0.115	0.063	0.038	0.059	0.074	0.001	0.064	0.035	0.001	0.138	0.074	0.003	0.114	0.063	0.043	0.056	0.078	0.002	0.067	0.035	0.002	0.141	0.075
	-5	0.080	0.074	0.075	0.055	0.060	0.094	0.005	0.064	0.035	0.010	0.134	0.081	0.076	0.073	0.073	0.062	0.054	0.094	0.006	0.064	0.033	0.012	0.135	0.079
	-10	0.104	0.066	0.105	0.060	0.062	0.102	0.014	0.064	0.040	0.023	0.125	0.084	0.104	0.063	0.103	0.065	0.053	0.100	0.014	0.062	0.038	0.024	0.128	0.083
	-20	0.075	0.063	0.078	0.060	0.063	0.105	0.027	0.063	0.046	0.035	0.113	0.083	0.077	0.059	0.079	0.071	0.054	0.105	0.029	0.061	0.045	0.037	0.115	0.085
	-50	0.061	0.057	0.063	0.066	0.075	0.119	0.041	0.059	0.049	0.046	0.094	0.081	0.061	0.056	0.064	0.082	0.050	0.112	0.039	0.059	0.049	0.051	0.095	0.084
5	0	0.000	0.090	0.044	0.028	0.058	0.066	0.001	0.067	0.036	0.001	0.136	0.071	0.000	0.100	0.054	0.038	0.055	0.071	0.002	0.067	0.036	0.002	0.140	0.077
	-5	0.069	0.059	0.055	0.046	0.057	0.082	0.003	0.068	0.037	0.009	0.133	0.079	0.071	0.065	0.064	0.057	0.053	0.086	0.006	0.066	0.034	0.011	0.139	0.078
	-10	0.100	0.055	0.088	0.051	0.057	0.083	0.012	0.069	0.041	0.021	0.125	0.079	0.101	0.058	0.097	0.061	0.051	0.091	0.013	0.064	0.040	0.023	0.130	0.083
	-20	0.070	0.049	0.064	0.050	0.054	0.080	0.026	0.071	0.050	0.032	0.112	0.082	0.074	0.053	0.071	0.066	0.050	0.093	0.027	0.068	0.047	0.035	0.115	0.084
	-50	0.054	0.050	0.050	0.044	0.050	0.072	0.043	0.079	0.067	0.043	0.100	0.083	0.059	0.051	0.057	0.070	0.041	0.089	0.040	0.071	0.060	0.046	0.101	0.083
10	0	0.000	0.068	0.030	0.020	0.054	0.055	0.001	0.066	0.035	0.001	0.132	0.072	0.000	0.087	0.042	0.032	0.053	0.065	0.002	0.066	0.036	0.003	0.138	0.074
	-5	0.058	0.047	0.039	0.038	0.052	0.068	0.003	0.066	0.037	0.008	0.128	0.077	0.064	0.055	0.050	0.051	0.051	0.080	0.005	0.065	0.034	0.011	0.133	0.077
	-10	0.095	0.045	0.078	0.043	0.049	0.068	0.011	0.071	0.040	0.019	0.121	0.077	0.099	0.050	0.086	0.056	0.048	0.083	0.013	0.064	0.038	0.021	0.128	0.079
	-20	0.064	0.045	0.058	0.040	0.040	0.058	0.026	0.074	0.052	0.030	0.114	0.077	0.071	0.047	0.064	0.058	0.042	0.079	0.027	0.069	0.047	0.033	0.111	0.079
	-50	0.051	0.054	0.052	0.026	0.021	0.030	0.042	0.082	0.066	0.038	0.095	0.075	0.056	0.048	0.053	0.056	0.029	0.062	0.042	0.076	0.061	0.043	0.095	0.082
20	0	0.000	0.054	0.033	0.010	0.047	0.040	0.001	0.058	0.032	0.001	0.132	0.071	0.000	0.067	0.029	0.022	0.052	0.054	0.002	0.062	0.033	0.002	0.134	0.075
	-5	0.039	0.051	0.042	0.025	0.040	0.045	0.003	0.058	0.032	0.007	0.131	0.076	0.055	0.045	0.035	0.042	0.044	0.064	0.005	0.062	0.034	0.009	0.134	0.075
	-10	0.083	0.052	0.079	0.029	0.031	0.039	0.011	0.062	0.036	0.013	0.124	0.075	0.094	0.041	0.079	0.046	0.040	0.064	0.014	0.063	0.037	0.019	0.125	0.079
	-20	0.052	0.055	0.060	0.025	0.018	0.026	0.023	0.064	0.045	0.022	0.112	0.077	0.065	0.043	0.059	0.045	0.032	0.057	0.027	0.066	0.047	0.029	0.112	0.078
	-50	0.049	0.058	0.056	0.008	0.003	0.005	0.041	0.064	0.052	0.030	0.103	0.073	0.052	0.052	0.050	0.033	0.013	0.031	0.044	0.070	0.060	0.039	0.103	0.077
50	0	0.001	0.160	0.139	0.002	0.021	0.012	0.002	0.041	0.024	0.001	0.127	0.069	0.000	0.065	0.049	0.009	0.043	0.036	0.002	0.053	0.028	0.002	0.126	0.072
	-5	0.009	0.117	0.104	0.006	0.012	0.006	0.007	0.040	0.023	0.004	0.129	0.074	0.028	0.059	0.050	0.024	0.028	0.034	0.005	0.052	0.028	0.007	0.127	0.076
	-10	0.054	0.086	0.083	0.008	0.006	0.006	0.016	0.040	0.025	0.007	0.130	0.077	0.078	0.060	0.079	0.025	0.020	0.029	0.013	0.050	0.030	0.013	0.119	0.077
	-20	0.043	0.067	0.060	0.004	0.001	0.002	0.029	0.041	0.031	0.010	0.128	0.080	0.051	0.058	0.060	0.020	0.010	0.017	0.026	0.051	0.038	0.022	0.117	0.078
	-50	0.061	0.060	0.071	0.000	0.000	0.000	0.032	0.039	0.033	0.014	0.119	0.074	0.052	0.057	0.059	0.007	0.001	0.003	0.042	0.051	0.048	0.027	0.114	0.082

See note under Table S.1.

Table S.3: Empirical rejection frequencies of one-sided (left and right tail) and two-sided predictability tests, for sample sizes $T = 250$ and $T = 500$. **DGP (Unconditional Heteroskedasticity):** $y_t = \beta x_{t-1} + u_t, x_t = \rho x_{t-1} + v_t$ and $v_t = \psi v_{t-1} + \nu_t$, where $\beta = 0$, $\rho = 1 - c/T, \psi = 0$ and $(u_t, \nu_t)' \sim NIID(\mathbf{0}, \mathbf{\Sigma_t})$, with $\mathbf{\Sigma}_t = \begin{bmatrix}\sigma^2_{ut} & -0.95\sigma_{ut}\sigma_{\nu t}; & -0.95\sigma_{ut}\sigma_{\nu t} & \sigma^2_{\nu t}\end{bmatrix}$ and $\sigma^2_{ut} = \sigma^2_{vt} = \sigma_1 \mathbb{I}(t \leq \lfloor \lambda T \rfloor) + \sigma_2 \mathbb{I}(t > \lfloor \lambda T \rfloor)$.

		t_h^{Xu}			t_h^{Bonf}			$t_{h,ivx}^{trf,res}$			$t_h^{rev,PL}$			t_h^{Xu}			t_h^{Bonf}			$t_{h,ivx}^{trf,res}$			$t_h^{rev,PL}$		
		$T = 250$												$T = 500$											
h	c	$\beta_h < 0$	$\beta_h > 0$	$\beta_h \neq 0$	$\beta_h < 0$	$\beta_h > 0$	$\beta_h \neq 0$	$\beta_h < 0$	$\beta_h > 0$	$\beta_h \neq 0$	$\beta_h < 0$	$\beta_h > 0$	$\beta_h \neq 0$	$\beta_h < 0$	$\beta_h > 0$	$\beta_h \neq 0$	$\beta_h < 0$	$\beta_h > 0$	$\beta_h \neq 0$	$\beta_h < 0$	$\beta_h > 0$	$\beta_h \neq 0$	$\beta_h < 0$	$\beta_h > 0$	$\beta_h \neq 0$
		$\lambda = 1/2$, and $\sigma_1 = 1$ and $\sigma_2 = 4$												$\lambda = 1/2$, and $\sigma_1 = 1$ and $\sigma_2 = 4$											
1	0	0.005	0.145	0.093	0.044	0.059	0.078	0.003	0.065	0.036	0.004	0.138	0.077	0.006	0.141	0.088	0.047	0.051	0.078	0.004	0.061	0.033	0.004	0.138	0.077
	-5	0.055	0.080	0.067	0.077	0.063	0.114	0.007	0.070	0.040	0.013	0.145	0.088	0.056	0.079	0.067	0.079	0.055	0.114	0.007	0.065	0.035	0.014	0.140	0.085
	-10	0.090	0.070	0.093	0.076	0.062	0.117	0.014	0.068	0.043	0.024	0.132	0.088	0.090	0.067	0.092	0.081	0.054	0.112	0.013	0.064	0.038	0.023	0.130	0.087
	-20	0.076	0.063	0.080	0.083	0.062	0.125	0.023	0.066	0.046	0.035	0.116	0.089	0.073	0.059	0.076	0.084	0.050	0.113	0.023	0.060	0.043	0.036	0.117	0.089
	-50	0.062	0.060	0.063	0.123	0.063	0.161	0.038	0.062	0.049	0.051	0.095	0.084	0.060	0.055	0.060	0.123	0.040	0.142	0.037	0.062	0.048	0.051	0.104	0.089
5	0	0.000	0.124	0.069	0.033	0.057	0.068	0.003	0.068	0.038	0.005	0.138	0.077	0.000	0.128	0.076	0.041	0.053	0.072	0.003	0.063	0.034	0.004	0.140	0.078
	-5	0.039	0.065	0.045	0.067	0.060	0.103	0.007	0.074	0.041	0.013	0.140	0.089	0.048	0.070	0.053	0.074	0.055	0.108	0.006	0.068	0.038	0.013	0.138	0.084
	-10	0.084	0.056	0.075	0.068	0.057	0.101	0.012	0.074	0.046	0.023	0.134	0.089	0.088	0.058	0.081	0.075	0.052	0.105	0.012	0.069	0.040	0.023	0.131	0.087
	-20	0.069	0.051	0.063	0.069	0.054	0.098	0.023	0.077	0.053	0.032	0.120	0.089	0.069	0.054	0.068	0.076	0.046	0.101	0.022	0.070	0.045	0.035	0.120	0.086
	-50	0.055	0.051	0.052	0.092	0.041	0.105	0.040	0.081	0.066	0.045	0.099	0.084	0.056	0.049	0.054	0.105	0.034	0.116	0.037	0.070	0.054	0.048	0.105	0.085
10	0	0.000	0.098	0.048	0.023	0.055	0.058	0.003	0.068	0.038	0.004	0.133	0.074	0.000	0.115	0.065	0.035	0.051	0.065	0.003	0.062	0.034	0.004	0.138	0.076
	-5	0.026	0.052	0.032	0.057	0.054	0.086	0.006	0.072	0.042	0.011	0.136	0.082	0.039	0.061	0.042	0.069	0.052	0.100	0.006	0.067	0.037	0.013	0.137	0.082
	-10	0.077	0.047	0.060	0.058	0.048	0.081	0.011	0.074	0.047	0.019	0.128	0.082	0.085	0.053	0.071	0.070	0.050	0.097	0.012	0.068	0.042	0.023	0.126	0.084
	-20	0.062	0.047	0.058	0.057	0.038	0.070	0.023	0.078	0.054	0.029	0.117	0.085	0.066	0.048	0.061	0.066	0.042	0.088	0.022	0.070	0.045	0.036	0.117	0.083
	-50	0.051	0.055	0.053	0.056	0.017	0.050	0.042	0.079	0.066	0.040	0.096	0.077	0.053	0.047	0.049	0.086	0.026	0.087	0.038	0.073	0.058	0.047	0.101	0.084
20	0	0.000	0.082	0.052	0.013	0.049	0.043	0.001	0.065	0.035	0.004	0.126	0.072	0.000	0.092	0.051	0.027	0.049	0.056	0.002	0.060	0.032	0.005	0.135	0.074
	-5	0.013	0.058	0.043	0.039	0.041	0.055	0.006	0.067	0.036	0.009	0.134	0.077	0.027	0.049	0.029	0.058	0.048	0.085	0.005	0.065	0.038	0.012	0.133	0.080
	-10	0.059	0.056	0.060	0.040	0.031	0.048	0.011	0.069	0.042	0.015	0.129	0.078	0.077	0.046	0.059	0.061	0.043	0.080	0.011	0.067	0.041	0.020	0.124	0.083
	-20	0.050	0.057	0.061	0.032	0.017	0.032	0.023	0.067	0.046	0.024	0.116	0.078	0.061	0.045	0.055	0.058	0.031	0.066	0.021	0.066	0.042	0.032	0.116	0.082
	-50	0.051	0.061	0.057	0.020	0.003	0.011	0.040	0.061	0.055	0.030	0.097	0.069	0.049	0.051	0.049	0.052	0.010	0.043	0.038	0.069	0.053	0.043	0.103	0.079
50	0	0.002	0.182	0.162	0.003	0.024	0.013	0.004	0.049	0.026	0.004	0.114	0.066	0.000	0.088	0.065	0.013	0.042	0.037	0.002	0.055	0.030	0.004	0.124	0.070
	-5	0.001	0.134	0.120	0.007	0.015	0.009	0.010	0.049	0.028	0.009	0.130	0.079	0.009	0.069	0.055	0.034	0.033	0.044	0.005	0.056	0.033	0.010	0.129	0.078
	-10	0.023	0.094	0.079	0.006	0.007	0.006	0.016	0.047	0.032	0.010	0.134	0.085	0.052	0.063	0.062	0.034	0.022	0.037	0.011	0.055	0.034	0.015	0.123	0.079
	-20	0.040	0.071	0.060	0.003	0.002	0.002	0.026	0.044	0.034	0.011	0.134	0.084	0.048	0.059	0.057	0.025	0.009	0.019	0.020	0.053	0.035	0.023	0.115	0.080
	-50	0.064	0.060	0.072	0.001	0.000	0.000	0.030	0.037	0.033	0.014	0.106	0.065	0.050	0.057	0.056	0.011	0.000	0.004	0.034	0.049	0.040	0.029	0.106	0.073
		$\lambda = 1/2$, and $\sigma_1 = 1$ and $\sigma_2 = 10$												$\lambda = 1/2$, and $\sigma_1 = 1$ and $\sigma_2 = 10$											
1	0	0.007	0.120	0.072	0.050	0.082	0.109	0.004	0.065	0.039	0.007	0.175	0.108	0.007	0.112	0.069	0.054	0.073	0.104	0.004	0.064	0.034	0.007	0.173	0.107
	-5	0.038	0.077	0.058	0.094	0.092	0.157	0.007	0.073	0.040	0.015	0.177	0.121	0.041	0.072	0.051	0.096	0.078	0.148	0.006	0.064	0.037	0.016	0.173	0.114
	-10	0.089	0.070	0.087	0.092	0.092	0.158	0.012	0.071	0.042	0.029	0.167	0.122	0.088	0.064	0.084	0.097	0.076	0.149	0.012	0.064	0.039	0.030	0.167	0.119
	-20	0.079	0.063	0.084	0.093	0.090	0.159	0.023	0.070	0.046	0.047	0.150	0.122	0.076	0.060	0.077	0.097	0.077	0.146	0.022	0.062	0.044	0.047	0.154	0.121
	-50	0.064	0.059	0.064	0.100	0.101	0.176	0.037	0.063	0.051	0.068	0.133	0.123	0.061	0.058	0.062	0.115	0.069	0.161	0.036	0.064	0.049	0.065	0.140	0.129
5	0	0.000	0.100	0.051	0.041	0.081	0.095	0.004	0.067	0.039	0.006	0.173	0.105	0.000	0.102	0.056	0.048	0.073	0.098	0.004	0.067	0.037	0.007	0.173	0.106
	-5	0.023	0.060	0.036	0.083	0.089	0.144	0.006	0.077	0.044	0.015	0.173	0.118	0.031	0.063	0.038	0.090	0.076	0.142	0.006	0.068	0.039	0.016	0.171	0.113
	-10	0.080	0.056	0.067	0.083	0.084	0.142	0.010	0.079	0.048	0.027	0.166	0.117	0.086	0.056	0.072	0.091	0.073	0.140	0.011	0.069	0.041	0.028	0.164	0.117
	-20	0.072	0.052	0.065	0.080	0.080	0.133	0.022	0.079	0.053	0.041	0.151	0.120	0.071	0.052	0.070	0.090	0.070	0.135	0.021	0.069	0.047	0.045	0.151	0.127
	-50	0.055	0.049	0.051	0.076	0.072	0.119	0.040	0.084	0.069	0.062	0.135	0.124	0.057	0.050	0.055	0.100	0.059	0.133	0.035	0.069	0.055	0.062	0.139	0.123
10	0	0.000	0.079	0.039	0.031	0.077	0.082	0.003	0.068	0.039	0.007	0.171	0.102	0.000	0.090	0.046	0.042	0.072	0.089	0.003	0.066	0.037	0.006	0.167	0.105
	-5	0.012	0.049	0.026	0.072	0.081	0.122	0.006	0.077	0.044	0.014	0.167	0.109	0.023	0.054	0.029	0.086	0.074	0.133	0.006	0.068	0.041	0.015	0.166	0.110
	-10	0.070	0.046	0.053	0.071	0.073	0.116	0.010	0.079	0.048	0.023	0.158	0.113	0.081	0.048	0.061	0.085	0.071	0.129	0.011	0.068	0.042	0.028	0.157	0.112
	-20	0.064	0.050	0.062	0.067	0.061	0.098	0.021	0.081	0.053	0.038	0.149	0.115	0.070	0.047	0.062	0.081	0.064	0.119	0.022	0.071	0.048	0.044	0.149	0.116
	-50	0.051	0.056	0.054	0.049	0.036	0.056	0.041	0.084	0.068	0.053	0.132	0.113	0.054	0.049	0.052	0.082	0.048	0.103	0.037	0.075	0.060	0.060	0.136	0.119
20	0	0.000	0.082	0.057	0.019	0.070	0.063	0.002	0.065	0.036	0.007	0.154	0.094	0.000	0.073	0.037	0.033	0.070	0.080	0.003	0.063	0.036	0.007	0.164	0.101
	-5	0.004	0.065	0.050	0.049	0.062	0.079	0.005	0.071	0.037	0.011	0.161	0.103	0.013	0.043	0.022	0.073	0.068	0.114	0.005	0.066	0.039	0.013	0.161	0.105
	-10	0.049	0.064	0.060	0.052	0.048	0.071	0.010	0.071	0.044	0.018	0.156	0.107	0.069	0.042	0.050	0.074	0.063	0.108	0.010	0.065	0.040	0.024	0.153	0.108
	-20	0.053	0.065	0.067	0.042	0.030	0.048	0.021	0.069	0.047	0.028	0.148	0.108	0.063	0.045	0.058	0.066	0.051	0.089	0.021	0.068	0.046	0.040	0.147	0.111
	-50	0.051	0.063	0.058	0.017	0.008	0.012	0.040	0.067	0.054	0.040	0.136	0.105	0.049	0.055	0.053	0.055	0.024	0.053	0.038	0.069	0.055	0.053	0.134	0.115
50	0	0.003	0.220	0.187	0.007	0.030	0.021	0.006	0.050	0.030	0.007	0.133	0.084	0.000	0.094	0.073	0.018	0.061	0.055	0.002	0.056	0.029	0.007	0.148	0.090
	-5	0.000	0.151	0.134	0.008	0.019	0.013	0.009	0.048	0.029	0.009	0.153	0.096	0.002	0.077	0.064	0.044	0.045	0.060	0.005	0.055	0.032	0.011	0.150	0.096
	-10	0.012	0.108	0.090	0.006	0.010	0.005	0.015	0.048	0.029	0.010	0.158	0.104	0.040	0.071	0.062	0.044	0.033	0.052	0.010	0.054	0.032	0.016	0.146	0.097
	-20	0.040	0.079	0.068	0.005	0.002	0.002	0.025	0.044	0.033	0.012	0.162	0.106	0.049	0.065	0.064	0.034	0.017	0.028	0.019	0.052	0.034	0.025	0.143	0.101
	-50	0.068	0.064	0.078	0.001	0.000	0.000	0.028	0.042	0.033	0.016	0.143	0.096	0.051	0.063	0.061	0.012	0.002	0.007	0.036	0.049	0.038	0.035	0.143	0.104

See note under Table S.1.

Table S.4: Empirical rejection frequencies of one-sided (left and right tail) and two-sided predictability tests, for sample sizes $T = 250$ and $T = 500$. **DGP (Unconditional Heteroskedasticity):** $y_t = \beta x_{t-1} + u_t, x_t = \rho x_{t-1} + v_t$ and $v_t = \psi v_{t-1} + \nu_t$, where $\beta = 0$, $\rho = 1 - c/T, \psi = 0$ and $(u_t, \nu_t)' \sim NIID(\mathbf{0}, \mathbf{\Sigma_t})$, with $\mathbf{\Sigma}_t = \begin{bmatrix}\sigma^2_{ut} & -0.95\sigma_{ut}\sigma_{\nu t}; & -0.95\sigma_{ut}\sigma_{\nu t} & \sigma^2_{\nu t}\end{bmatrix}$ and $\sigma^2_{ut} = \sigma^2_{vt} = \sigma_1 \mathbb{I}(t \leq \lfloor \lambda T \rfloor) + \sigma_2 \mathbb{I}(t > \lfloor \lambda T \rfloor)$.

T = 250

		t_h^{Xu}			t_h^{Bonf}			$t_{h,ivx}^{trf,res}$			$t_h^{rev,PL}$		
h	c	$\beta_h < 0$	$\beta_h > 0$	$\beta_h \neq 0$	$\beta_h < 0$	$\beta_h > 0$	$\beta_h \neq 0$	$\beta_h < 0$	$\beta_h > 0$	$\beta_h \neq 0$	$\beta_h < 0$	$\beta_h > 0$	$\beta_h \neq 0$
						$\lambda = 3/4$, and $\sigma_1 = 1$ and $\sigma_2 = 4$							
1	0	0.007	0.152	0.094	0.048	0.054	0.080	0.004	0.059	0.032	0.012	0.128	0.074
	-5	0.041	0.079	0.061	0.112	0.059	0.147	0.012	0.062	0.038	0.025	0.139	0.095
	-10	0.079	0.067	0.078	0.111	0.061	0.151	0.017	0.063	0.040	0.032	0.135	0.095
	-20	0.074	0.060	0.074	0.113	0.061	0.151	0.027	0.063	0.045	0.045	0.119	0.096
	-50	0.067	0.054	0.068	0.177	0.059	0.210	0.039	0.059	0.051	0.059	0.102	0.093
5	0	0.000	0.128	0.072	0.036	0.053	0.067	0.003	0.060	0.033	0.011	0.130	0.072
	-5	0.021	0.062	0.035	0.100	0.058	0.132	0.011	0.066	0.040	0.025	0.138	0.094
	-10	0.067	0.052	0.053	0.101	0.057	0.134	0.017	0.070	0.043	0.030	0.132	0.093
	-20	0.066	0.047	0.056	0.099	0.052	0.124	0.025	0.073	0.051	0.041	0.122	0.096
	-50	0.058	0.047	0.055	0.137	0.038	0.144	0.042	0.077	0.064	0.051	0.105	0.091
10	0	0.000	0.107	0.057	0.025	0.051	0.058	0.003	0.059	0.033	0.011	0.122	0.074
	-5	0.012	0.051	0.024	0.087	0.050	0.110	0.010	0.066	0.041	0.025	0.133	0.088
	-10	0.057	0.043	0.041	0.088	0.047	0.108	0.016	0.072	0.046	0.029	0.127	0.091
	-20	0.060	0.046	0.051	0.082	0.034	0.092	0.025	0.074	0.053	0.035	0.117	0.090
	-50	0.056	0.051	0.056	0.092	0.018	0.079	0.043	0.082	0.064	0.045	0.101	0.081
20	0	0.000	0.096	0.062	0.015	0.045	0.041	0.002	0.056	0.031	0.011	0.118	0.070
	-5	0.005	0.059	0.042	0.061	0.038	0.071	0.011	0.061	0.038	0.026	0.126	0.084
	-10	0.037	0.054	0.046	0.062	0.029	0.065	0.016	0.066	0.041	0.027	0.121	0.087
	-20	0.047	0.057	0.053	0.050	0.015	0.044	0.024	0.066	0.049	0.031	0.114	0.083
	-50	0.054	0.056	0.058	0.029	0.002	0.019	0.043	0.060	0.053	0.034	0.092	0.073
50	0	0.002	0.209	0.182	0.004	0.024	0.014	0.007	0.050	0.030	0.008	0.112	0.065
	-5	0.001	0.140	0.124	0.007	0.014	0.009	0.020	0.043	0.033	0.024	0.128	0.086
	-10	0.011	0.097	0.078	0.007	0.007	0.005	0.028	0.042	0.037	0.028	0.126	0.090
	-20	0.035	0.069	0.056	0.003	0.001	0.001	0.031	0.043	0.038	0.028	0.120	0.084
	-50	0.069	0.058	0.076	0.000	0.000	0.000	0.036	0.040	0.040	0.024	0.087	0.064
						$\lambda = 3/4$, and $\sigma_1 = 1$ and $\sigma_2 = 10$							
1	0	0.012	0.129	0.081	0.070	0.089	0.131	0.007	0.062	0.037	0.029	0.187	0.131
	-5	0.026	0.069	0.049	0.149	0.107	0.228	0.014	0.068	0.042	0.043	0.210	0.162
	-10	0.066	0.063	0.062	0.152	0.112	0.237	0.018	0.071	0.047	0.056	0.198	0.169
	-20	0.077	0.062	0.076	0.150	0.111	0.236	0.026	0.072	0.048	0.072	0.187	0.176
	-50	0.069	0.060	0.077	0.170	0.116	0.259	0.039	0.066	0.055	0.097	0.171	0.186
5	0	0.000	0.118	0.062	0.059	0.085	0.114	0.006	0.060	0.034	0.030	0.180	0.129
	-5	0.005	0.053	0.024	0.140	0.102	0.213	0.014	0.069	0.042	0.043	0.200	0.154
	-10	0.046	0.046	0.033	0.141	0.104	0.215	0.017	0.074	0.045	0.049	0.197	0.162
	-20	0.067	0.046	0.056	0.138	0.099	0.209	0.023	0.079	0.052	0.066	0.189	0.170
	-50	0.063	0.051	0.061	0.136	0.084	0.188	0.040	0.081	0.066	0.086	0.174	0.179
10	0	0.000	0.104	0.056	0.044	0.083	0.098	0.005	0.057	0.034	0.029	0.170	0.121
	-5	0.002	0.048	0.023	0.125	0.093	0.184	0.013	0.071	0.044	0.044	0.186	0.148
	-10	0.031	0.044	0.031	0.128	0.091	0.183	0.015	0.074	0.049	0.047	0.181	0.147
	-20	0.056	0.049	0.055	0.121	0.076	0.161	0.023	0.080	0.055	0.059	0.177	0.155
	-50	0.055	0.062	0.064	0.099	0.047	0.112	0.038	0.087	0.066	0.074	0.165	0.158
20	0	0.000	0.122	0.086	0.025	0.074	0.071	0.005	0.056	0.033	0.029	0.153	0.113
	-5	0.000	0.080	0.061	0.091	0.072	0.122	0.012	0.066	0.040	0.043	0.170	0.135
	-10	0.013	0.073	0.058	0.091	0.061	0.113	0.015	0.071	0.043	0.044	0.168	0.141
	-20	0.041	0.071	0.063	0.076	0.039	0.079	0.023	0.074	0.048	0.048	0.166	0.141
	-50	0.056	0.068	0.068	0.038	0.009	0.026	0.038	0.068	0.054	0.056	0.152	0.132
50	0	0.003	0.275	0.242	0.006	0.034	0.022	0.016	0.058	0.043	0.024	0.125	0.092
	-5	0.000	0.188	0.167	0.007	0.020	0.012	0.029	0.053	0.044	0.040	0.157	0.127
	-10	0.002	0.126	0.106	0.005	0.009	0.005	0.031	0.051	0.045	0.043	0.160	0.131
	-20	0.025	0.088	0.067	0.002	0.002	0.001	0.034	0.053	0.047	0.042	0.164	0.130
	-50	0.079	0.068	0.096	0.001	0.000	0.000	0.037	0.049	0.050	0.038	0.132	0.105

T = 500

		t_h^{Xu}			t_h^{Bonf}			$t_{h,ivx}^{trf,res}$			$t_h^{rev,PL}$		
h	c	$\beta_h < 0$	$\beta_h > 0$	$\beta_h \neq 0$	$\beta_h < 0$	$\beta_h > 0$	$\beta_h \neq 0$	$\beta_h < 0$	$\beta_h > 0$	$\beta_h \neq 0$	$\beta_h < 0$	$\beta_h > 0$	$\beta_h \neq 0$
						$\lambda = 3/4$, and $\sigma_1 = 1$ and $\sigma_2 = 4$							
1	0	0.006	0.152	0.096	0.052	0.052	0.081	0.004	0.060	0.034	0.011	0.133	0.079
	-5	0.039	0.079	0.056	0.114	0.056	0.147	0.011	0.067	0.039	0.024	0.146	0.098
	-10	0.073	0.067	0.076	0.112	0.057	0.147	0.015	0.066	0.042	0.030	0.142	0.098
	-20	0.067	0.061	0.071	0.111	0.057	0.144	0.021	0.068	0.045	0.040	0.131	0.099
	-50	0.059	0.058	0.063	0.159	0.044	0.178	0.035	0.064	0.048	0.057	0.117	0.104
5	0	0.000	0.142	0.083	0.046	0.052	0.074	0.003	0.059	0.034	0.012	0.132	0.078
	-5	0.024	0.070	0.041	0.107	0.055	0.140	0.011	0.066	0.038	0.024	0.142	0.096
	-10	0.069	0.060	0.062	0.106	0.055	0.137	0.014	0.069	0.042	0.028	0.137	0.097
	-20	0.062	0.055	0.061	0.101	0.052	0.130	0.019	0.073	0.047	0.038	0.130	0.098
	-50	0.057	0.053	0.053	0.140	0.036	0.147	0.035	0.074	0.058	0.053	0.114	0.099
10	0	0.000	0.130	0.072	0.038	0.051	0.069	0.003	0.060	0.033	0.011	0.130	0.076
	-5	0.017	0.061	0.032	0.100	0.054	0.130	0.010	0.065	0.038	0.023	0.140	0.091
	-10	0.062	0.053	0.052	0.098	0.052	0.126	0.013	0.069	0.043	0.028	0.133	0.092
	-20	0.059	0.050	0.053	0.092	0.047	0.111	0.019	0.073	0.049	0.036	0.127	0.093
	-50	0.052	0.050	0.048	0.119	0.025	0.114	0.035	0.080	0.062	0.049	0.111	0.092
20	0	0.000	0.109	0.057	0.030	0.050	0.061	0.003	0.060	0.030	0.012	0.128	0.074
	-5	0.011	0.049	0.026	0.087	0.050	0.110	0.009	0.063	0.037	0.024	0.138	0.094
	-10	0.050	0.046	0.038	0.086	0.044	0.102	0.013	0.068	0.041	0.027	0.132	0.092
	-20	0.052	0.048	0.048	0.073	0.032	0.080	0.019	0.070	0.046	0.033	0.119	0.090
	-50	0.049	0.054	0.051	0.073	0.012	0.061	0.034	0.074	0.057	0.043	0.107	0.090
50	0	0.000	0.106	0.075	0.013	0.039	0.034	0.002	0.050	0.027	0.012	0.115	0.067
	-5	0.002	0.074	0.057	0.049	0.030	0.053	0.010	0.057	0.031	0.025	0.128	0.087
	-10	0.025	0.066	0.054	0.046	0.023	0.047	0.014	0.055	0.033	0.027	0.128	0.091
	-20	0.037	0.063	0.056	0.031	0.009	0.025	0.020	0.055	0.036	0.028	0.121	0.089
	-50	0.050	0.061	0.055	0.012	0.001	0.007	0.033	0.053	0.044	0.032	0.108	0.078
						$\lambda = 3/4$, and $\sigma_1 = 1$ and $\sigma_2 = 10$							
1	0	0.010	0.131	0.080	0.070	0.081	0.124	0.006	0.060	0.032	0.028	0.195	0.131
	-5	0.023	0.066	0.044	0.155	0.099	0.227	0.011	0.069	0.041	0.038	0.214	0.164
	-10	0.058	0.063	0.058	0.151	0.106	0.230	0.013	0.072	0.042	0.047	0.209	0.170
	-20	0.067	0.061	0.069	0.146	0.111	0.230	0.019	0.075	0.043	0.065	0.202	0.183
	-50	0.062	0.060	0.067	0.162	0.098	0.233	0.031	0.070	0.051	0.088	0.189	0.194
5	0	0.000	0.127	0.066	0.064	0.081	0.116	0.006	0.056	0.031	0.028	0.188	0.127
	-5	0.007	0.058	0.030	0.149	0.097	0.217	0.011	0.069	0.039	0.038	0.206	0.158
	-10	0.049	0.054	0.039	0.146	0.105	0.221	0.013	0.072	0.044	0.046	0.204	0.164
	-20	0.062	0.052	0.057	0.139	0.102	0.215	0.018	0.076	0.046	0.062	0.197	0.176
	-50	0.058	0.052	0.054	0.148	0.085	0.206	0.030	0.080	0.058	0.086	0.183	0.182
10	0	0.000	0.117	0.058	0.057	0.079	0.111	0.005	0.055	0.030	0.028	0.181	0.124
	-5	0.004	0.050	0.022	0.143	0.094	0.206	0.011	0.067	0.040	0.036	0.195	0.152
	-10	0.039	0.045	0.030	0.141	0.099	0.210	0.013	0.074	0.044	0.045	0.191	0.155
	-20	0.057	0.046	0.049	0.133	0.095	0.195	0.018	0.078	0.049	0.060	0.188	0.163
	-50	0.053	0.050	0.053	0.132	0.069	0.168	0.031	0.082	0.061	0.078	0.177	0.175
20	0	0.000	0.103	0.053	0.048	0.078	0.101	0.005	0.055	0.029	0.030	0.171	0.121
	-5	0.002	0.045	0.024	0.127	0.089	0.179	0.010	0.067	0.039	0.040	0.191	0.147
	-10	0.025	0.044	0.027	0.129	0.086	0.179	0.012	0.071	0.043	0.042	0.182	0.148
	-20	0.050	0.050	0.050	0.112	0.074	0.154	0.018	0.075	0.048	0.055	0.179	0.149
	-50	0.048	0.061	0.057	0.095	0.035	0.098	0.030	0.079	0.056	0.068	0.170	0.164
50	0	0.000	0.148	0.112	0.021	0.064	0.059	0.005	0.048	0.026	0.028	0.145	0.105
	-5	0.000	0.100	0.083	0.071	0.059	0.086	0.010	0.059	0.030	0.040	0.166	0.134
	-10	0.007	0.089	0.071	0.070	0.045	0.078	0.012	0.062	0.034	0.042	0.166	0.137
	-20	0.031	0.082	0.064	0.049	0.026	0.045	0.017	0.060	0.037	0.043	0.168	0.135
	-50	0.050	0.069	0.065	0.021	0.004	0.013	0.031	0.060	0.044	0.050	0.165	0.135

See note under Table S.1.

Table S.5: Empirical rejection frequencies of one-sided (left and right tail) and two-sided predictability tests, for sample sizes $T = 250$ and $T = 500$. **DGP (Unconditional Heteroskedasticity):** $y_t = \beta x_{t-1} + u_t, x_t = \rho x_{t-1} + v_t$ and $v_t = \psi v_{t-1} + \nu_t$, where $\beta = 0$, $\rho = 1 - c/T, \psi = 0$ and $(u_t, \nu_t)' \sim NIID(\mathbf{0}, \mathbf{\Sigma_t})$, with $\mathbf{\Sigma}_t = \begin{bmatrix}\sigma^2_{ut} & -0.95\sigma_{ut}\sigma_{\nu t}; & -0.95\sigma_{ut}\sigma_{\nu t} & \sigma^2_{\nu t}\end{bmatrix}$ and $\sigma^2_{ut} = \sigma^2_{vt} = \sigma_1 \mathbb{I}(t \leq \lfloor \lambda T \rfloor) + \sigma_2 \mathbb{I}(t > \lfloor \lambda T \rfloor)$.

		t_h^{Xu}			t_h^{Bonf}			$t_{h,ivx}^{trf,res}$			$t_h^{rev,PL}$			t_h^{Xu}			t_h^{Bonf}			$t_{h,ivx}^{trf,res}$			$t_h^{rev,PL}$		
		$T = 250$												$T = 500$											
h	c	$\beta_h < 0$	$\beta_h > 0$	$\beta_h \neq 0$	$\beta_h < 0$	$\beta_h > 0$	$\beta_h \neq 0$	$\beta_h < 0$	$\beta_h > 0$	$\beta_h \neq 0$	$\beta_h < 0$	$\beta_h > 0$	$\beta_h \neq 0$	$\beta_h < 0$	$\beta_h > 0$	$\beta_h \neq 0$	$\beta_h < 0$	$\beta_h > 0$	$\beta_h \neq 0$	$\beta_h < 0$	$\beta_h > 0$	$\beta_h \neq 0$	$\beta_h < 0$	$\beta_h > 0$	$\beta_h \neq 0$
		$\lambda = 1/4$, and $\sigma_1 = 4$ and $\sigma_2 = 1$												$\lambda = 1/4$, and $\sigma_1 = 4$ and $\sigma_2 = 1$											
1	0	0.002	0.247	0.149	0.022	0.047	0.052	0.000	0.060	0.033	0.000	0.127	0.067	0.002	0.247	0.150	0.027	0.043	0.055	0.001	0.057	0.030	0.000	0.123	0.066
	-5	0.154	0.079	0.160	0.030	0.055	0.068	0.007	0.066	0.035	0.015	0.130	0.080	0.150	0.079	0.158	0.033	0.054	0.069	0.007	0.065	0.037	0.017	0.133	0.085
	-10	0.122	0.067	0.126	0.070	0.047	0.096	0.021	0.066	0.043	0.032	0.116	0.082	0.120	0.067	0.127	0.065	0.050	0.095	0.018	0.067	0.042	0.033	0.125	0.088
	-20	0.083	0.059	0.083	0.152	0.037	0.164	0.031	0.061	0.050	0.043	0.097	0.080	0.083	0.063	0.088	0.124	0.043	0.141	0.029	0.065	0.048	0.046	0.112	0.091
	-50	0.064	0.056	0.067	0.391	0.032	0.399	0.042	0.059	0.051	0.053	0.078	0.072	0.065	0.057	0.064	0.329	0.027	0.332	0.041	0.062	0.054	0.062	0.096	0.092
5	0	0.001	0.202	0.115	0.010	0.046	0.043	0.000	0.060	0.034	0.000	0.128	0.069	0.001	0.225	0.131	0.020	0.044	0.048	0.000	0.059	0.031	0.000	0.128	0.069
	-5	0.153	0.062	0.143	0.024	0.052	0.057	0.005	0.070	0.039	0.015	0.134	0.084	0.150	0.071	0.152	0.027	0.052	0.062	0.006	0.069	0.038	0.016	0.132	0.083
	-10	0.117	0.052	0.110	0.060	0.043	0.084	0.020	0.073	0.047	0.031	0.119	0.084	0.120	0.060	0.119	0.061	0.048	0.085	0.016	0.070	0.043	0.030	0.124	0.088
	-20	0.076	0.048	0.068	0.136	0.032	0.140	0.034	0.074	0.056	0.043	0.101	0.084	0.082	0.055	0.078	0.116	0.038	0.130	0.027	0.070	0.052	0.045	0.111	0.087
	-50	0.057	0.047	0.052	0.347	0.021	0.335	0.048	0.074	0.066	0.046	0.080	0.069	0.060	0.051	0.058	0.305	0.022	0.298	0.044	0.070	0.062	0.060	0.095	0.084
10	0	0.001	0.160	0.081	0.004	0.047	0.037	0.000	0.059	0.034	0.000	0.127	0.070	0.001	0.195	0.114	0.014	0.041	0.040	0.000	0.058	0.030	0.000	0.126	0.068
	-5	0.150	0.050	0.135	0.018	0.049	0.048	0.005	0.070	0.041	0.013	0.129	0.082	0.150	0.060	0.143	0.024	0.049	0.055	0.006	0.069	0.037	0.014	0.134	0.082
	-10	0.115	0.044	0.105	0.050	0.039	0.067	0.019	0.076	0.051	0.028	0.117	0.085	0.118	0.053	0.111	0.054	0.044	0.076	0.015	0.070	0.043	0.030	0.122	0.088
	-20	0.070	0.044	0.064	0.113	0.025	0.109	0.034	0.077	0.060	0.040	0.101	0.083	0.077	0.049	0.069	0.105	0.033	0.113	0.027	0.073	0.052	0.042	0.111	0.086
	-50	0.053	0.052	0.055	0.280	0.011	0.251	0.051	0.071	0.065	0.043	0.075	0.066	0.058	0.049	0.052	0.275	0.015	0.255	0.044	0.073	0.065	0.052	0.090	0.082
20	0	0.002	0.124	0.081	0.001	0.044	0.034	0.000	0.055	0.031	0.000	0.128	0.075	0.001	0.155	0.084	0.006	0.040	0.033	0.000	0.056	0.029	0.000	0.126	0.069
	-5	0.149	0.058	0.146	0.011	0.039	0.033	0.005	0.067	0.039	0.011	0.125	0.076	0.149	0.048	0.138	0.019	0.044	0.043	0.006	0.065	0.036	0.013	0.128	0.081
	-10	0.106	0.055	0.111	0.037	0.025	0.043	0.019	0.068	0.047	0.025	0.110	0.076	0.115	0.045	0.102	0.044	0.037	0.059	0.015	0.067	0.041	0.028	0.120	0.081
	-20	0.060	0.056	0.067	0.075	0.011	0.061	0.036	0.067	0.054	0.034	0.095	0.072	0.072	0.046	0.065	0.084	0.024	0.083	0.026	0.068	0.048	0.040	0.106	0.086
	-50	0.053	0.058	0.058	0.151	0.001	0.109	0.051	0.049	0.055	0.033	0.066	0.056	0.052	0.052	0.053	0.209	0.007	0.180	0.045	0.064	0.056	0.048	0.087	0.075
50	0	0.011	0.248	0.232	0.000	0.039	0.023	0.000	0.044	0.024	0.000	0.104	0.062	0.002	0.133	0.101	0.001	0.039	0.024	0.000	0.046	0.026	0.000	0.128	0.073
	-5	0.133	0.140	0.206	0.003	0.024	0.014	0.006	0.048	0.027	0.011	0.130	0.080	0.142	0.074	0.160	0.011	0.029	0.026	0.006	0.054	0.030	0.012	0.132	0.081
	-10	0.092	0.092	0.132	0.013	0.013	0.013	0.020	0.047	0.032	0.017	0.121	0.078	0.103	0.065	0.119	0.027	0.020	0.029	0.015	0.056	0.035	0.025	0.117	0.080
	-20	0.051	0.067	0.066	0.016	0.003	0.010	0.036	0.039	0.038	0.020	0.099	0.068	0.058	0.064	0.068	0.041	0.009	0.032	0.027	0.056	0.039	0.034	0.102	0.077
	-50	0.065	0.058	0.074	0.017	0.000	0.007	0.040	0.028	0.035	0.017	0.062	0.042	0.056	0.059	0.061	0.065	0.000	0.044	0.042	0.041	0.041	0.038	0.077	0.066
		$\lambda = 1/4$, and $\sigma_1 = 10$ and $\sigma_2 = 1$												$\lambda = 1/4$, and $\sigma_1 = 10$ and $\sigma_2 = 1$											
1	0	0.004	0.278	0.169	0.023	0.069	0.073	0.000	0.057	0.027	0.000	0.170	0.100	0.004	0.268	0.163	0.031	0.067	0.076	0.001	0.053	0.027	0.000	0.164	0.094
	-5	0.167	0.075	0.177	0.038	0.088	0.103	0.009	0.071	0.039	0.024	0.177	0.127	0.163	0.070	0.171	0.039	0.083	0.101	0.008	0.068	0.037	0.025	0.184	0.128
	-10	0.136	0.062	0.142	0.092	0.078	0.148	0.021	0.073	0.047	0.048	0.161	0.135	0.137	0.063	0.140	0.086	0.080	0.142	0.017	0.067	0.044	0.051	0.178	0.144
	-20	0.094	0.058	0.094	0.206	0.059	0.238	0.030	0.070	0.054	0.068	0.148	0.138	0.094	0.058	0.096	0.164	0.070	0.208	0.028	0.068	0.049	0.076	0.163	0.160
	-50	0.068	0.061	0.071	0.460	0.050	0.483	0.040	0.066	0.056	0.086	0.127	0.133	0.070	0.058	0.067	0.398	0.045	0.419	0.039	0.066	0.056	0.096	0.146	0.161
5	0	0.004	0.228	0.131	0.007	0.071	0.064	0.000	0.052	0.027	0.000	0.169	0.101	0.004	0.245	0.138	0.020	0.066	0.068	0.000	0.054	0.028	0.000	0.170	0.099
	-5	0.166	0.052	0.161	0.029	0.085	0.093	0.007	0.073	0.040	0.023	0.176	0.127	0.162	0.058	0.163	0.033	0.081	0.091	0.006	0.069	0.037	0.023	0.182	0.129
	-10	0.134	0.045	0.127	0.084	0.074	0.130	0.020	0.078	0.051	0.045	0.164	0.135	0.135	0.054	0.130	0.081	0.078	0.134	0.015	0.070	0.046	0.049	0.174	0.146
	-20	0.086	0.044	0.076	0.191	0.054	0.216	0.032	0.080	0.061	0.064	0.145	0.137	0.090	0.050	0.086	0.156	0.064	0.195	0.026	0.071	0.052	0.073	0.163	0.154
	-50	0.061	0.052	0.059	0.414	0.038	0.422	0.046	0.076	0.068	0.076	0.119	0.124	0.066	0.050	0.058	0.379	0.040	0.388	0.042	0.071	0.061	0.088	0.143	0.156
10	0	0.005	0.184	0.103	0.002	0.071	0.057	0.000	0.050	0.027	0.000	0.169	0.101	0.004	0.214	0.118	0.012	0.066	0.059	0.000	0.052	0.027	0.000	0.169	0.099
	-5	0.165	0.044	0.160	0.024	0.079	0.080	0.006	0.072	0.041	0.019	0.172	0.121	0.162	0.047	0.156	0.029	0.077	0.085	0.005	0.067	0.037	0.022	0.179	0.125
	-10	0.131	0.042	0.124	0.073	0.064	0.109	0.020	0.082	0.054	0.044	0.158	0.130	0.134	0.044	0.124	0.075	0.073	0.121	0.015	0.072	0.044	0.046	0.171	0.137
	-20	0.079	0.046	0.078	0.167	0.044	0.181	0.033	0.084	0.064	0.063	0.140	0.129	0.088	0.044	0.077	0.147	0.059	0.177	0.025	0.077	0.052	0.069	0.160	0.147
	-50	0.054	0.061	0.063	0.352	0.022	0.334	0.050	0.074	0.070	0.066	0.109	0.112	0.060	0.049	0.056	0.348	0.031	0.344	0.042	0.080	0.064	0.081	0.136	0.143
20	0	0.006	0.173	0.118	0.000	0.070	0.053	0.000	0.047	0.026	0.000	0.163	0.101	0.004	0.174	0.094	0.004	0.065	0.053	0.000	0.049	0.026	0.000	0.171	0.099
	-5	0.163	0.074	0.187	0.016	0.066	0.058	0.005	0.068	0.039	0.019	0.158	0.109	0.162	0.042	0.156	0.023	0.071	0.073	0.005	0.063	0.035	0.020	0.167	0.119
	-10	0.126	0.067	0.147	0.054	0.046	0.074	0.019	0.075	0.050	0.042	0.141	0.114	0.131	0.042	0.124	0.064	0.062	0.101	0.015	0.067	0.042	0.044	0.160	0.127
	-20	0.068	0.066	0.085	0.121	0.027	0.110	0.036	0.072	0.058	0.056	0.127	0.114	0.079	0.047	0.077	0.126	0.044	0.139	0.025	0.072	0.048	0.063	0.149	0.136
	-50	0.054	0.068	0.068	0.212	0.005	0.163	0.052	0.050	0.058	0.055	0.090	0.091	0.055	0.058	0.061	0.282	0.016	0.258	0.045	0.066	0.057	0.075	0.127	0.133
50	0	0.016	0.343	0.328	0.000	0.067	0.043	0.000	0.038	0.019	0.000	0.110	0.075	0.007	0.188	0.153	0.000	0.063	0.045	0.000	0.044	0.022	0.000	0.163	0.108
	-5	0.150	0.174	0.270	0.002	0.046	0.029	0.005	0.050	0.026	0.016	0.153	0.110	0.160	0.096	0.209	0.012	0.054	0.045	0.005	0.055	0.030	0.017	0.163	0.115
	-10	0.113	0.118	0.180	0.017	0.024	0.022	0.018	0.055	0.034	0.031	0.140	0.111	0.120	0.083	0.160	0.042	0.039	0.056	0.014	0.059	0.036	0.039	0.145	0.117
	-20	0.055	0.081	0.084	0.026	0.007	0.019	0.039	0.048	0.046	0.037	0.120	0.098	0.066	0.076	0.093	0.070	0.018	0.061	0.026	0.057	0.042	0.054	0.131	0.116
	-50	0.074	0.068	0.089	0.034	0.000	0.017	0.045	0.033	0.045	0.029	0.076	0.067	0.056	0.067	0.066	0.108	0.002	0.074	0.044	0.045	0.044	0.056	0.104	0.103

See note under Table S.1.

Table S.6: Empirical rejection frequencies of one-sided (left and right tail) and two-sided predictability tests, for sample sizes $T = 250$ and $T = 500$. **DGP (Unconditional Heteroskedasticity):** $y_t = \beta x_{t-1} + u_t, x_t = \rho x_{t-1} + v_t$ and $v_t = \psi v_{t-1} + \nu_t$, where $\beta = 0$, $\rho = 1 - c/T, \psi = 0$ and $(u_t, \nu_t)' \sim NIID(\mathbf{0}, \mathbf{\Sigma_t})$, with $\mathbf{\Sigma}_t = \begin{bmatrix}\sigma^2_{ut} & -0.95\sigma_{ut}\sigma_{\nu t}; & -0.95\sigma_{ut}\sigma_{\nu t} & \sigma^2_{\nu t}\end{bmatrix}$ and $\sigma^2_{ut} = \sigma^2_{vt} = \sigma_1 \mathbb{I}(t \leq \lfloor \lambda T \rfloor) + \sigma_2 \mathbb{I}(t > \lfloor \lambda T \rfloor)$.

		t_h^{Xu}			t_h^{Bonf}			$t_{h,ivx}^{trf,res}$			$t_h^{rev,PL}$			t_h^{Xu}			t_h^{Bonf}			$t_{h,ivx}^{trf,res}$			$t_h^{rev,PL}$		
		$T = 250$												$T = 500$											
h	c	$\beta_h < 0$	$\beta_h > 0$	$\beta_h \neq 0$	$\beta_h < 0$	$\beta_h > 0$	$\beta_h \neq 0$	$\beta_h < 0$	$\beta_h > 0$	$\beta_h \neq 0$	$\beta_h < 0$	$\beta_h > 0$	$\beta_h \neq 0$	$\beta_h < 0$	$\beta_h > 0$	$\beta_h \neq 0$	$\beta_h < 0$	$\beta_h > 0$	$\beta_h \neq 0$	$\beta_h < 0$	$\beta_h > 0$	$\beta_h \neq 0$	$\beta_h < 0$	$\beta_h > 0$	$\beta_h \neq 0$
		$\lambda = 1/2$, and $\sigma_1 = 4$ and $\sigma_2 = 1$												$\lambda = 1/2$, and $\sigma_1 = 4$ and $\sigma_2 = 1$											
1	0	0.002	0.265	0.159	0.014	0.048	0.048	0.000	0.058	0.032	0.000	0.126	0.067	0.002	0.265	0.161	0.024	0.045	0.053	0.000	0.056	0.030	0.000	0.125	0.064
	-5	0.160	0.081	0.169	0.019	0.054	0.059	0.005	0.067	0.034	0.009	0.124	0.073	0.160	0.079	0.172	0.026	0.051	0.060	0.003	0.063	0.033	0.009	0.128	0.074
	-10	0.128	0.068	0.131	0.048	0.052	0.083	0.018	0.067	0.040	0.025	0.108	0.075	0.127	0.067	0.134	0.054	0.050	0.083	0.015	0.067	0.039	0.024	0.118	0.076
	-20	0.083	0.060	0.086	0.109	0.043	0.127	0.031	0.065	0.046	0.038	0.097	0.073	0.080	0.062	0.082	0.095	0.045	0.116	0.028	0.069	0.045	0.038	0.107	0.081
	-50	0.063	0.057	0.065	0.315	0.037	0.330	0.042	0.061	0.051	0.048	0.079	0.067	0.058	0.057	0.059	0.260	0.029	0.262	0.038	0.063	0.048	0.048	0.093	0.076
5	0	0.002	0.212	0.122	0.006	0.047	0.040	0.000	0.060	0.032	0.000	0.123	0.066	0.002	0.233	0.138	0.017	0.043	0.046	0.000	0.057	0.030	0.000	0.122	0.064
	-5	0.159	0.063	0.154	0.014	0.051	0.049	0.004	0.066	0.037	0.007	0.123	0.073	0.162	0.071	0.165	0.021	0.049	0.054	0.002	0.063	0.034	0.009	0.128	0.074
	-10	0.125	0.052	0.114	0.041	0.046	0.070	0.017	0.072	0.044	0.023	0.111	0.073	0.126	0.061	0.126	0.048	0.047	0.074	0.013	0.067	0.040	0.023	0.115	0.076
	-20	0.077	0.049	0.070	0.091	0.036	0.104	0.030	0.073	0.053	0.036	0.098	0.073	0.078	0.054	0.074	0.086	0.041	0.104	0.026	0.070	0.047	0.037	0.103	0.078
	-50	0.056	0.048	0.054	0.269	0.024	0.262	0.043	0.076	0.063	0.039	0.077	0.064	0.056	0.051	0.050	0.237	0.023	0.228	0.038	0.068	0.055	0.048	0.090	0.074
10	0	0.002	0.159	0.086	0.002	0.046	0.038	0.000	0.060	0.033	0.000	0.126	0.070	0.003	0.205	0.117	0.011	0.043	0.041	0.000	0.056	0.030	0.000	0.121	0.064
	-5	0.160	0.050	0.144	0.010	0.046	0.041	0.004	0.068	0.037	0.007	0.123	0.074	0.161	0.064	0.156	0.017	0.046	0.048	0.002	0.063	0.033	0.008	0.129	0.073
	-10	0.121	0.043	0.108	0.036	0.039	0.051	0.015	0.071	0.045	0.020	0.112	0.075	0.125	0.053	0.119	0.043	0.044	0.067	0.013	0.067	0.040	0.023	0.117	0.079
	-20	0.070	0.044	0.063	0.075	0.026	0.076	0.030	0.075	0.055	0.032	0.097	0.074	0.075	0.049	0.066	0.077	0.036	0.091	0.025	0.073	0.050	0.038	0.102	0.080
	-50	0.051	0.052	0.053	0.204	0.011	0.182	0.044	0.074	0.065	0.036	0.075	0.060	0.051	0.047	0.047	0.205	0.017	0.190	0.038	0.072	0.057	0.044	0.088	0.074
20	0	0.003	0.115	0.075	0.001	0.045	0.033	0.000	0.056	0.031	0.000	0.126	0.067	0.003	0.157	0.087	0.004	0.042	0.035	0.000	0.052	0.029	0.000	0.124	0.062
	-5	0.159	0.056	0.159	0.007	0.038	0.029	0.003	0.062	0.033	0.005	0.120	0.070	0.161	0.051	0.148	0.013	0.042	0.039	0.002	0.063	0.032	0.007	0.124	0.072
	-10	0.115	0.052	0.115	0.022	0.025	0.030	0.015	0.064	0.041	0.017	0.108	0.069	0.119	0.044	0.113	0.035	0.039	0.051	0.012	0.064	0.036	0.021	0.115	0.077
	-20	0.061	0.055	0.066	0.046	0.011	0.040	0.031	0.064	0.048	0.028	0.092	0.063	0.068	0.045	0.059	0.061	0.026	0.064	0.026	0.068	0.047	0.037	0.105	0.080
	-50	0.050	0.058	0.059	0.098	0.002	0.065	0.045	0.052	0.048	0.029	0.067	0.051	0.047	0.051	0.049	0.148	0.008	0.122	0.040	0.064	0.055	0.044	0.086	0.074
50	0	0.020	0.211	0.199	0.000	0.031	0.017	0.000	0.043	0.021	0.000	0.130	0.075	0.005	0.118	0.088	0.000	0.037	0.027	0.000	0.046	0.023	0.000	0.123	0.066
	-5	0.152	0.133	0.221	0.002	0.020	0.009	0.005	0.045	0.024	0.004	0.132	0.077	0.158	0.072	0.176	0.006	0.032	0.023	0.003	0.051	0.024	0.005	0.122	0.068
	-10	0.100	0.089	0.138	0.007	0.011	0.007	0.016	0.043	0.030	0.010	0.122	0.079	0.112	0.063	0.127	0.017	0.021	0.023	0.011	0.052	0.031	0.014	0.112	0.072
	-20	0.050	0.065	0.066	0.010	0.002	0.005	0.033	0.039	0.035	0.014	0.106	0.070	0.055	0.060	0.065	0.025	0.008	0.020	0.026	0.053	0.035	0.028	0.101	0.070
	-50	0.062	0.059	0.071	0.008	0.000	0.003	0.033	0.027	0.027	0.015	0.067	0.046	0.049	0.057	0.054	0.036	0.001	0.021	0.040	0.046	0.039	0.034	0.081	0.063
		$\lambda = 1/2$, and $\sigma_1 = 10$ and $\sigma_2 = 1$												$\lambda = 1/2$, and $\sigma_1 = 10$ and $\sigma_2 = 1$											
1	0	0.005	0.300	0.190	0.013	0.059	0.058	0.000	0.057	0.029	0.000	0.149	0.080	0.005	0.299	0.183	0.024	0.054	0.061	0.000	0.053	0.027	0.000	0.147	0.079
	-5	0.171	0.077	0.179	0.020	0.075	0.077	0.005	0.070	0.035	0.011	0.146	0.096	0.169	0.076	0.179	0.026	0.068	0.077	0.004	0.063	0.033	0.010	0.153	0.095
	-10	0.137	0.064	0.140	0.057	0.070	0.103	0.018	0.071	0.042	0.030	0.134	0.097	0.136	0.064	0.138	0.059	0.068	0.105	0.015	0.069	0.039	0.032	0.141	0.103
	-20	0.087	0.059	0.088	0.127	0.055	0.157	0.030	0.069	0.051	0.047	0.118	0.099	0.087	0.057	0.086	0.108	0.061	0.144	0.028	0.067	0.046	0.049	0.133	0.110
	-50	0.066	0.059	0.067	0.343	0.048	0.368	0.042	0.062	0.054	0.062	0.096	0.095	0.060	0.057	0.062	0.288	0.040	0.301	0.038	0.063	0.049	0.063	0.119	0.108
5	0	0.005	0.242	0.140	0.005	0.059	0.051	0.000	0.056	0.029	0.000	0.145	0.081	0.005	0.268	0.158	0.015	0.054	0.053	0.000	0.055	0.027	0.000	0.143	0.079
	-5	0.169	0.058	0.165	0.015	0.072	0.069	0.003	0.068	0.038	0.009	0.147	0.091	0.169	0.065	0.171	0.022	0.067	0.070	0.002	0.065	0.034	0.010	0.150	0.094
	-10	0.133	0.048	0.125	0.048	0.063	0.086	0.015	0.074	0.046	0.027	0.133	0.093	0.134	0.056	0.131	0.056	0.064	0.096	0.013	0.067	0.040	0.029	0.137	0.100
	-20	0.081	0.046	0.073	0.109	0.047	0.131	0.029	0.076	0.056	0.044	0.123	0.098	0.084	0.051	0.076	0.101	0.056	0.130	0.026	0.070	0.048	0.047	0.127	0.103
	-50	0.057	0.049	0.053	0.297	0.031	0.297	0.043	0.079	0.065	0.053	0.096	0.084	0.056	0.050	0.053	0.264	0.032	0.267	0.038	0.069	0.058	0.060	0.113	0.102
10	0	0.005	0.186	0.101	0.002	0.060	0.047	0.000	0.057	0.029	0.000	0.148	0.085	0.006	0.231	0.132	0.009	0.052	0.047	0.000	0.054	0.026	0.000	0.143	0.079
	-5	0.170	0.045	0.158	0.010	0.067	0.057	0.003	0.069	0.038	0.007	0.146	0.090	0.167	0.056	0.163	0.018	0.065	0.063	0.002	0.066	0.036	0.008	0.148	0.093
	-10	0.132	0.042	0.120	0.042	0.055	0.071	0.014	0.076	0.047	0.024	0.131	0.093	0.133	0.048	0.123	0.050	0.060	0.087	0.012	0.069	0.042	0.029	0.135	0.099
	-20	0.074	0.044	0.072	0.091	0.037	0.098	0.029	0.080	0.058	0.044	0.118	0.094	0.079	0.045	0.070	0.093	0.050	0.114	0.026	0.073	0.050	0.046	0.124	0.105
	-50	0.052	0.056	0.056	0.237	0.016	0.215	0.048	0.077	0.068	0.048	0.094	0.081	0.052	0.047	0.049	0.234	0.026	0.227	0.040	0.073	0.061	0.057	0.108	0.098
20	0	0.008	0.141	0.095	0.000	0.057	0.043	0.000	0.053	0.027	0.000	0.141	0.079	0.007	0.173	0.097	0.002	0.051	0.043	0.000	0.051	0.027	0.000	0.140	0.077
	-5	0.169	0.061	0.177	0.007	0.053	0.040	0.003	0.066	0.035	0.004	0.138	0.084	0.169	0.044	0.158	0.011	0.059	0.053	0.002	0.063	0.033	0.007	0.142	0.090
	-10	0.124	0.056	0.132	0.027	0.036	0.042	0.014	0.069	0.043	0.019	0.124	0.082	0.128	0.041	0.119	0.040	0.051	0.070	0.011	0.065	0.040	0.027	0.133	0.095
	-20	0.063	0.059	0.074	0.058	0.016	0.051	0.030	0.067	0.050	0.034	0.107	0.082	0.073	0.044	0.065	0.073	0.036	0.084	0.025	0.068	0.048	0.045	0.123	0.106
	-50	0.052	0.062	0.062	0.120	0.003	0.086	0.047	0.051	0.050	0.038	0.082	0.066	0.048	0.052	0.051	0.175	0.014	0.151	0.041	0.068	0.059	0.054	0.107	0.097
50	0	0.031	0.252	0.247	0.000	0.045	0.027	0.000	0.039	0.019	0.000	0.146	0.090	0.011	0.145	0.115	0.000	0.048	0.034	0.000	0.042	0.021	0.000	0.142	0.076
	-5	0.167	0.145	0.261	0.002	0.031	0.016	0.003	0.047	0.025	0.002	0.142	0.088	0.167	0.078	0.197	0.005	0.042	0.031	0.002	0.052	0.025	0.005	0.139	0.085
	-10	0.111	0.100	0.157	0.008	0.015	0.011	0.016	0.045	0.030	0.010	0.129	0.086	0.121	0.069	0.146	0.020	0.030	0.032	0.011	0.055	0.031	0.017	0.129	0.087
	-20	0.054	0.071	0.073	0.010	0.003	0.006	0.033	0.042	0.039	0.018	0.111	0.077	0.058	0.066	0.071	0.036	0.014	0.030	0.025	0.051	0.036	0.035	0.118	0.087
	-50	0.067	0.061	0.076	0.010	0.000	0.004	0.034	0.028	0.029	0.017	0.074	0.052	0.050	0.061	0.058	0.050	0.002	0.031	0.040	0.046	0.042	0.043	0.096	0.080

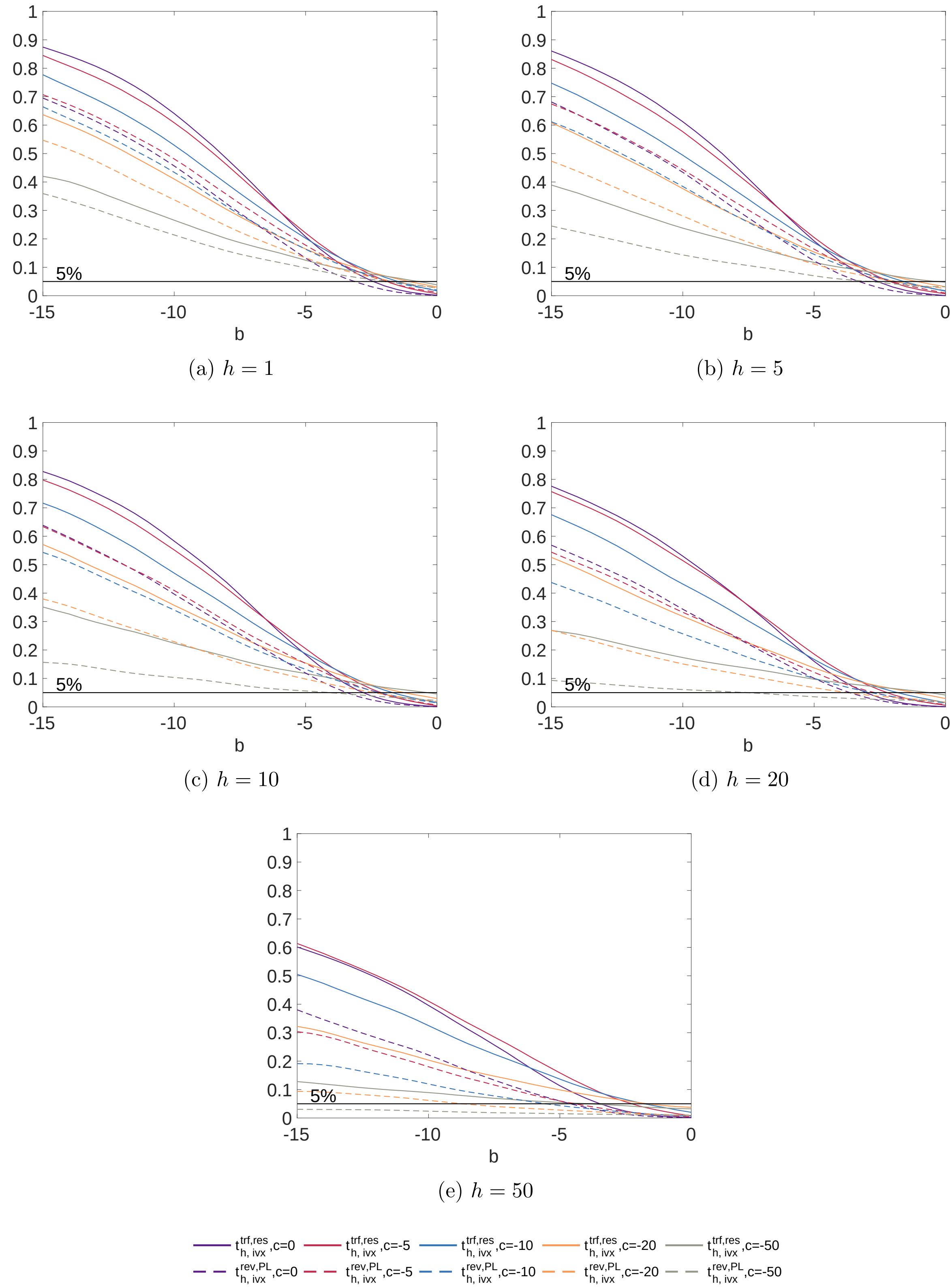
See note under Table S.1.

Table S.7: Empirical rejection frequencies of one-sided (left and right tail) and two-sided predictability tests, for sample sizes $T = 250$ and $T = 500$. **DGP (Unconditional Heteroskedasticity):** $y_t = \beta x_{t-1} + u_t, x_t = \rho x_{t-1} + v_t$ and $v_t = \psi v_{t-1} + \nu_t$, where $\beta = 0$, $\rho = 1 - c/T, \psi = 0$ and $(u_t, \nu_t)' \sim NIID(\mathbf{0}, \mathbf{\Sigma_t})$, with $\mathbf{\Sigma}_t = \begin{bmatrix}\sigma^2_{ut} & -0.95\sigma_{ut}\sigma_{\nu t}; & -0.95\sigma_{ut}\sigma_{\nu t} & \sigma^2_{\nu t}\end{bmatrix}$ and $\sigma^2_{ut} = \sigma^2_{vt} = \sigma_1 \mathbb{I}(t \leq \lfloor \lambda T \rfloor) + \sigma_2 \mathbb{I}(t > \lfloor \lambda T \rfloor)$.

		t_h^{Xu}			t_h^{Bonf}			$t_{h,ivx}^{trf,res}$			$t_h^{rev,PL}$			t_h^{Xu}			t_h^{Bonf}			$t_{h,ivx}^{trf,res}$			$t_h^{rev,PL}$		
		$T = 250$												$T = 500$											
h	c	$\beta_h < 0$	$\beta_h > 0$	$\beta_h \neq 0$	$\beta_h < 0$	$\beta_h > 0$	$\beta_h \neq 0$	$\beta_h < 0$	$\beta_h > 0$	$\beta_h \neq 0$	$\beta_h < 0$	$\beta_h > 0$	$\beta_h \neq 0$	$\beta_h < 0$	$\beta_h > 0$	$\beta_h \neq 0$	$\beta_h < 0$	$\beta_h > 0$	$\beta_h \neq 0$	$\beta_h < 0$	$\beta_h > 0$	$\beta_h \neq 0$	$\beta_h < 0$	$\beta_h > 0$	$\beta_h \neq 0$
		$\lambda = 3/4$, and $\sigma_1 = 4$ and $\sigma_2 = 1$												$\lambda = 3/4$, and $\sigma_1 = 4$ and $\sigma_2 = 1$											
1	0	0.002	0.242	0.149	0.018	0.045	0.047	0.000	0.061	0.033	0.000	0.115	0.062	0.002	0.230	0.137	0.024	0.040	0.047	0.000	0.060	0.034	0.000	0.114	0.057
	-5	0.141	0.086	0.158	0.016	0.050	0.053	0.004	0.065	0.037	0.007	0.114	0.067	0.148	0.085	0.161	0.022	0.043	0.051	0.005	0.060	0.032	0.009	0.117	0.066
	-10	0.118	0.069	0.125	0.038	0.048	0.067	0.017	0.067	0.042	0.019	0.101	0.065	0.120	0.072	0.126	0.041	0.045	0.068	0.016	0.062	0.038	0.020	0.107	0.068
	-20	0.079	0.061	0.080	0.081	0.038	0.100	0.029	0.064	0.048	0.029	0.086	0.060	0.081	0.063	0.083	0.076	0.038	0.094	0.030	0.064	0.044	0.032	0.095	0.068
	-50	0.059	0.054	0.062	0.254	0.032	0.262	0.041	0.057	0.050	0.038	0.066	0.055	0.057	0.061	0.064	0.210	0.027	0.212	0.039	0.058	0.053	0.038	0.077	0.064
5	0	0.002	0.193	0.111	0.009	0.045	0.039	0.000	0.061	0.033	0.000	0.115	0.060	0.001	0.205	0.116	0.018	0.040	0.042	0.000	0.060	0.034	0.000	0.113	0.056
	-5	0.142	0.071	0.139	0.012	0.047	0.045	0.003	0.071	0.039	0.007	0.117	0.064	0.148	0.078	0.152	0.018	0.041	0.047	0.005	0.064	0.034	0.009	0.114	0.066
	-10	0.114	0.057	0.110	0.031	0.043	0.053	0.015	0.072	0.043	0.017	0.106	0.066	0.120	0.064	0.120	0.037	0.041	0.060	0.016	0.066	0.041	0.020	0.106	0.068
	-20	0.073	0.049	0.066	0.067	0.030	0.076	0.027	0.073	0.054	0.026	0.090	0.061	0.078	0.056	0.076	0.069	0.034	0.083	0.029	0.067	0.048	0.031	0.094	0.066
	-50	0.053	0.048	0.053	0.207	0.019	0.199	0.043	0.074	0.062	0.034	0.068	0.052	0.054	0.055	0.056	0.188	0.023	0.179	0.040	0.071	0.060	0.038	0.080	0.060
10	0	0.001	0.147	0.079	0.004	0.042	0.035	0.000	0.061	0.034	0.000	0.116	0.060	0.002	0.176	0.096	0.012	0.039	0.038	0.000	0.059	0.033	0.000	0.112	0.056
	-5	0.141	0.057	0.131	0.009	0.042	0.038	0.003	0.069	0.038	0.006	0.114	0.063	0.148	0.071	0.143	0.015	0.041	0.041	0.004	0.065	0.035	0.008	0.113	0.064
	-10	0.111	0.049	0.100	0.024	0.034	0.041	0.015	0.071	0.046	0.016	0.104	0.063	0.117	0.056	0.113	0.032	0.038	0.052	0.015	0.069	0.041	0.019	0.103	0.066
	-20	0.067	0.045	0.058	0.053	0.023	0.056	0.028	0.074	0.053	0.025	0.088	0.061	0.074	0.052	0.069	0.059	0.030	0.070	0.029	0.068	0.050	0.031	0.095	0.064
	-50	0.049	0.050	0.052	0.151	0.009	0.123	0.043	0.069	0.060	0.028	0.067	0.049	0.050	0.052	0.053	0.157	0.017	0.145	0.042	0.074	0.063	0.037	0.078	0.060
20	0	0.002	0.101	0.062	0.001	0.039	0.028	0.000	0.056	0.030	0.000	0.112	0.058	0.002	0.133	0.069	0.006	0.038	0.033	0.000	0.058	0.032	0.000	0.112	0.056
	-5	0.140	0.058	0.139	0.006	0.033	0.025	0.003	0.063	0.035	0.004	0.109	0.058	0.148	0.055	0.136	0.010	0.037	0.034	0.004	0.062	0.033	0.007	0.113	0.065
	-10	0.104	0.053	0.104	0.016	0.024	0.025	0.015	0.064	0.040	0.013	0.102	0.060	0.114	0.048	0.104	0.024	0.033	0.040	0.015	0.064	0.040	0.019	0.104	0.064
	-20	0.059	0.054	0.059	0.032	0.010	0.027	0.028	0.062	0.046	0.022	0.086	0.058	0.068	0.046	0.062	0.044	0.023	0.047	0.029	0.065	0.048	0.029	0.091	0.064
	-50	0.049	0.055	0.056	0.064	0.001	0.043	0.044	0.047	0.045	0.024	0.064	0.043	0.047	0.054	0.052	0.106	0.007	0.087	0.043	0.067	0.057	0.037	0.078	0.060
50	0	0.022	0.180	0.173	0.000	0.022	0.011	0.000	0.042	0.022	0.000	0.115	0.063	0.003	0.093	0.067	0.001	0.034	0.025	0.000	0.050	0.028	0.000	0.110	0.057
	-5	0.134	0.129	0.194	0.002	0.017	0.009	0.004	0.042	0.023	0.001	0.115	0.063	0.144	0.068	0.155	0.005	0.026	0.020	0.004	0.053	0.028	0.003	0.111	0.061
	-10	0.086	0.086	0.120	0.005	0.008	0.006	0.015	0.041	0.025	0.004	0.110	0.061	0.101	0.062	0.110	0.012	0.018	0.016	0.013	0.052	0.030	0.012	0.105	0.063
	-20	0.046	0.065	0.060	0.006	0.002	0.003	0.030	0.035	0.033	0.007	0.097	0.057	0.054	0.059	0.065	0.017	0.006	0.012	0.027	0.050	0.039	0.023	0.098	0.060
	-50	0.059	0.055	0.067	0.004	0.000	0.001	0.031	0.027	0.026	0.010	0.065	0.041	0.049	0.059	0.059	0.023	0.001	0.012	0.039	0.049	0.043	0.028	0.077	0.051
		$\lambda = 3/4$, and $\sigma_1 = 10$ and $\sigma_2 = 1$												$\lambda = 3/4$, and $\sigma_1 = 10$ and $\sigma_2 = 1$											
1	0	0.005	0.263	0.163	0.016	0.048	0.049	0.000	0.061	0.032	0.000	0.123	0.067	0.005	0.251	0.149	0.022	0.043	0.050	0.000	0.059	0.033	0.000	0.120	0.060
	-5	0.149	0.087	0.164	0.015	0.057	0.056	0.004	0.065	0.037	0.008	0.123	0.073	0.155	0.083	0.167	0.019	0.048	0.053	0.005	0.061	0.031	0.009	0.122	0.072
	-10	0.120	0.070	0.129	0.038	0.054	0.075	0.017	0.068	0.043	0.021	0.110	0.071	0.126	0.070	0.128	0.042	0.050	0.075	0.016	0.062	0.038	0.023	0.111	0.075
	-20	0.080	0.062	0.083	0.086	0.042	0.107	0.029	0.065	0.048	0.031	0.095	0.069	0.080	0.062	0.085	0.077	0.044	0.104	0.029	0.064	0.046	0.035	0.100	0.076
	-50	0.060	0.056	0.063	0.263	0.037	0.275	0.041	0.057	0.051	0.041	0.072	0.063	0.058	0.059	0.063	0.220	0.030	0.226	0.039	0.060	0.052	0.043	0.084	0.071
5	0	0.005	0.209	0.122	0.006	0.046	0.041	0.000	0.061	0.033	0.000	0.121	0.067	0.005	0.219	0.128	0.016	0.043	0.044	0.000	0.058	0.033	0.000	0.118	0.061
	-5	0.149	0.071	0.149	0.011	0.052	0.049	0.003	0.069	0.040	0.007	0.127	0.073	0.154	0.076	0.156	0.015	0.047	0.048	0.005	0.063	0.034	0.009	0.121	0.070
	-10	0.116	0.056	0.114	0.031	0.048	0.060	0.014	0.072	0.046	0.018	0.113	0.071	0.122	0.062	0.121	0.038	0.047	0.068	0.015	0.065	0.040	0.022	0.112	0.072
	-20	0.075	0.050	0.068	0.071	0.034	0.082	0.028	0.075	0.054	0.029	0.097	0.067	0.079	0.056	0.076	0.071	0.040	0.092	0.030	0.067	0.048	0.035	0.102	0.075
	-50	0.053	0.049	0.052	0.218	0.022	0.210	0.043	0.075	0.062	0.037	0.073	0.058	0.055	0.054	0.056	0.197	0.025	0.192	0.040	0.071	0.059	0.044	0.086	0.070
10	0	0.005	0.156	0.086	0.003	0.044	0.037	0.000	0.060	0.032	0.000	0.122	0.065	0.004	0.187	0.103	0.010	0.041	0.039	0.000	0.057	0.033	0.000	0.119	0.059
	-5	0.149	0.055	0.137	0.008	0.048	0.042	0.003	0.069	0.039	0.006	0.122	0.068	0.152	0.066	0.150	0.012	0.045	0.045	0.005	0.065	0.034	0.008	0.119	0.070
	-10	0.113	0.048	0.104	0.025	0.040	0.045	0.014	0.073	0.047	0.017	0.108	0.069	0.123	0.055	0.115	0.033	0.043	0.059	0.015	0.067	0.041	0.023	0.110	0.071
	-20	0.069	0.046	0.060	0.056	0.025	0.061	0.028	0.075	0.054	0.027	0.096	0.067	0.076	0.051	0.069	0.063	0.036	0.079	0.030	0.069	0.050	0.036	0.100	0.071
	-50	0.049	0.051	0.053	0.159	0.010	0.135	0.043	0.070	0.061	0.032	0.072	0.055	0.052	0.051	0.049	0.168	0.018	0.157	0.042	0.075	0.063	0.042	0.083	0.070
20	0	0.006	0.110	0.072	0.000	0.041	0.029	0.000	0.057	0.028	0.000	0.116	0.060	0.005	0.140	0.075	0.005	0.040	0.034	0.000	0.055	0.031	0.000	0.116	0.060
	-5	0.149	0.059	0.150	0.005	0.038	0.027	0.003	0.064	0.033	0.004	0.114	0.064	0.153	0.053	0.142	0.008	0.042	0.038	0.004	0.062	0.032	0.007	0.118	0.069
	-10	0.105	0.054	0.109	0.017	0.025	0.026	0.015	0.065	0.041	0.014	0.106	0.063	0.117	0.045	0.107	0.025	0.038	0.044	0.014	0.065	0.040	0.021	0.106	0.070
	-20	0.058	0.055	0.063	0.035	0.012	0.029	0.028	0.063	0.048	0.024	0.092	0.062	0.070	0.047	0.062	0.048	0.027	0.053	0.028	0.065	0.049	0.033	0.096	0.072
	-50	0.048	0.057	0.058	0.069	0.001	0.048	0.043	0.048	0.045	0.026	0.069	0.047	0.048	0.055	0.052	0.114	0.008	0.096	0.043	0.066	0.058	0.041	0.084	0.067
50	0	0.045	0.190	0.191	0.000	0.026	0.012	0.000	0.043	0.021	0.000	0.117	0.065	0.011	0.098	0.075	0.000	0.035	0.025	0.000	0.048	0.025	0.000	0.114	0.056
	-5	0.145	0.133	0.215	0.001	0.020	0.011	0.003	0.043	0.022	0.000	0.117	0.066	0.150	0.071	0.169	0.004	0.031	0.022	0.004	0.053	0.026	0.003	0.117	0.065
	-10	0.089	0.090	0.128	0.005	0.010	0.007	0.015	0.043	0.025	0.003	0.111	0.063	0.107	0.064	0.118	0.013	0.023	0.019	0.013	0.052	0.031	0.012	0.109	0.066
	-20	0.048	0.068	0.062	0.006	0.003	0.004	0.029	0.037	0.032	0.007	0.100	0.058	0.057	0.060	0.067	0.020	0.008	0.015	0.027	0.051	0.041	0.025	0.100	0.065
	-50	0.060	0.056	0.070	0.005	0.000	0.002	0.029	0.028	0.026	0.010	0.067	0.042	0.048	0.060	0.060	0.026	0.001	0.014	0.040	0.047	0.043	0.031	0.079	0.057

See note under Table S.1.

Power plots when innovations are iid



(a) $h = 1$ (b) $h = 5$ (c) $h = 10$ (d) $h = 20$ (e) $h = 50$

Figure S.1: Power curves of the **LEFT**-sided tests $t^{trf,res}_{h,ivx}$ and $t^{rev,PL}_{h,ivx}$ for prediction horizon $h = \{1, 5, 10, 20, 50\}$ and $T = 250$. **DGP:** $y_{t+1} = \beta x_t + u_{t+1}, x_{t+1} = \rho x_t + v_{t+1}$ and $v_{t+1} = \psi v_t + \nu_{t+1}$, where $\beta = b/T$, $\rho = 1 + c/T$, with $c = \{0, -5, -10, -20, -50\}$, $\psi = 0$ and $(u_{t+1}, \nu_{t+1})' \sim NIID(\mathbf{0}, \mathbf{\Sigma})$, with $\mathbf{\Sigma} = [1 \quad -0.95; \quad -0.95 \quad 1]$.

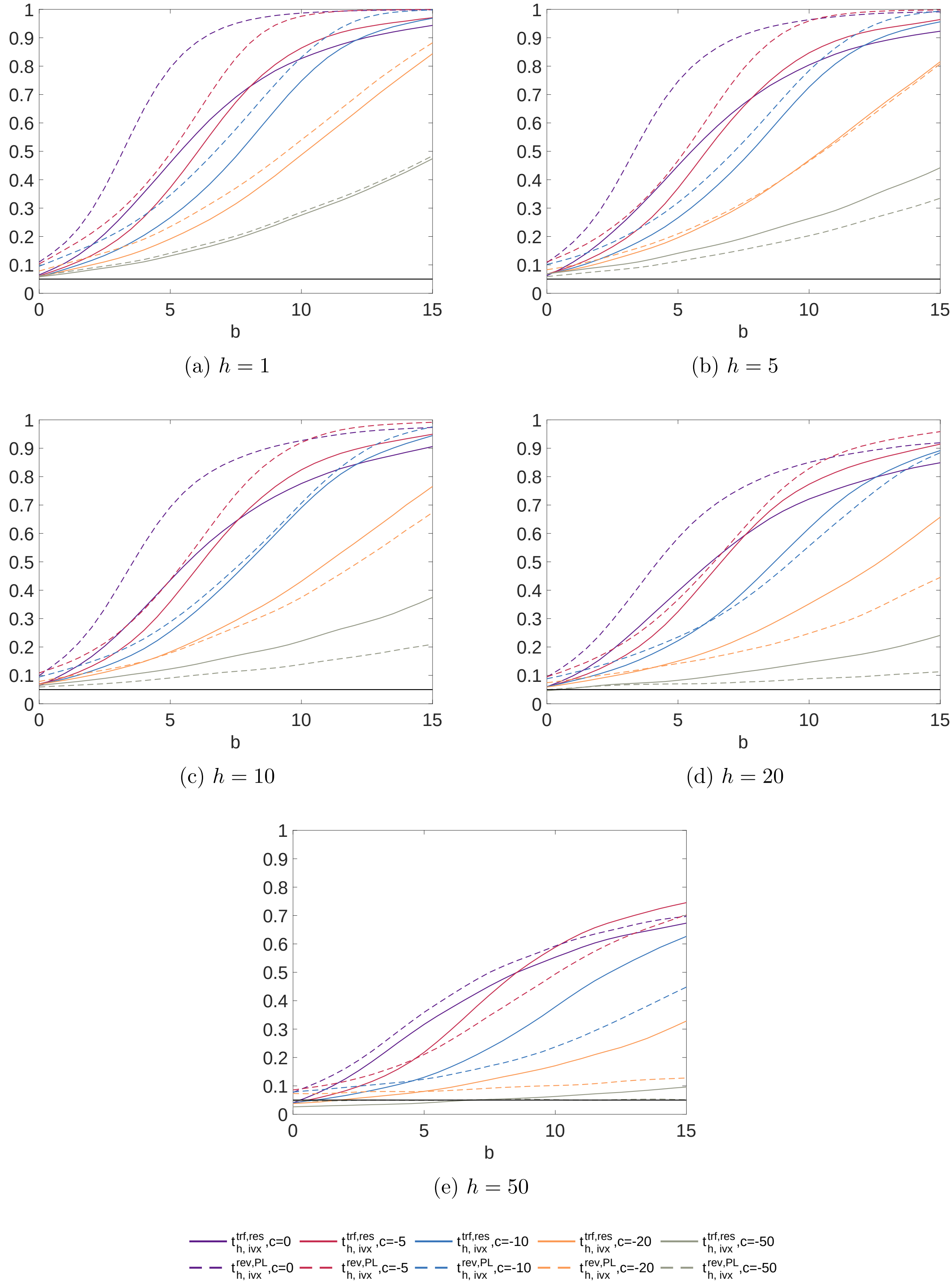


Figure S.2: Power curves of the **RIGHT**-sided tests $t^{trf,res}_{h,ivx}$ and $t^{rev,PL}_{h,ivx}$ for prediction horizon $h = \{1, 5, 10, 20, 50\}$ and $T = 250$. **DGP:** $y_{t+1} = \beta x_t + u_{t+1}, x_{t+1} = \rho x_t + v_{t+1}$ and $v_{t+1} = \psi v_t + \nu_{t+1}$, where $\beta = b/T$, $\rho = 1 + c/T$, with $c = \{0, -5, -10, -20, -50\}$, $\psi = 0$ and $(u_{t+1}, \nu_{t+1})' \sim NIID(\mathbf{0}, \mathbf{\Sigma})$, with $\mathbf{\Sigma} = [1 \quad -0.95; \quad -0.95 \quad 1]$.

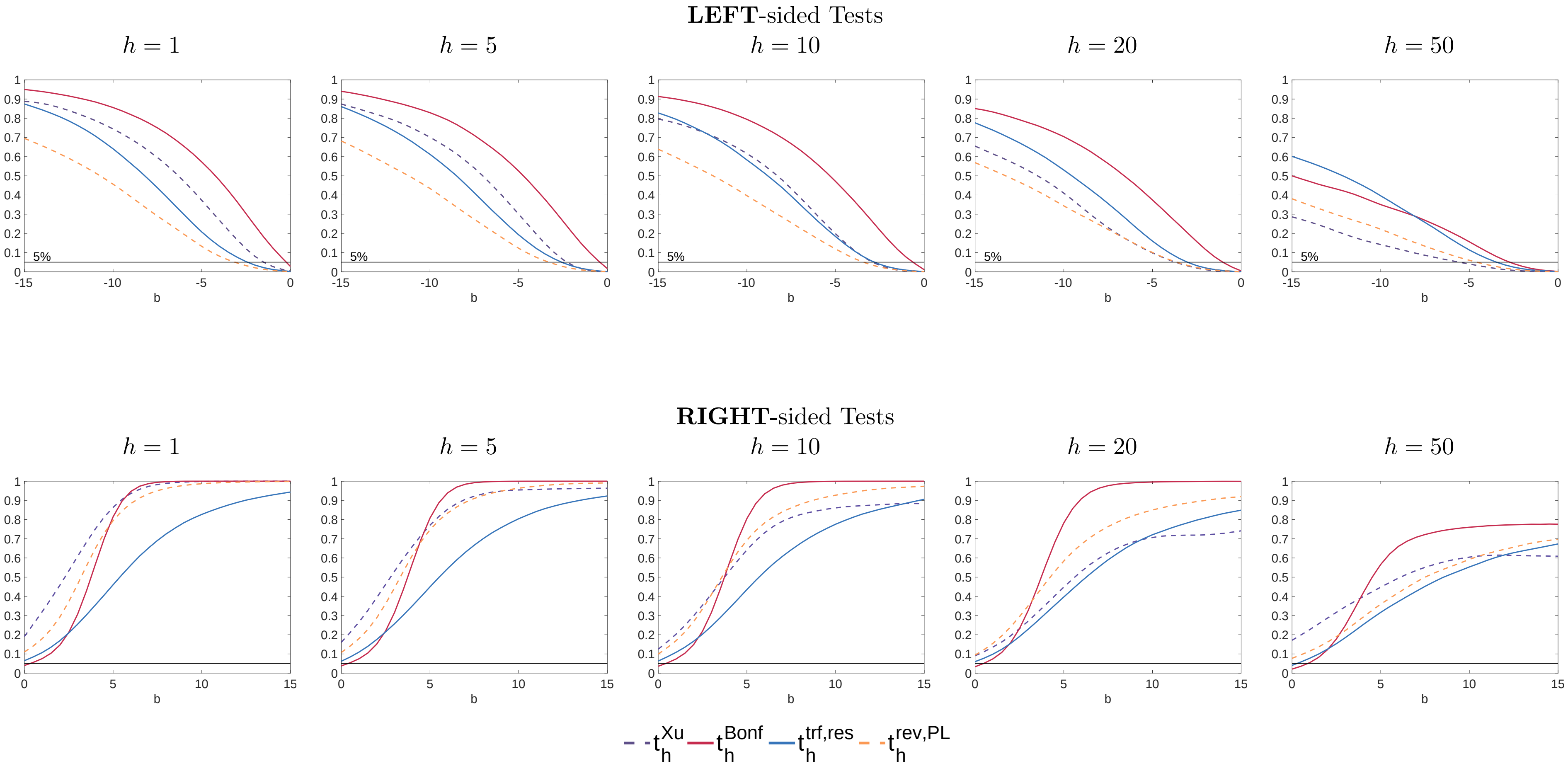


Figure S.3 Power curves of the t_h^{Xu}, t_h^{Bonf}, $t_{h,ivx}^{trf,res}$ and $t_{h,ivx}^{rev,PL}$ tests for prediction horizon $h = \{1, 5, 10, 20, 50\}$ and $T = 250$. **DGP:** $y_{t+1} = \beta x_t + u_{t+1}, x_{t+1} = \rho x_t + v_{t+1}$ and $v_{t+1} = \psi v_t + \nu_{t+1}$, where $\beta = b/T$ with $b \in \{-15, -14.5, ..., 0, ..., 15\}$, $\rho = 1$, $\psi = 0$ and $(u_{t+1}, \nu_{t+1})' \sim NIID(\mathbf{0}, \mathbf{\Sigma})$, with $\mathbf{\Sigma} = [1 \quad -0.95; \quad -0.95 \quad 1]$.

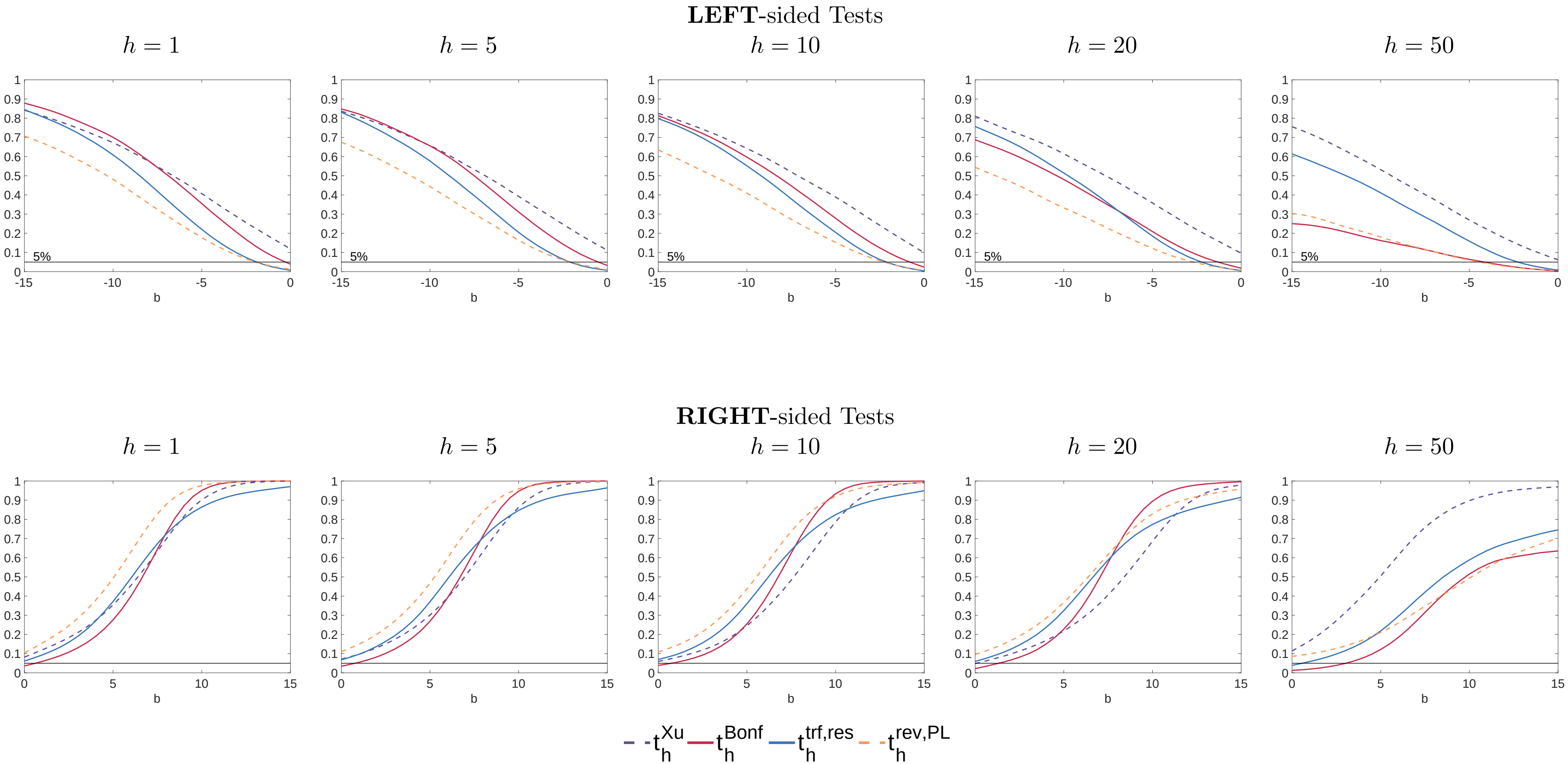


Figure S.4: Power curves of the t_h^{Xu}, t_h^{Bonf}, $t_{h,ivx}^{trf,res}$ and $t_{h,ivx}^{rev,PL}$ tests for prediction horizon $h = \{1, 5, 10, 20, 50\}$ and $T = 250$. **DGP:** $y_{t+1} = \beta x_t + u_{t+1}, x_{t+1} = \rho x_t + v_{t+1}$ and $v_{t+1} = \psi v_t + \nu_{t+1}$, where $\beta = b/T$ with $b \in \{-15, -14.5, ..., 0, ..., 15\}$, $\rho = 1 - 5/T$, $\psi = 0$ and $(u_{t+1}, \nu_{t+1})' \sim NIID(\mathbf{0}, \mathbf{\Sigma})$, with $\mathbf{\Sigma} = [1 \quad -0.95; \quad -0.95 \quad 1]$.

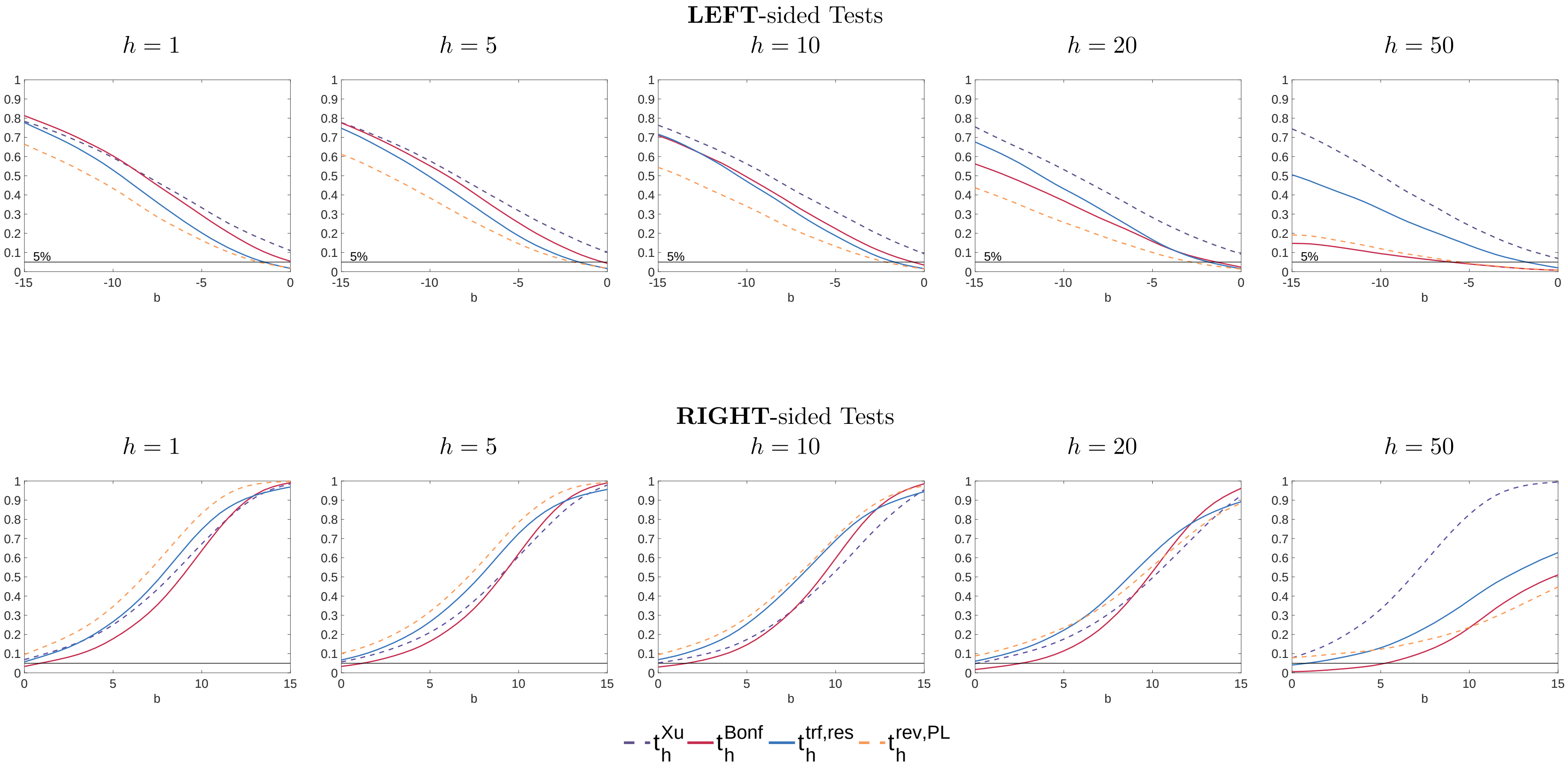


Figure S.5: Power curves of the t_h^{Xu}, t_h^{Bonf}, $t_{h,ivx}^{trf,res}$ and $t_{h,ivx}^{rev,PL}$ tests for prediction horizon $h = \{1, 5, 10, 20, 50\}$ and $T = 250$. **DGP:** $y_{t+1} = \beta x_t + u_{t+1}, x_{t+1} = \rho x_t + v_{t+1}$ and $v_{t+1} = \psi v_t + \nu_{t+1}$, where $\beta = b/T$ with $b \in \{-15, -14.5, ..., 0, ..., 15\}$, $\rho = 1 - 10/T$, $\psi = 0$ and $(u_{t+1}, \nu_{t+1})' \sim NIID(\mathbf{0}, \mathbf{\Sigma})$, with $\mathbf{\Sigma} = [1 \quad -0.95; \quad -0.95 \quad 1]$.

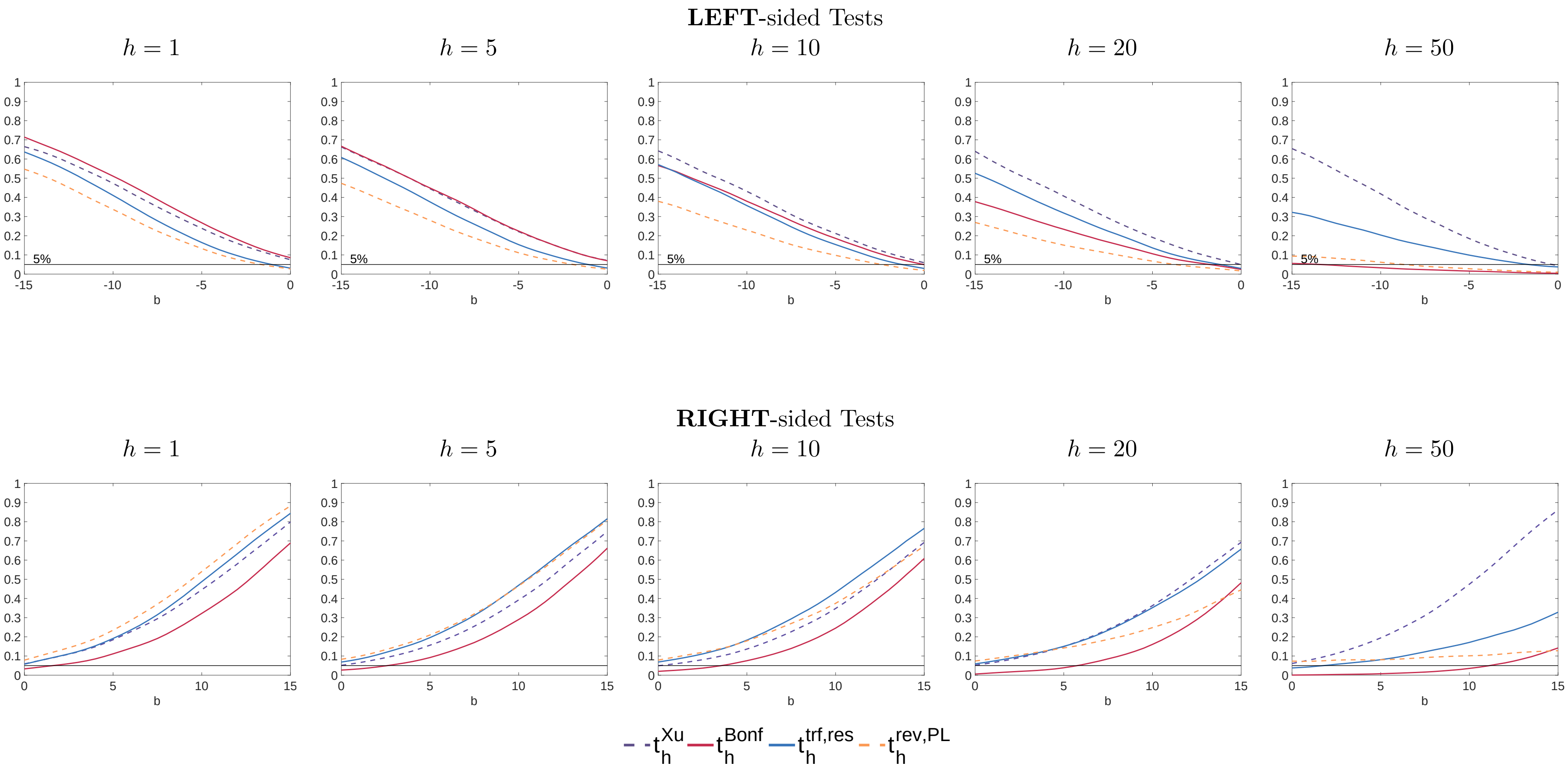


Figure S.6: Power curves of the t_h^{Xu}, t_h^{Bonf}, $t_{h,ivx}^{trf,res}$ and $t_{h,ivx}^{rev,PL}$ tests for prediction horizon $h = \{1, 5, 10, 20, 50\}$ and $T = 250$. **DGP:** $y_{t+1} = \beta x_t + u_{t+1}, x_{t+1} = \rho x_t + v_{t+1}$ and $v_{t+1} = \psi v_t + \nu_{t+1}$, where $\beta = b/T$ with $b \in \{-15, -14.5, ..., 0, ..., 15\}$, $\rho = 1 - 20/T$, $\psi = 0$ and $(u_{t+1}, \nu_{t+1})' \sim NIID(\mathbf{0}, \mathbf{\Sigma})$, with $\mathbf{\Sigma} = [1 \quad -0.95; \quad -0.95 \quad 1]$.

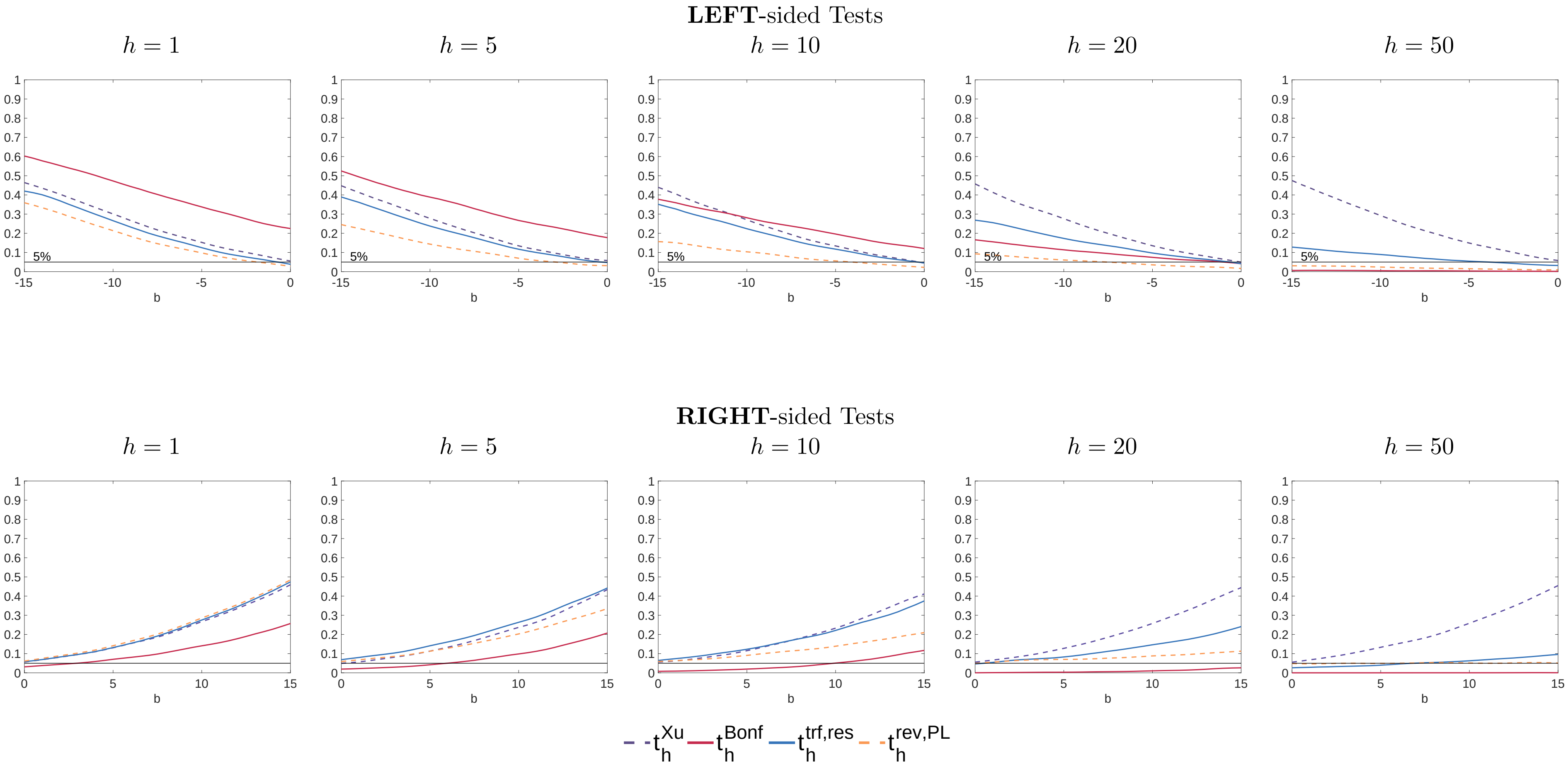


Figure S.7: Power curves of the t_h^{Xu}, t_h^{Bonf}, $t_{h,ivx}^{trf,res}$ and $t_{h,ivx}^{rev,PL}$ tests for prediction horizon $h = \{1, 5, 10, 20, 50\}$ and $T = 250$. **DGP:** $y_{t+1} = \beta x_t + u_{t+1}, x_{t+1} = \rho x_t + v_{t+1}$ and $v_{t+1} = \psi v_t + \nu_{t+1}$, where $\beta = b/T$ with $b \in \{-15, -14.5, ..., 0, ..., 15\}$, $\rho = 1 - 50/T$, $\psi = 0$ and $(u_{t+1}, \nu_{t+1})' \sim NIID(\mathbf{0}, \mathbf{\Sigma})$, with $\mathbf{\Sigma} = [1 \quad -0.95; \quad -0.95 \quad 1]$.

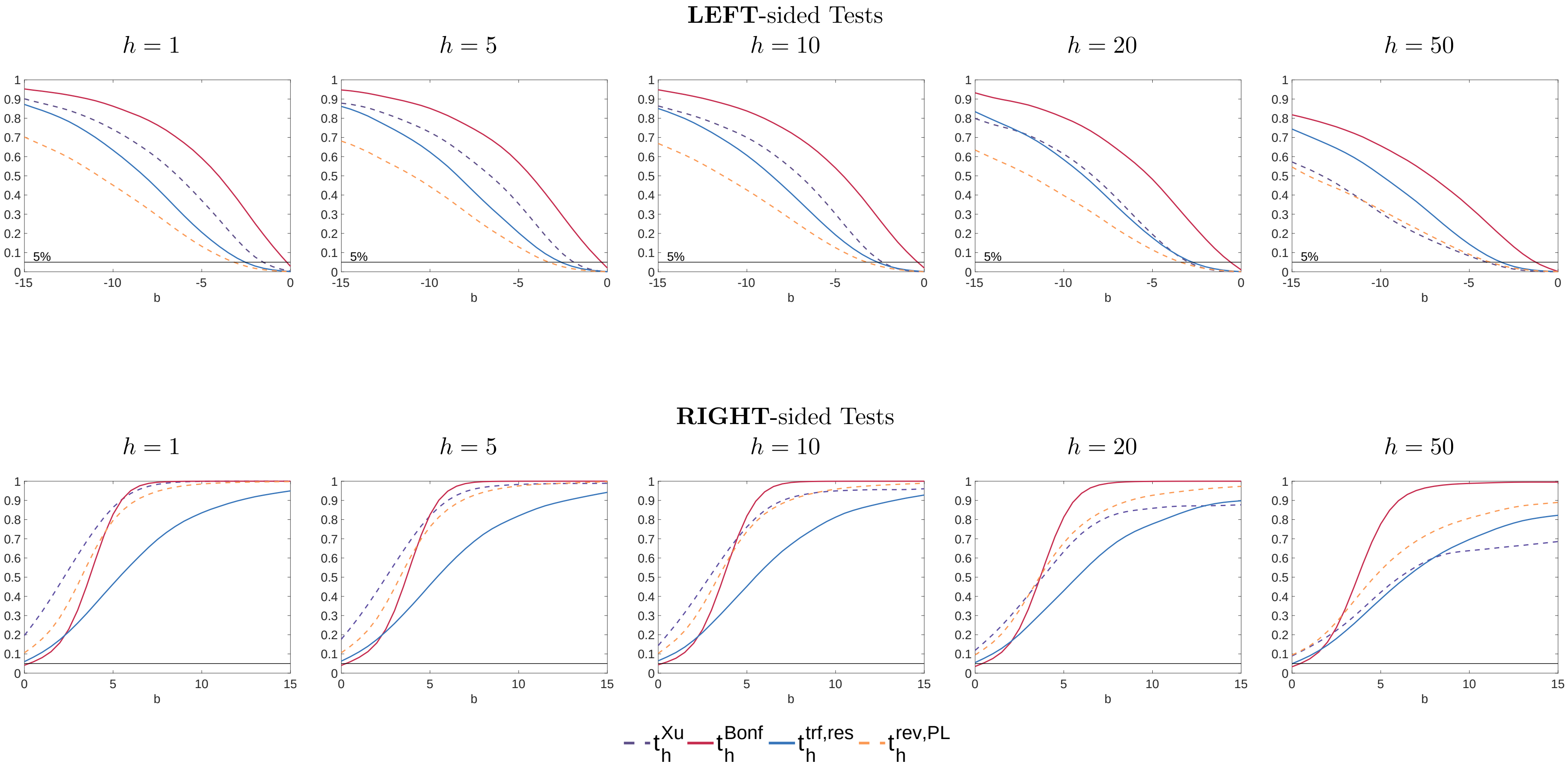


Figure S.8: Power curves of the t_h^{Xu}, t_h^{Bonf}, $t_{h,ivx}^{trf,res}$ and $t_{h,ivx}^{rev,PL}$ tests for prediction horizon $h = \{1, 5, 10, 20, 50\}$ and $T = 500$. **DGP:** $y_{t+1} = \beta x_t + u_{t+1}, x_{t+1} = \rho x_t + v_{t+1}$ and $v_{t+1} = \psi v_t + \nu_{t+1}$, where $\beta = b/T$ with $b \in \{-15, -14.5, ..., 0, ..., 15\}$, $\rho = 1$, $\psi = 0$ and $(u_{t+1}, \nu_{t+1})' \sim NIID(\mathbf{0}, \mathbf{\Sigma})$, with $\mathbf{\Sigma} = [1 \quad -0.95; \quad -0.95 \quad 1]$.

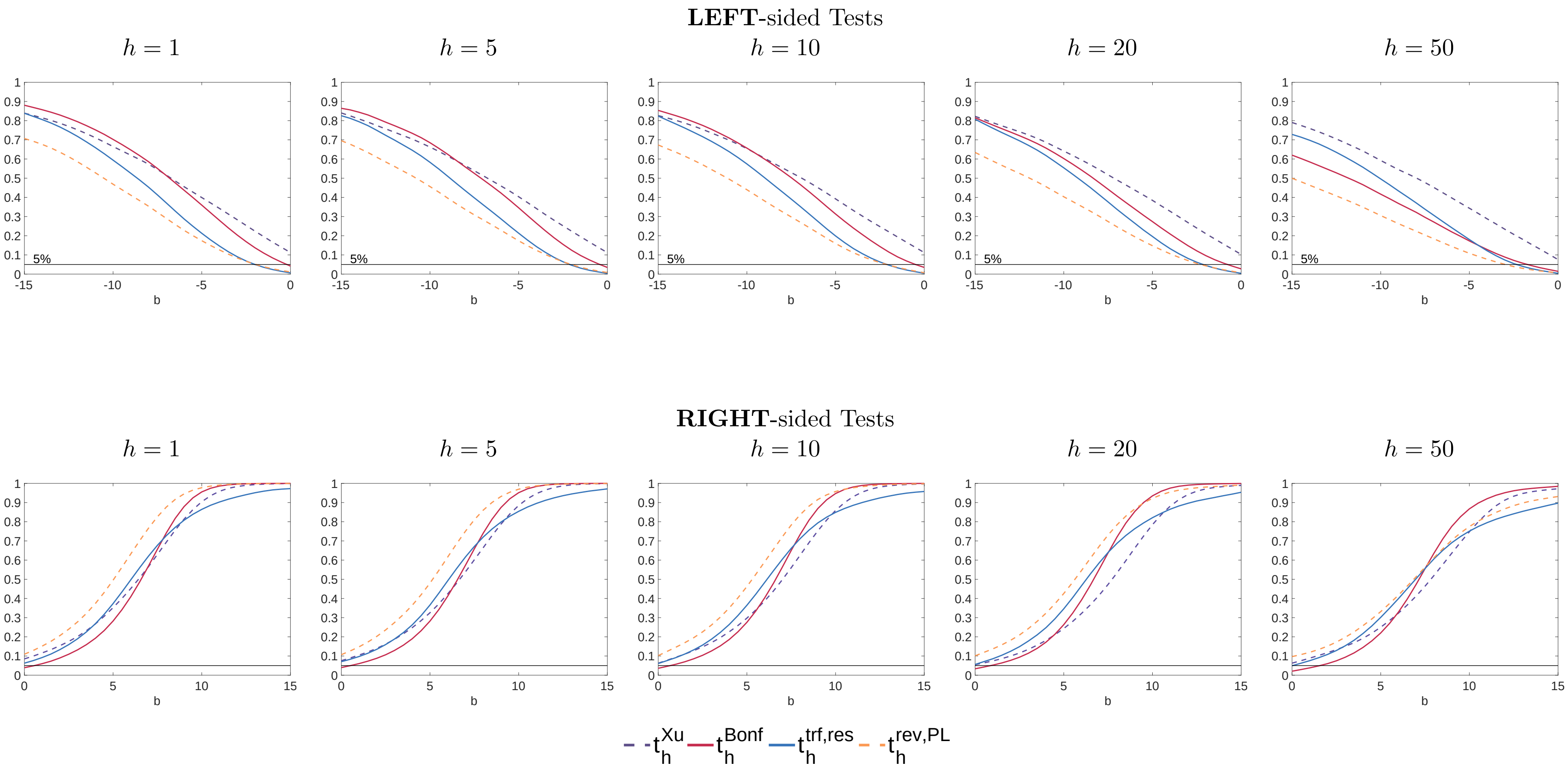


Figure S.9: Power curves of the t_h^{Xu}, t_h^{Bonf}, $t_{h,ivx}^{trf,res}$ and $t_{h,ivx}^{rev,PL}$ tests for prediction horizon $h = \{1, 5, 10, 20, 50\}$ and $T = 500$. **DGP:** $y_{t+1} = \beta x_t + u_{t+1}, x_{t+1} = \rho x_t + v_{t+1}$ and $v_{t+1} = \psi v_t + \nu_{t+1}$, where $\beta = b/T$ with $b \in \{-15, -14.5, ..., 0, ..., 15\}$, $\rho = 1 - 5/T$, $\psi = 0$ and $(u_{t+1}, \nu_{t+1})' \sim NIID(\mathbf{0}, \mathbf{\Sigma})$, with $\mathbf{\Sigma} = [1 \quad -0.95; \quad -0.95 \quad 1]$.

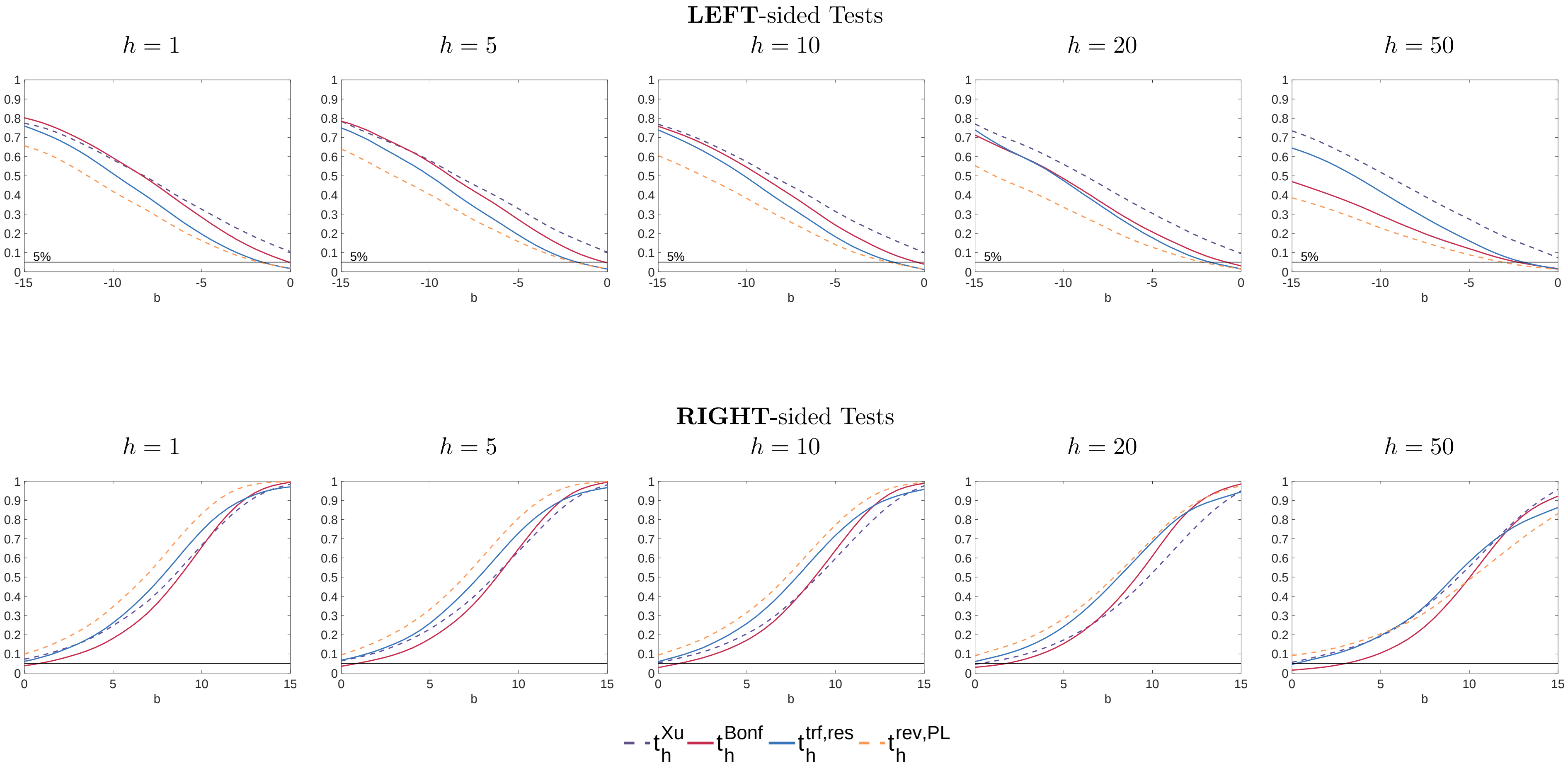


Figure S.10:Power curves of the t_h^{Xu}, t_h^{Bonf}, $t_{h,ivx}^{trf,res}$ and $t_{h,ivx}^{rev,PL}$ tests for prediction horizon $h = \{1, 5, 10, 20, 50\}$ and $T = 500$. **DGP:** $y_{t+1} = \beta x_t + u_{t+1}, x_{t+1} = \rho x_t + v_{t+1}$ and $v_{t+1} = \psi v_t + \nu_{t+1}$, where $\beta = b/T$ with $b \in \{-15, -14.5, ..., 0, ..., 15\}$, $\rho = 1 - 10/T$, $\psi = 0$ and $(u_{t+1}, \nu_{t+1})' \sim NIID(\mathbf{0}, \mathbf{\Sigma})$, with $\mathbf{\Sigma} = [1 \quad -0.95; \quad -0.95 \quad 1]$.

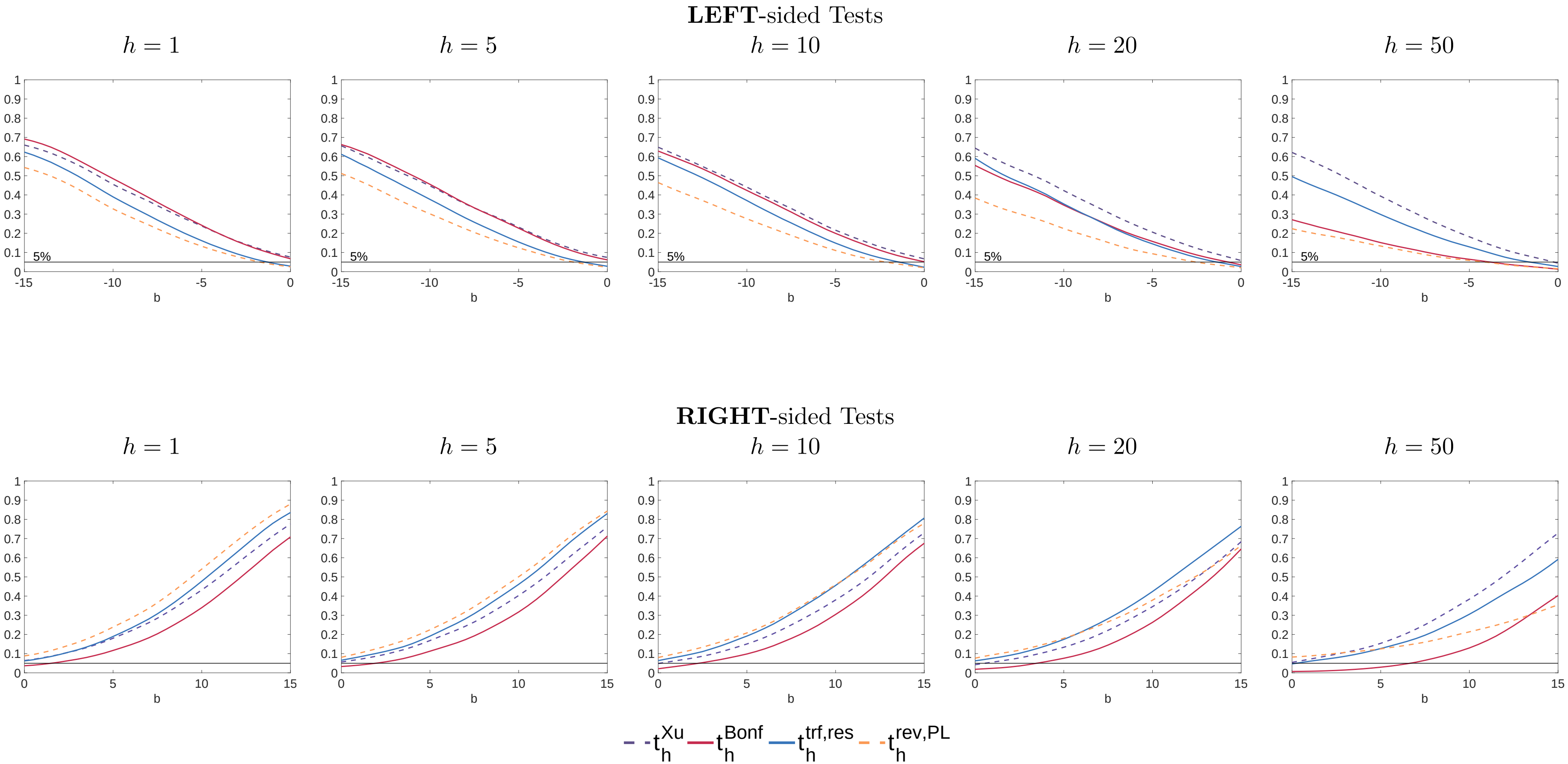


Figure S.11:Power curves of the t_h^{Xu}, t_h^{Bonf}, $t_{h,ivx}^{trf,res}$ and $t_{h,ivx}^{rev,PL}$ tests for prediction horizon $h = \{1, 5, 10, 20, 50\}$ and $T = 500$. **DGP:** $y_{t+1} = \beta x_t + u_{t+1}, x_{t+1} = \rho x_t + v_{t+1}$ and $v_{t+1} = \psi v_t + \nu_{t+1}$, where $\beta = b/T$ with $b \in \{-15, -14.5, ..., 0, ..., 15\}$, $\rho = 1 - 20/T$, $\psi = 0$ and $(u_{t+1}, \nu_{t+1})' \sim NIID(\mathbf{0}, \mathbf{\Sigma})$, with $\mathbf{\Sigma} = [1 \quad -0.95; \quad -0.95 \quad 1]$.

Additional Power plots when innovations are positively autocorrelated

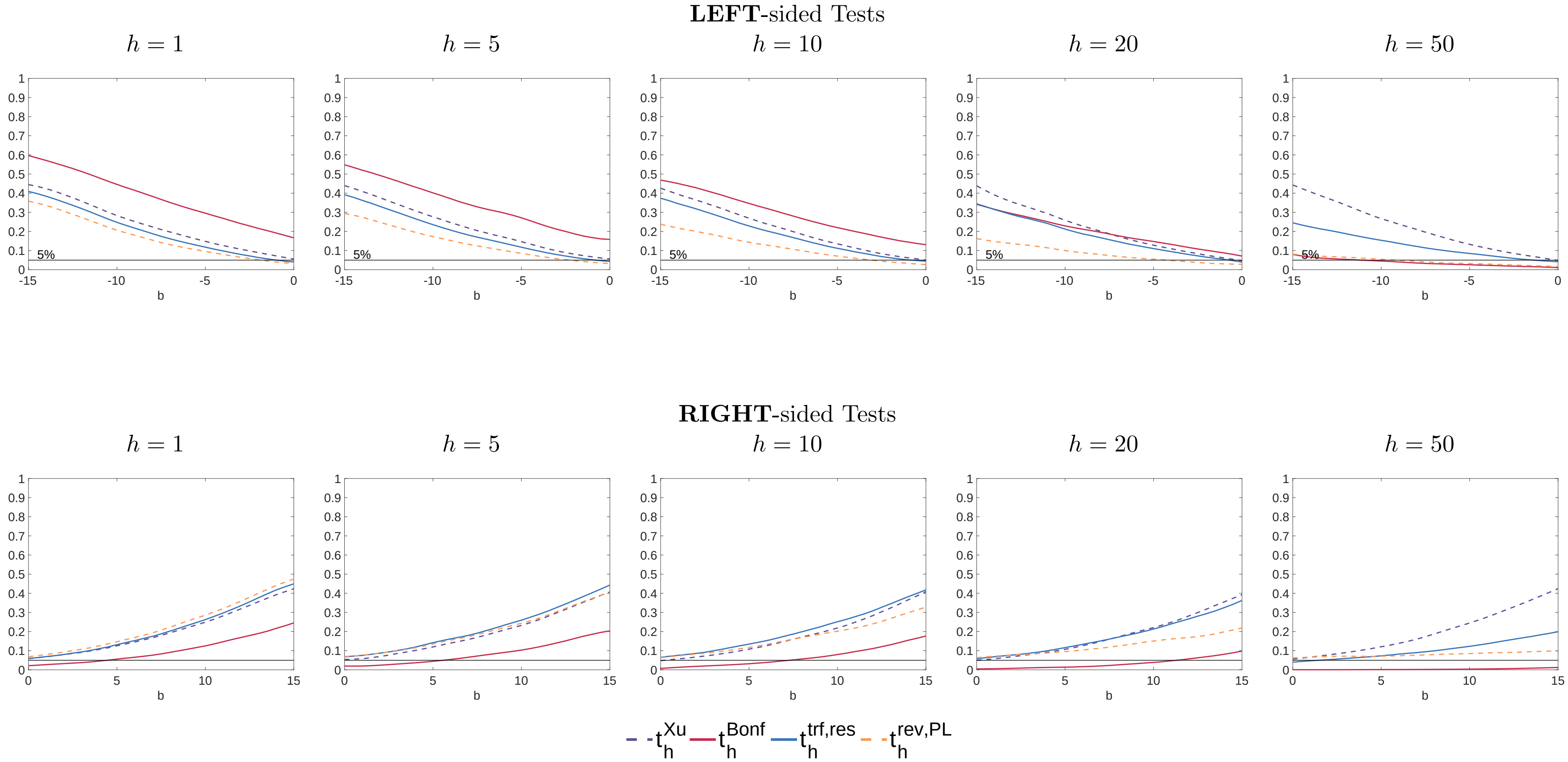


Figure S.12: Power curves of the t_h^{Xu}, t_h^{Bonf}, $t_{h,ivx}^{trf,res}$ and $t_{h,ivx}^{rev,PL}$ tests for prediction horizon $h = \{1, 5, 10, 20, 50\}$ and $T = 500$. **DGP:** $y_{t+1} = \beta x_t + u_{t+1}, x_{t+1} = \rho x_t + v_{t+1}$ and $v_{t+1} = \psi v_t + \nu_{t+1}$, where $\beta = b/T$ with $b \in \{-15, -14.5, ..., 0, ..., 15\}$, $\rho = 1 - 50/T$, $\psi = 0$ and $(u_{t+1}, \nu_{t+1})' \sim NIID(\mathbf{0}, \mathbf{\Sigma})$, with $\mathbf{\Sigma} = [1 \quad -0.95; \quad -0.95 \quad 1]$.

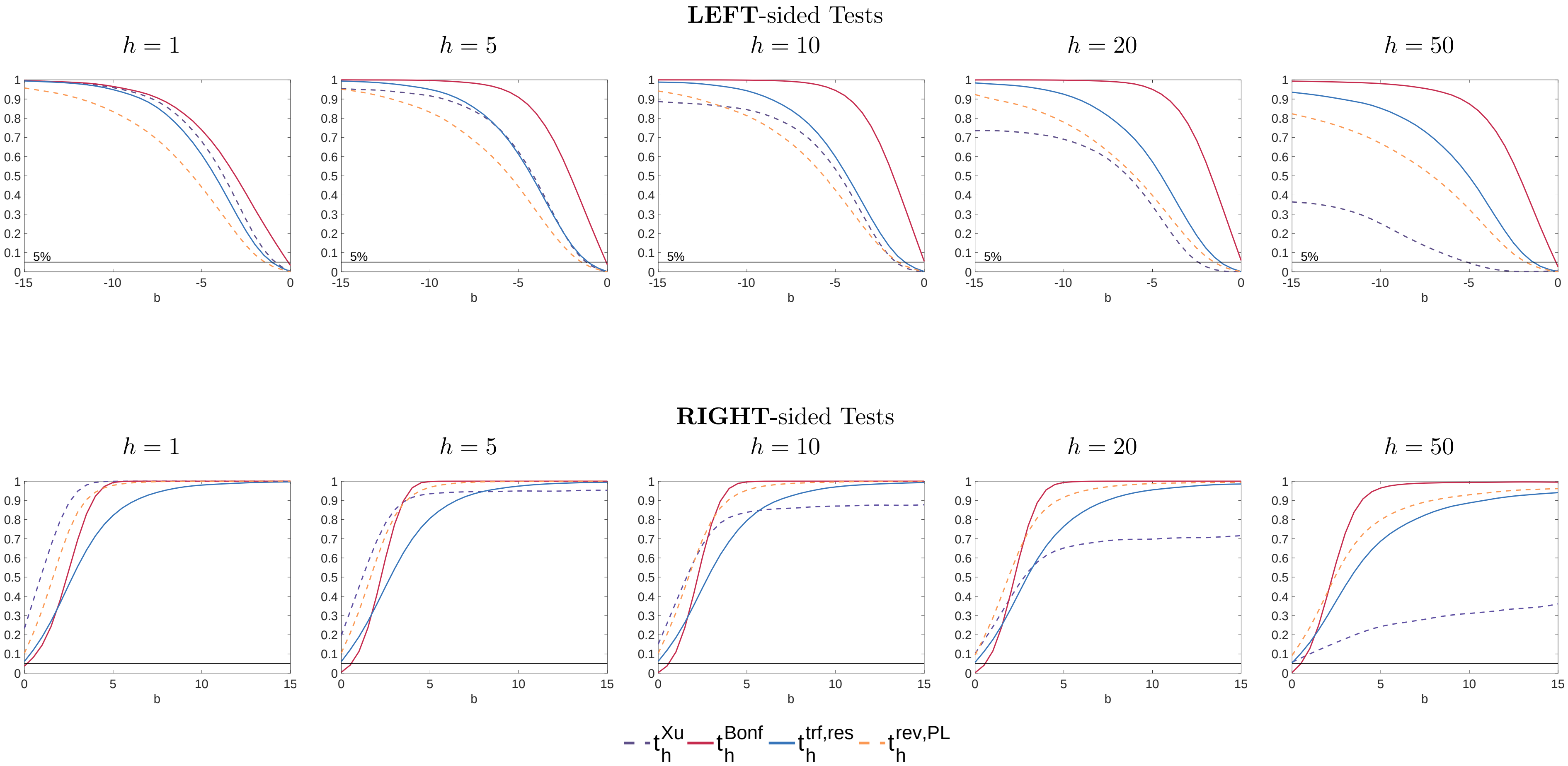


Figure S.13: Power curves of the t_h^{Xu}, t_h^{Bonf}, $t_{h,ivx}^{trf,res}$ and $t_{h,ivx}^{rev,PL}$ tests for prediction horizon $h = \{1, 5, 10, 20, 50\}$ and $T = 500$. **DGP:** $y_{t+1} = \beta x_t + u_{t+1}, x_{t+1} = \rho x_t + v_{t+1}$ and $v_{t+1} = \psi v_t + \nu_{t+1}$, where $\beta = b/T$ with $b \in \{-15, -14.5, ..., 0, ..., 15\}$, $\rho = 1$, $\psi = 0.5$ and $(u_{t+1}, \nu_{t+1})' \sim NIID(\mathbf{0}, \mathbf{\Sigma})$, with $\mathbf{\Sigma} = [1 \quad -0.95; \quad -0.95 \quad 1]$.

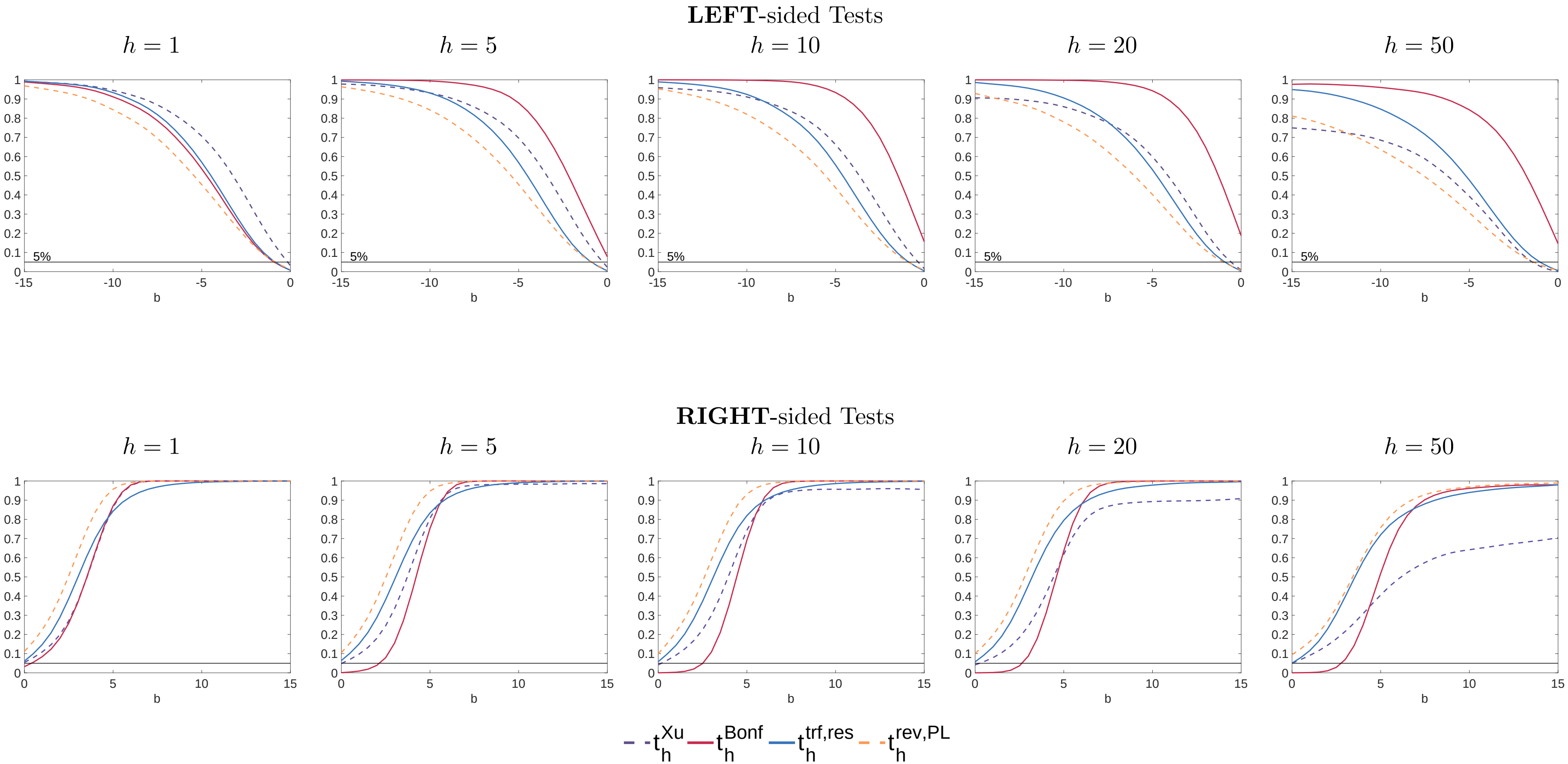


Figure S.14: Power curves of the t_h^{Xu}, t_h^{Bonf}, $t_{h,ivx}^{trf,res}$ and $t_{h,ivx}^{rev,PL}$ tests for prediction horizon $h = \{1, 5, 10, 20, 50\}$ and $T = 500$. **DGP:** $y_{t+1} = \beta x_t + u_{t+1}, x_{t+1} = \rho x_t + v_{t+1}$ and $v_{t+1} = \psi v_t + \nu_{t+1}$, where $\beta = b/T$ with $b \in \{-15, -14.5, ..., 0, ..., 15\}$, $\rho = 1 - 5/T$, $\psi = 0.5$ and $(u_{t+1}, \nu_{t+1})' \sim NIID(\mathbf{0}, \mathbf{\Sigma})$, with $\mathbf{\Sigma} = [1 \quad -0.95; \quad -0.95 \quad 1]$.

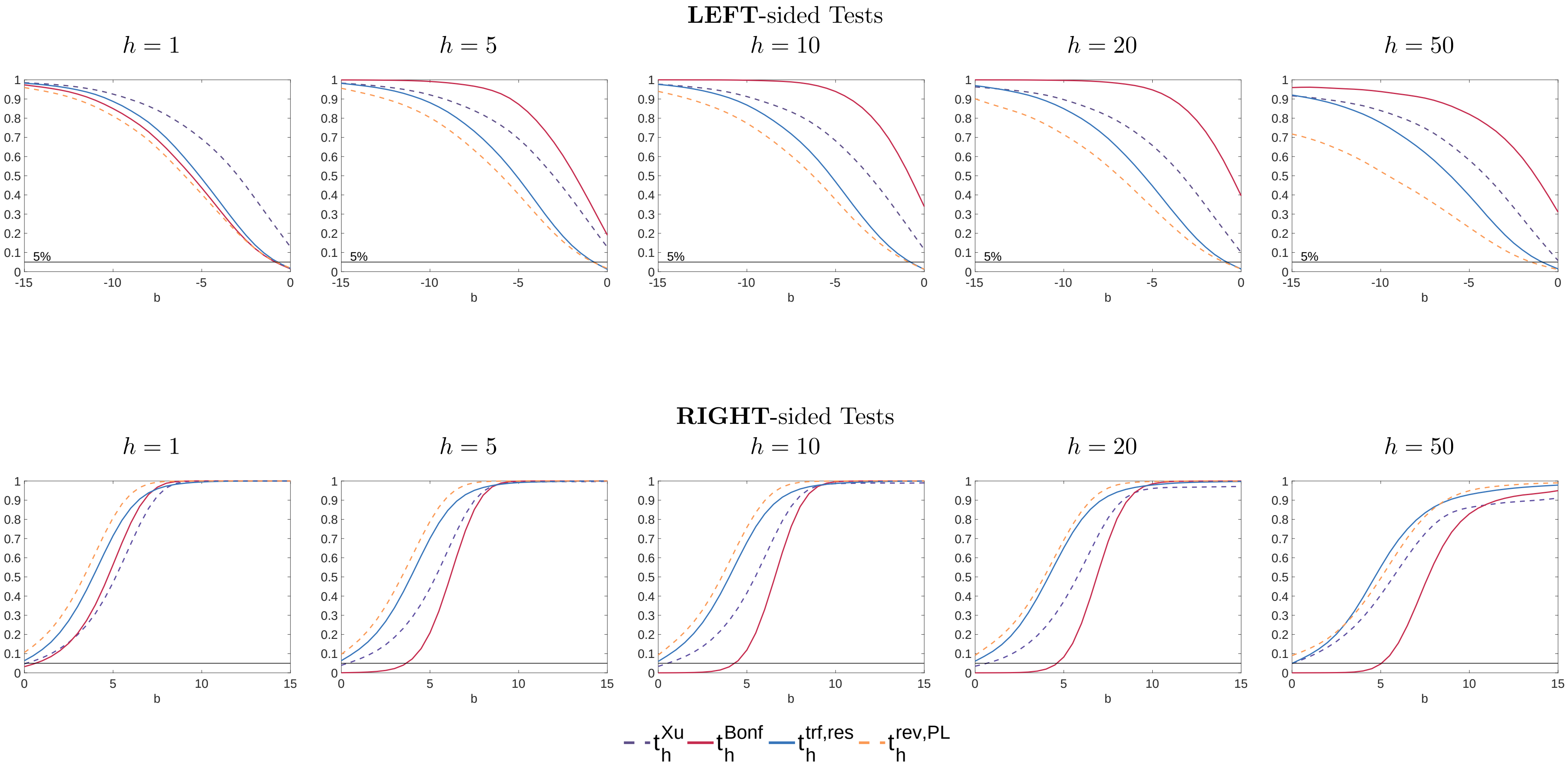


Figure S.15: Power curves of the t_h^{Xu}, t_h^{Bonf}, $t_{h,ivx}^{trf,res}$ and $t_{h,ivx}^{rev,PL}$ tests for prediction horizon $h = \{1, 5, 10, 20, 50\}$ and $T = 500$. **DGP:** $y_{t+1} = \beta x_t + u_{t+1}, x_{t+1} = \rho x_t + v_{t+1}$ and $v_{t+1} = \psi v_t + \nu_{t+1}$, where $\beta = b/T$ with $b \in \{-15, -14.5, ..., 0, ..., 15\}$, $\rho = 1 - 10/T$, $\psi = 0.5$ and $(u_{t+1}, \nu_{t+1})' \sim NIID(\mathbf{0}, \mathbf{\Sigma})$, with $\mathbf{\Sigma} = [1 \quad -0.95; \quad -0.95 \quad 1]$.

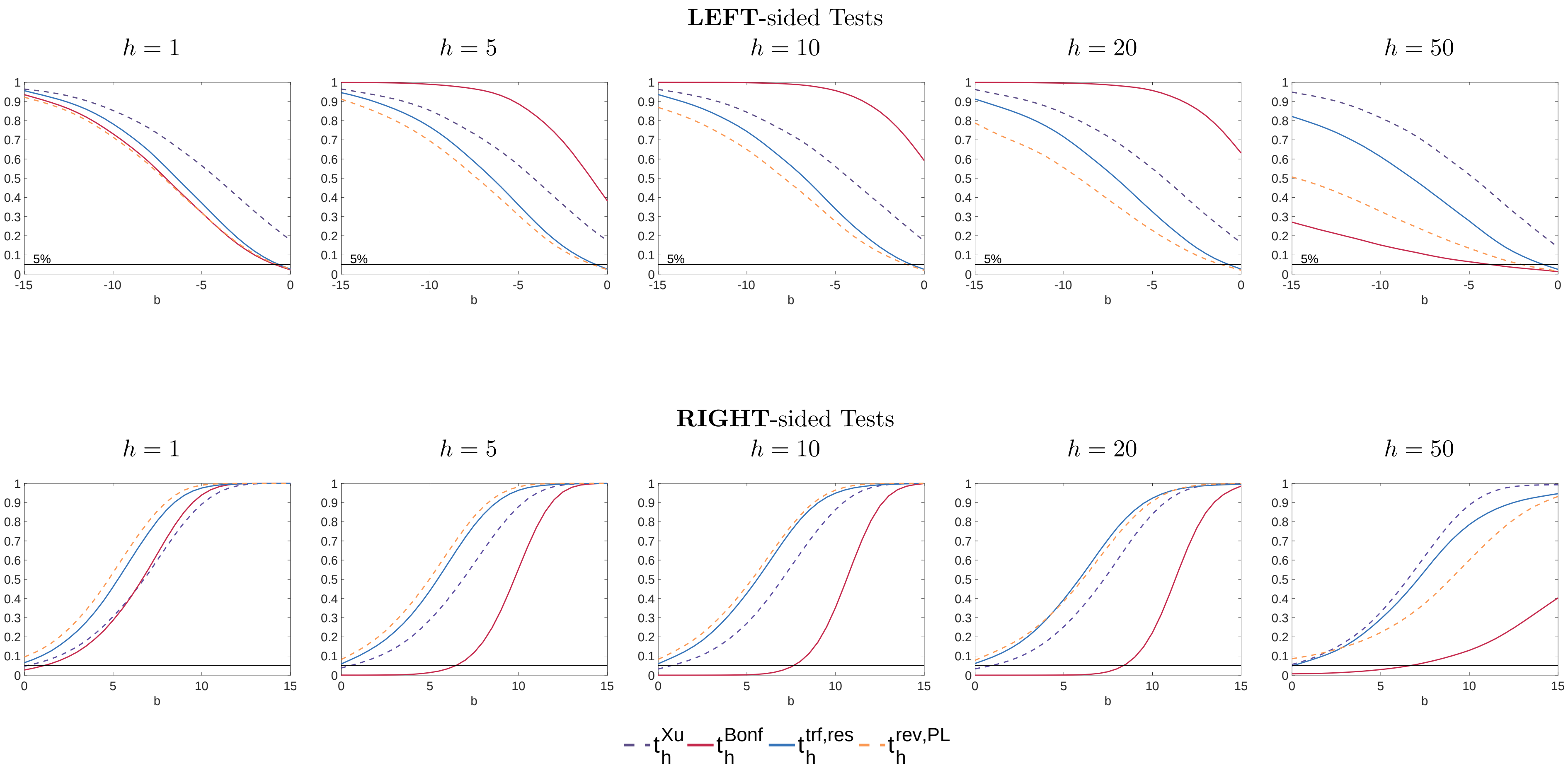


Figure S.16: Power curves of the t_h^{Xu}, t_h^{Bonf}, $t_{h,ivx}^{trf,res}$ and $t_{h,ivx}^{rev,PL}$ tests for prediction horizon $h = \{1, 5, 10, 20, 50\}$ and $T = 500$. **DGP:** $y_{t+1} = \beta x_t + u_{t+1}, x_{t+1} = \rho x_t + v_{t+1}$ and $v_{t+1} = \psi v_t + \nu_{t+1}$, where $\beta = b/T$ with $b \in \{-15, -14.5, ..., 0, ..., 15\}$, $\rho = 1-20/T$, $\psi = 0.5$ and $(u_{t+1}, \nu_{t+1})' \sim NIID(\mathbf{0}, \mathbf{\Sigma})$, with $\mathbf{\Sigma} = [1 \quad -0.95; \quad -0.95 \quad 1]$.

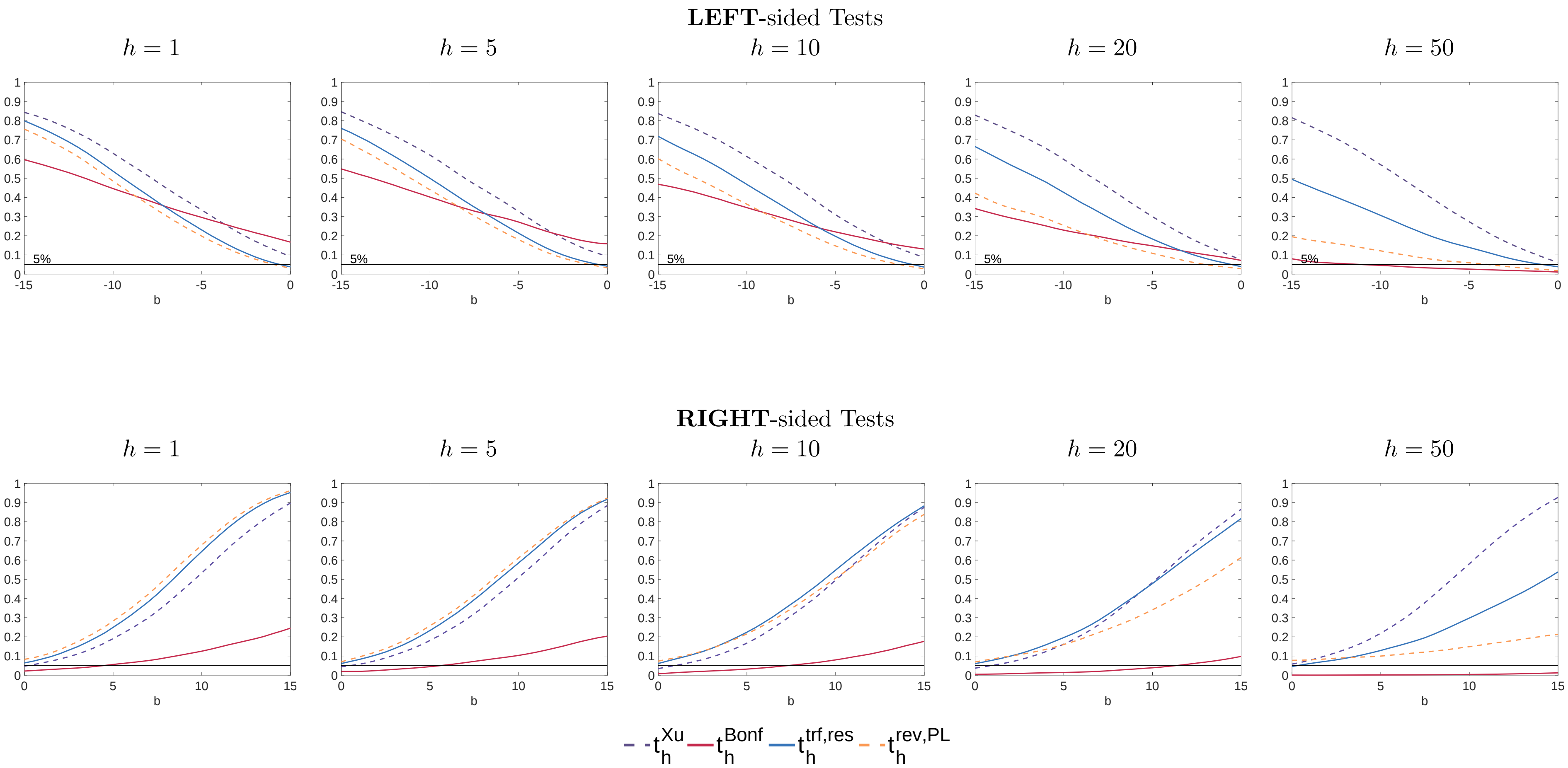
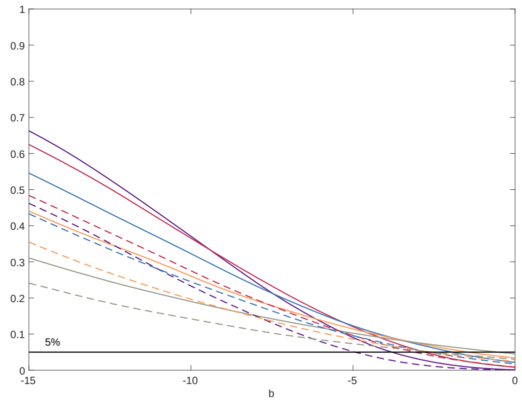
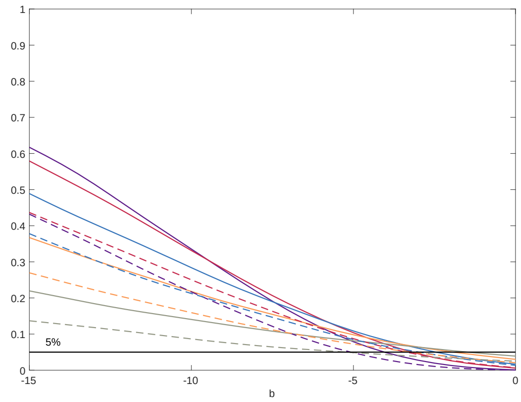


Figure S.17: Power curves of the t_h^{Xu}, t_h^{Bonf}, $t_{h,ivx}^{trf,res}$ and $t_{h,ivx}^{rev,PL}$ tests for prediction horizon $h = \{1, 5, 10, 20, 50\}$ and $T = 500$. **DGP:** $y_{t+1} = \beta x_t + u_{t+1}, x_{t+1} = \rho x_t + v_{t+1}$ and $v_{t+1} = \psi v_t + \nu_{t+1}$, where $\beta = b/T$ with $b \in \{-15, -14.5, ..., 0, ..., 15\}$, $\rho = 1 - 50/T$, $\psi = 0.5$ and $(u_{t+1}, \nu_{t+1})' \sim NIID(\mathbf{0}, \mathbf{\Sigma})$, with $\mathbf{\Sigma} = [1 \quad -0.95; \quad -0.95 \quad 1]$.

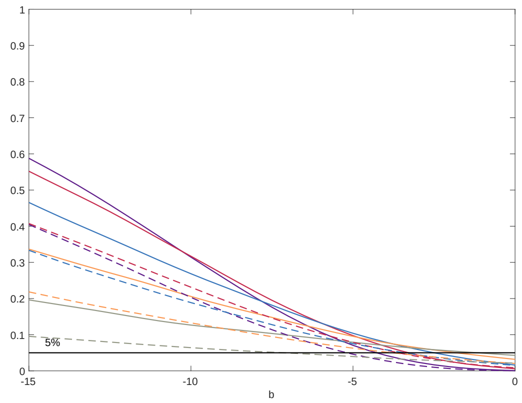
Power plots when innovations are negatively autocorrelated



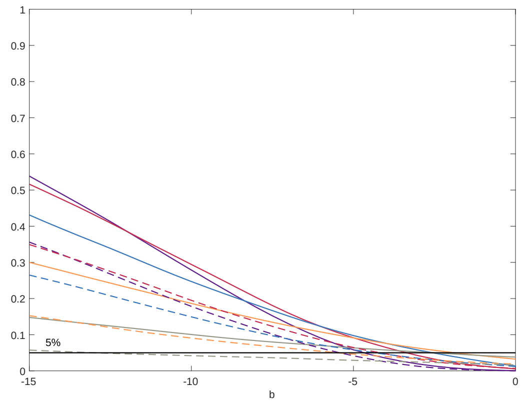
(a) $h = 1$



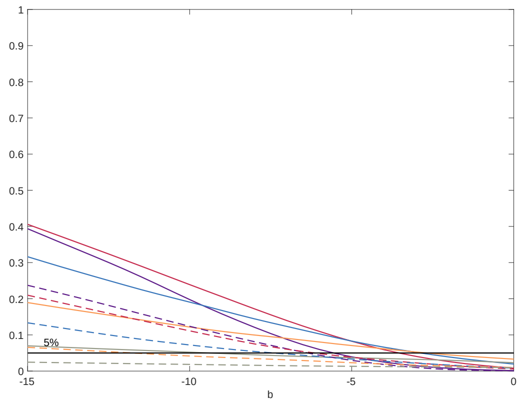
(b) $h = 5$



(c) $h = 10$



(d) $h = 20$



(e) $h = 50$

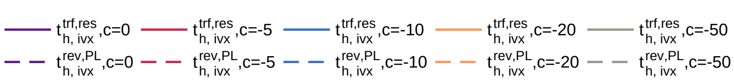
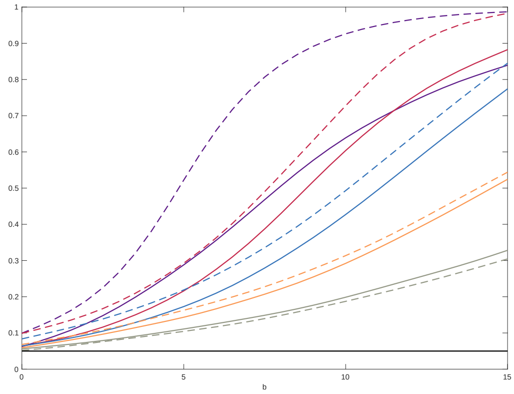
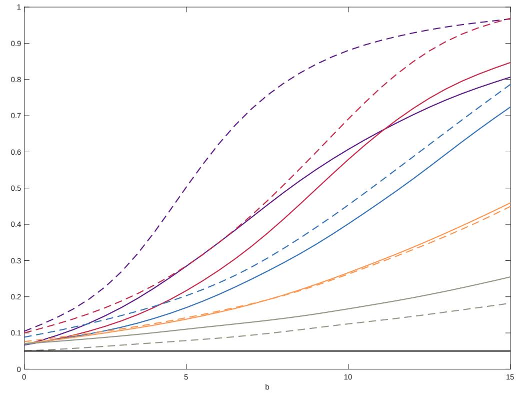


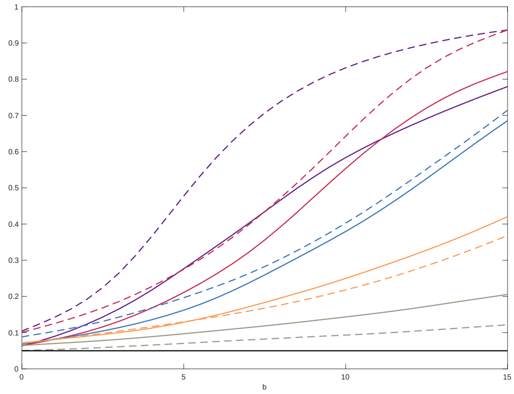
Figure S.3: Power curves of the **LEFT**-sided tests $t^{trf,res}_{h,ivx}$ and $t^{rev,PL}_{h,ivx}$ for prediction horizon $h = \{1, 5, 10, 20, 50\}$ and $T = 250$. **DGP:** $y_{t+1} = \beta x_t + u_{t+1}, x_{t+1} = \rho x_t + v_{t+1}$ and $v_{t+1} = \psi v_t + \nu_{t+1}$, where $\beta = b/T$, $\rho = 1 + c/T$, with $c = \{0, -5, -10, -20, -50\}$, $\psi = -0.5$ and $(u_{t+1}, \nu_{t+1})' \sim NIID(\mathbf{0}, \mathbf{\Sigma})$, with $\mathbf{\Sigma} = [1 \quad -0.95; \quad -0.95 \quad 1]$.



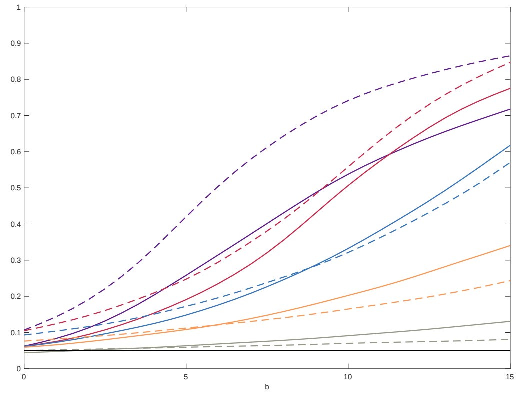
(a) $h = 1$



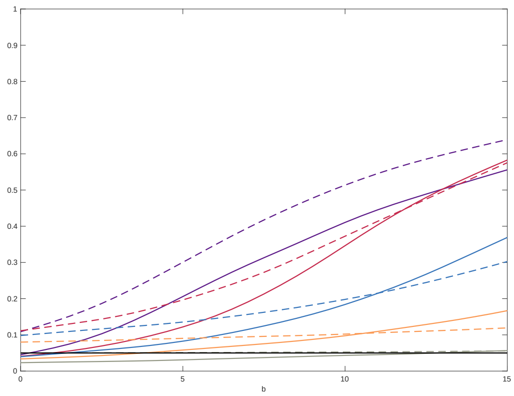
(b) $h = 5$



(c) $h = 10$



(d) $h = 20$



(e) $h = 50$

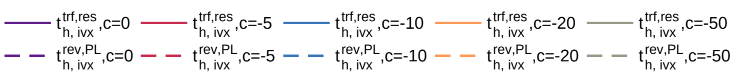


Figure S.4: Power curves of the **RIGHT**-sided tests $t_{h,ivx}^{trf,res}$ and $t_{h,ivx}^{rev,PL}$ for prediction horizon $h = \{1, 5, 10, 20, 50\}$ and $T = 250$. **DGP:** $y_{t+1} = \beta x_t + u_{t+1}, x_{t+1} = \rho x_t + v_{t+1}$ and $v_{t+1} = \psi v_t + \nu_{t+1}$, where $\beta = b/T$, $\rho = 1 + c/T$, with $c = \{0, -5, -10, -20, -50\}$, $\psi = -0.5$ and $(u_{t+1}, \nu_{t+1})' \sim NIID(\mathbf{0}, \mathbf{\Sigma})$, with $\mathbf{\Sigma} = [1 \quad -0.95; \quad -0.95 \quad 1]$.

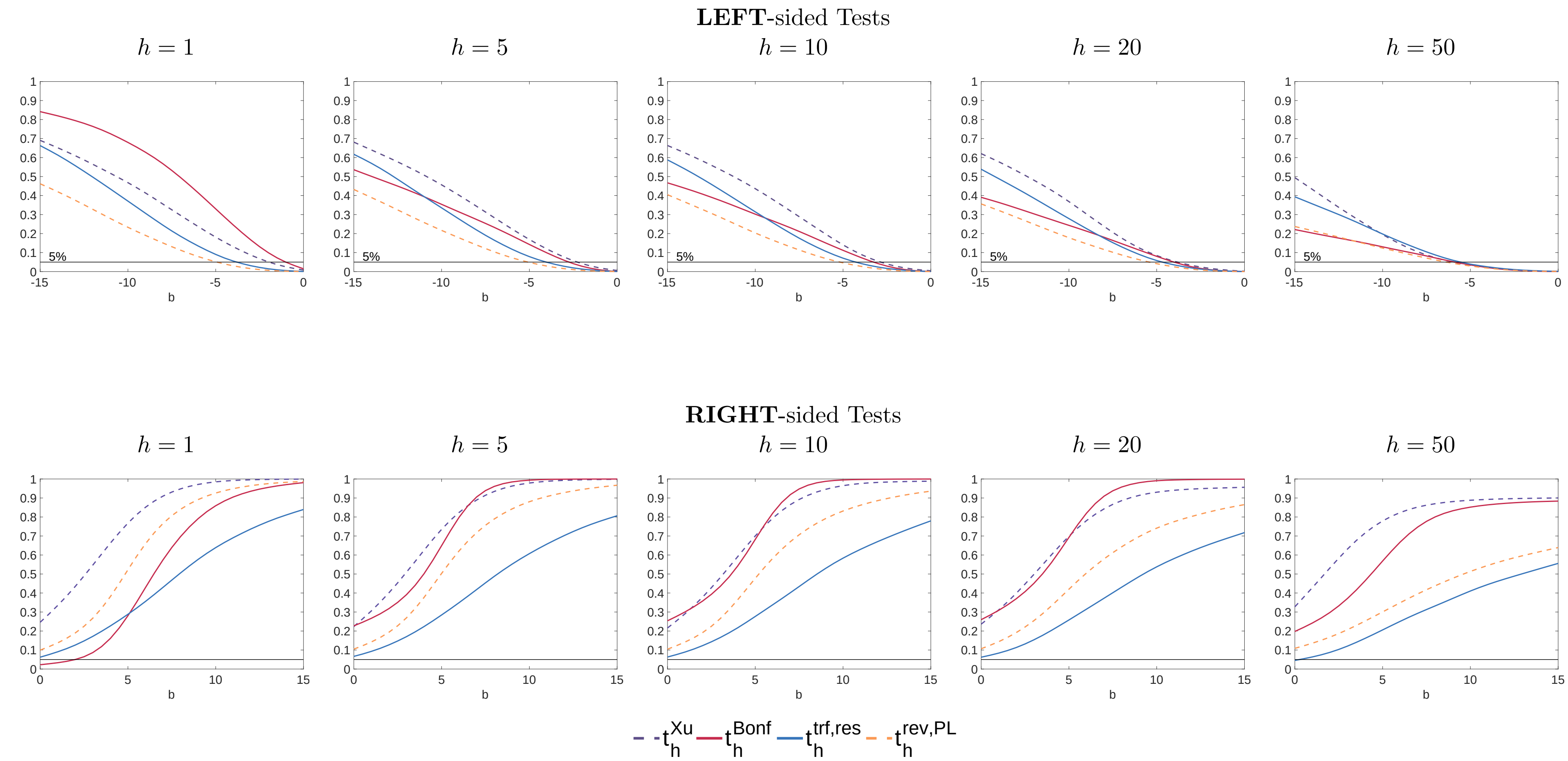


Figure S.3 Power curves of the t_h^{Xu}, t_h^{Bonf}, $t_{h,ivx}^{trf,res}$ and $t_{h,ivx}^{rev,PL}$ tests for prediction horizon $h = \{1, 5, 10, 20, 50\}$ and $T = 250$. **DGP:** $y_{t+1} = \beta x_t + u_{t+1}, x_{t+1} = \rho x_t + v_{t+1}$ and $v_{t+1} = \psi v_t + \nu_{t+1}$, where $\beta = b/T$ with $b \in \{-15, -14.5, ..., 0, ..., 15\}$, $\rho = 1$, $\psi = -0.5$ and $(u_{t+1}, \nu_{t+1})' \sim NIID(\mathbf{0}, \mathbf{\Sigma})$, with $\mathbf{\Sigma} = [1 \quad -0.95; \quad -0.95 \quad 1]$.

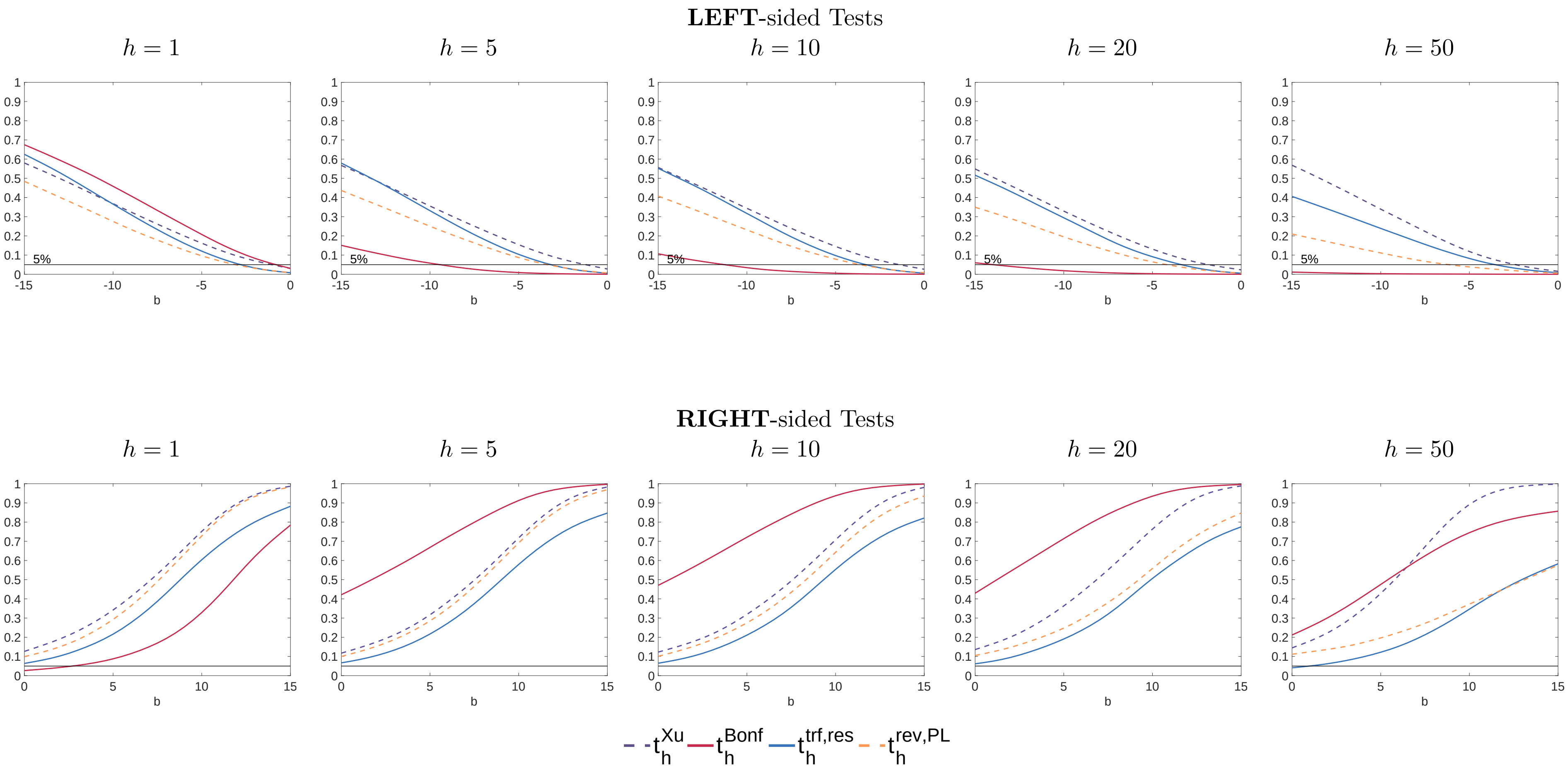


Figure S.4: Power curves of the t_h^{Xu}, t_h^{Bonf}, $t_{h,ivx}^{trf,res}$ and $t_{h,ivx}^{rev,PL}$ tests for prediction horizon $h = \{1, 5, 10, 20, 50\}$ and $T = 250$. **DGP:** $y_{t+1} = \beta x_t + u_{t+1}, x_{t+1} = \rho x_t + v_{t+1}$ and $v_{t+1} = \psi v_t + \nu_{t+1}$, where $\beta = b/T$ with $b \in \{-15, -14.5, ..., 0, ..., 15\}$, $\rho = 1 - 5/T$, $\psi = -0.5$ and $(u_{t+1}, \nu_{t+1})' \sim NIID(\mathbf{0}, \mathbf{\Sigma})$, with $\mathbf{\Sigma} = [1 \quad -0.95; \quad -0.95 \quad 1]$.

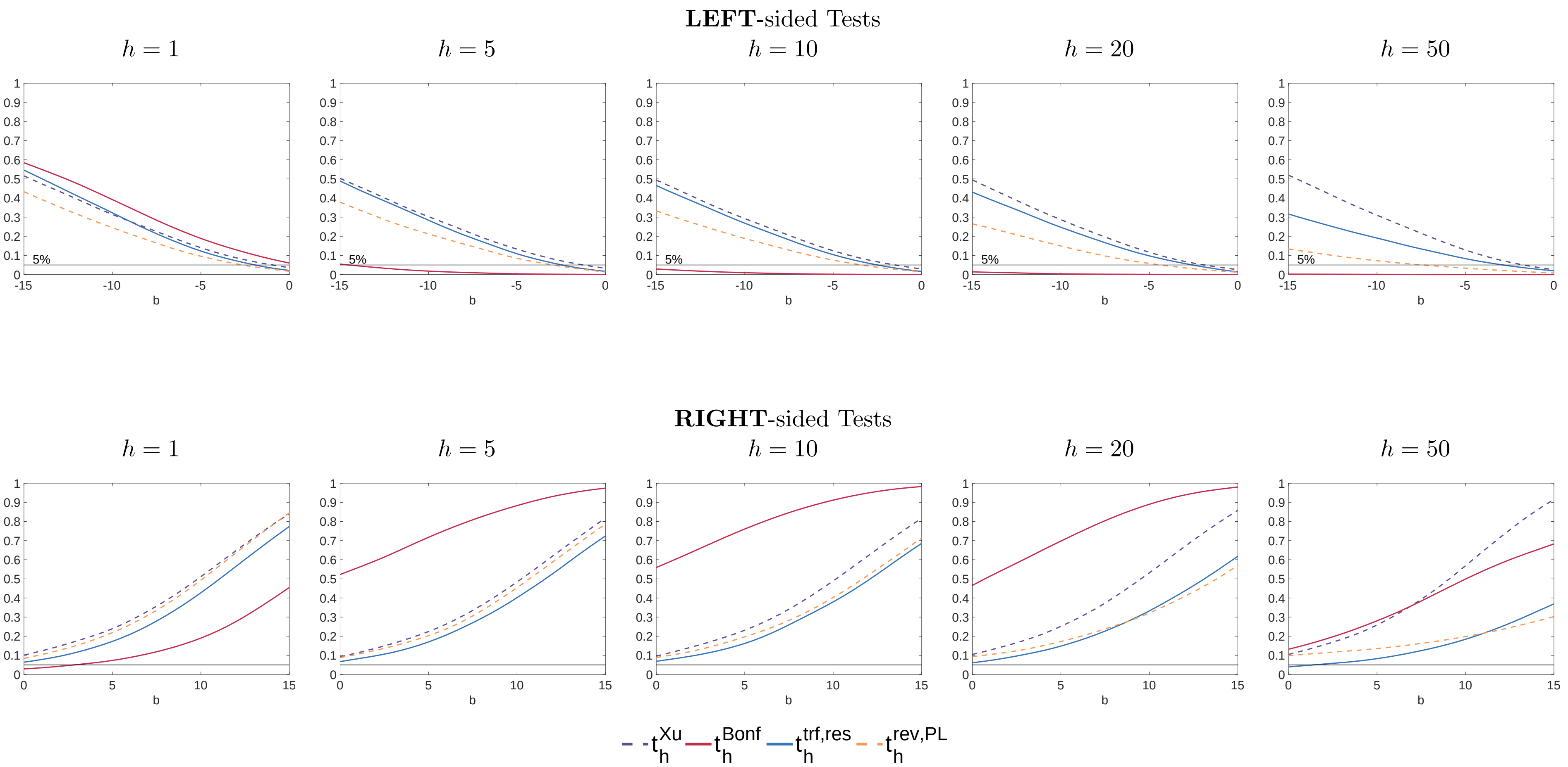


Figure S.5: Power curves of the t_h^{Xu}, t_h^{Bonf}, $t_{h,ivx}^{trf,res}$ and $t_{h,ivx}^{rev,PL}$ tests for prediction horizon $h = \{1, 5, 10, 20, 50\}$ and $T = 250$. **DGP:** $y_{t+1} = \beta x_t + u_{t+1}, x_{t+1} = \rho x_t + v_{t+1}$ and $v_{t+1} = \psi v_t + \nu_{t+1}$, where $\beta = b/T$ with $b \in \{-15, -14.5, ..., 0, ..., 15\}$, $\rho = 1 - 10/T$, $\psi = -0.5$ and $(u_{t+1}, \nu_{t+1})' \sim NIID(\mathbf{0}, \mathbf{\Sigma})$, with $\mathbf{\Sigma} = [1 \quad -0.95; \quad -0.95 \quad 1]$.

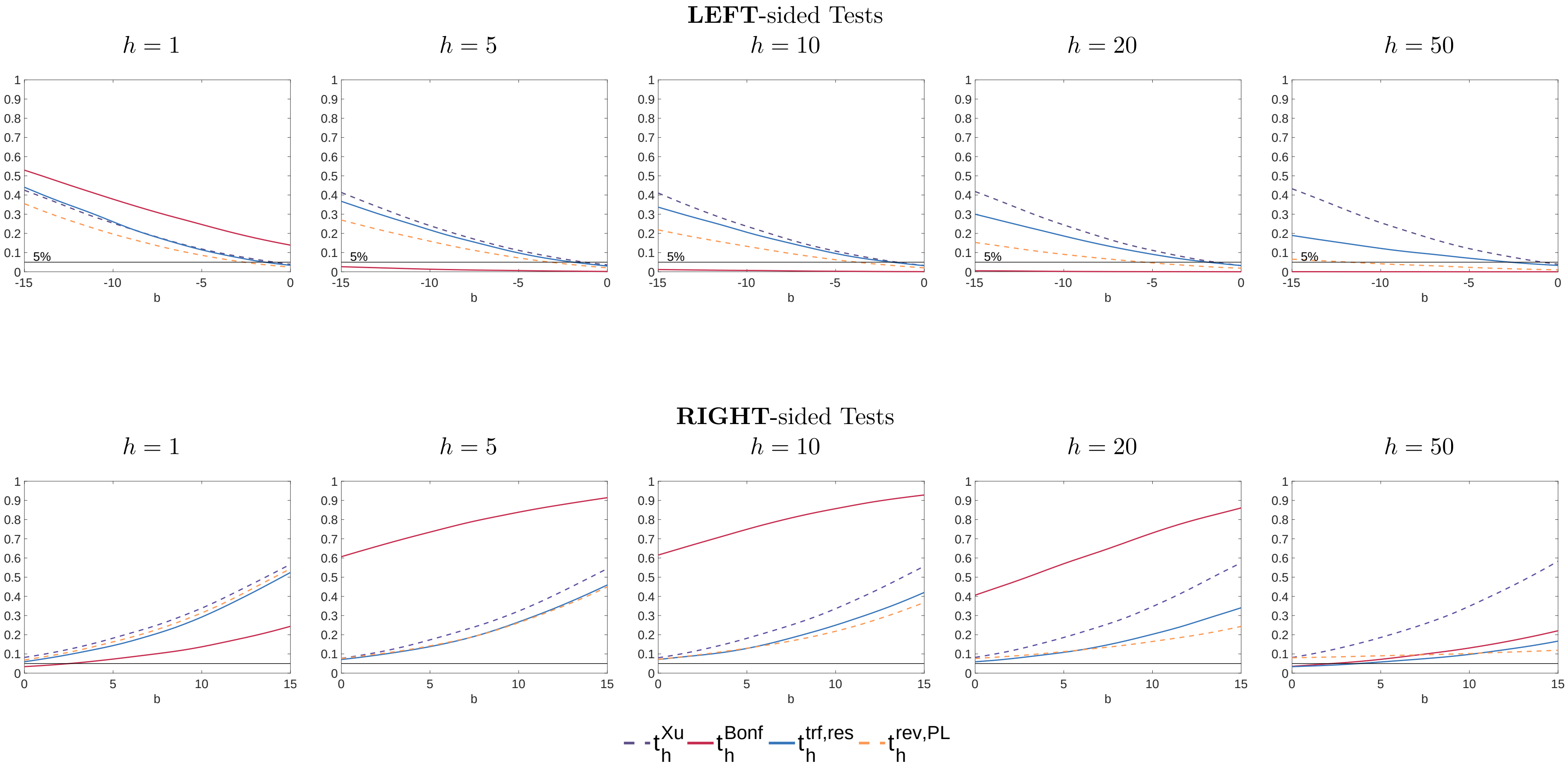


Figure S.6: Power curves of the t_h^{Xu}, t_h^{Bonf}, $t_{h,ivx}^{trf,res}$ and $t_{h,ivx}^{rev,PL}$ tests for prediction horizon $h = \{1, 5, 10, 20, 50\}$ and $T = 250$. **DGP:** $y_{t+1} = \beta x_t + u_{t+1}, x_{t+1} = \rho x_t + v_{t+1}$ and $v_{t+1} = \psi v_t + \nu_{t+1}$, where $\beta = b/T$ with $b \in \{-15, -14.5, ..., 0, ..., 15\}$, $\rho = 1 - 20/T$, $\psi = -0.5$ and $(u_{t+1}, \nu_{t+1})' \sim NIID(\mathbf{0}, \mathbf{\Sigma})$, with $\mathbf{\Sigma} = [1 \quad -0.95; \quad -0.95 \quad 1]$.

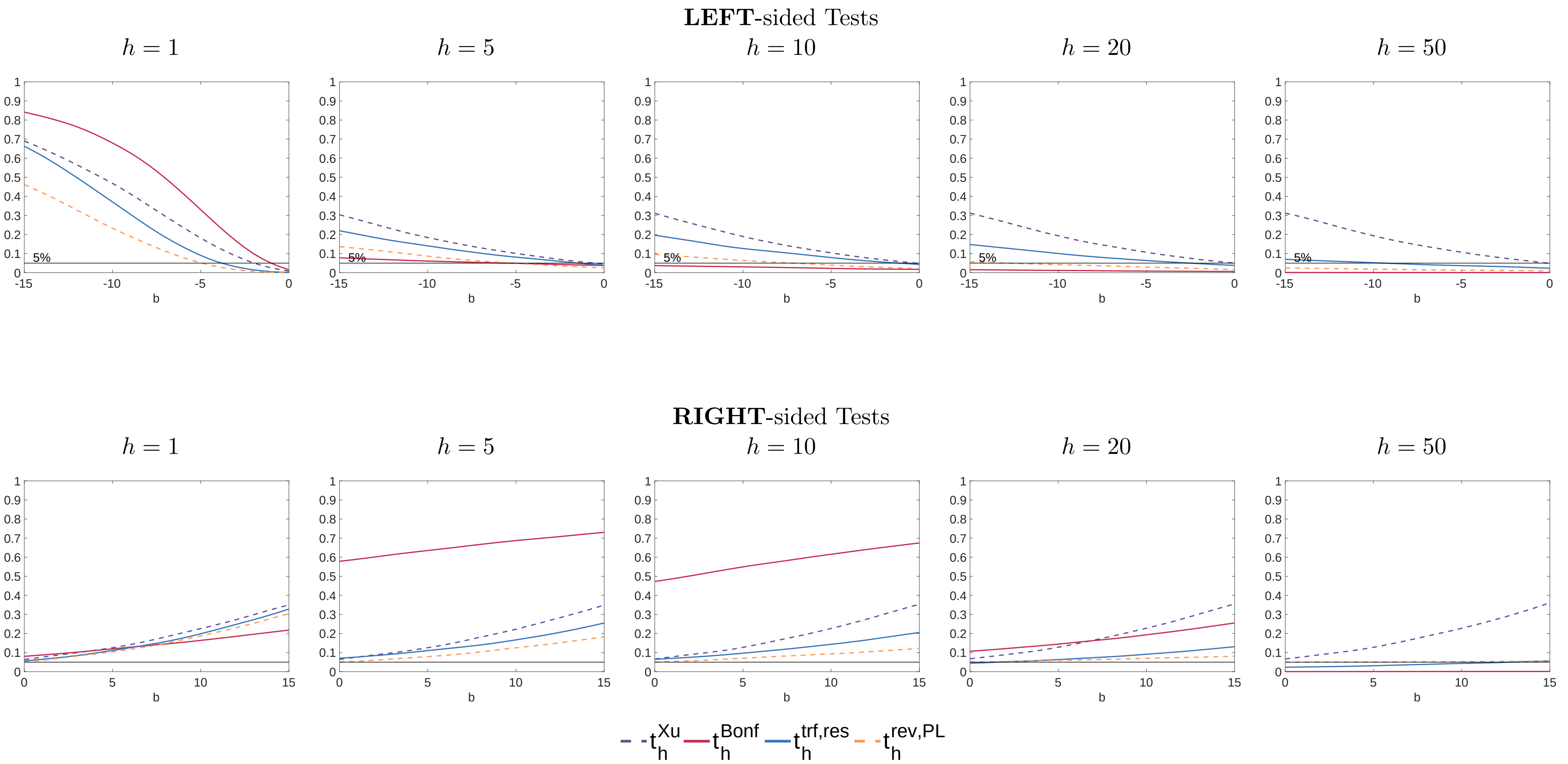


Figure S.7: Power curves of the t_h^{Xu}, t_h^{Bonf}, $t_{h,ivx}^{trf,res}$ and $t_{h,ivx}^{rev,PL}$ tests for prediction horizon $h = \{1, 5, 10, 20, 50\}$ and $T = 250$. **DGP:** $y_{t+1} = \beta x_t + u_{t+1}, x_{t+1} = \rho x_t + v_{t+1}$ and $v_{t+1} = \psi v_t + \nu_{t+1}$, where $\beta = b/T$ with $b \in \{-15, -14.5, ..., 0, ..., 15\}$, $\rho = 1 - 50/T$, $\psi = -0.5$ and $(u_{t+1}, \nu_{t+1})' \sim NIID(\mathbf{0}, \mathbf{\Sigma})$, with $\mathbf{\Sigma} = [1 \quad -0.95; \quad -0.95 \quad 1]$.

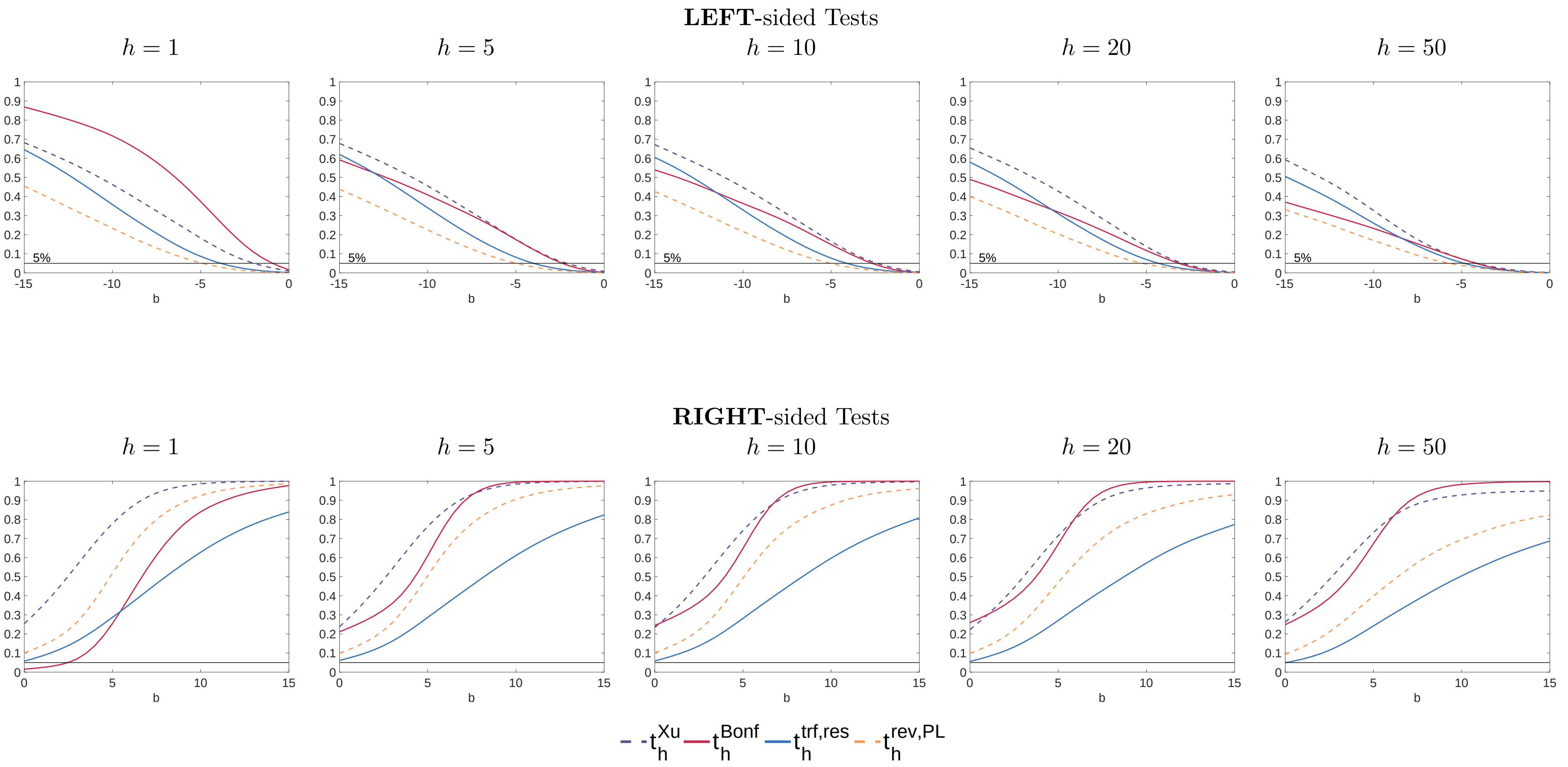


Figure S.8: Power curves of the t_h^{Xu}, t_h^{Bonf}, $t_{h,ivx}^{trf,res}$ and $t_{h,ivx}^{rev,PL}$ tests for prediction horizon $h = \{1, 5, 10, 20, 50\}$ and $T = 500$. **DGP:** $y_{t+1} = \beta x_t + u_{t+1}, x_{t+1} = \rho x_t + v_{t+1}$ and $v_{t+1} = \psi v_t + \nu_{t+1}$, where $\beta = b/T$ with $b \in \{-15, -14.5, ..., 0, ..., 15\}$, $\rho = 1$, $\psi = -0.5$ and $(u_{t+1}, \nu_{t+1})' \sim NIID(\mathbf{0}, \mathbf{\Sigma})$, with $\mathbf{\Sigma} = [1 \quad -0.95; \quad -0.95 \quad 1]$.

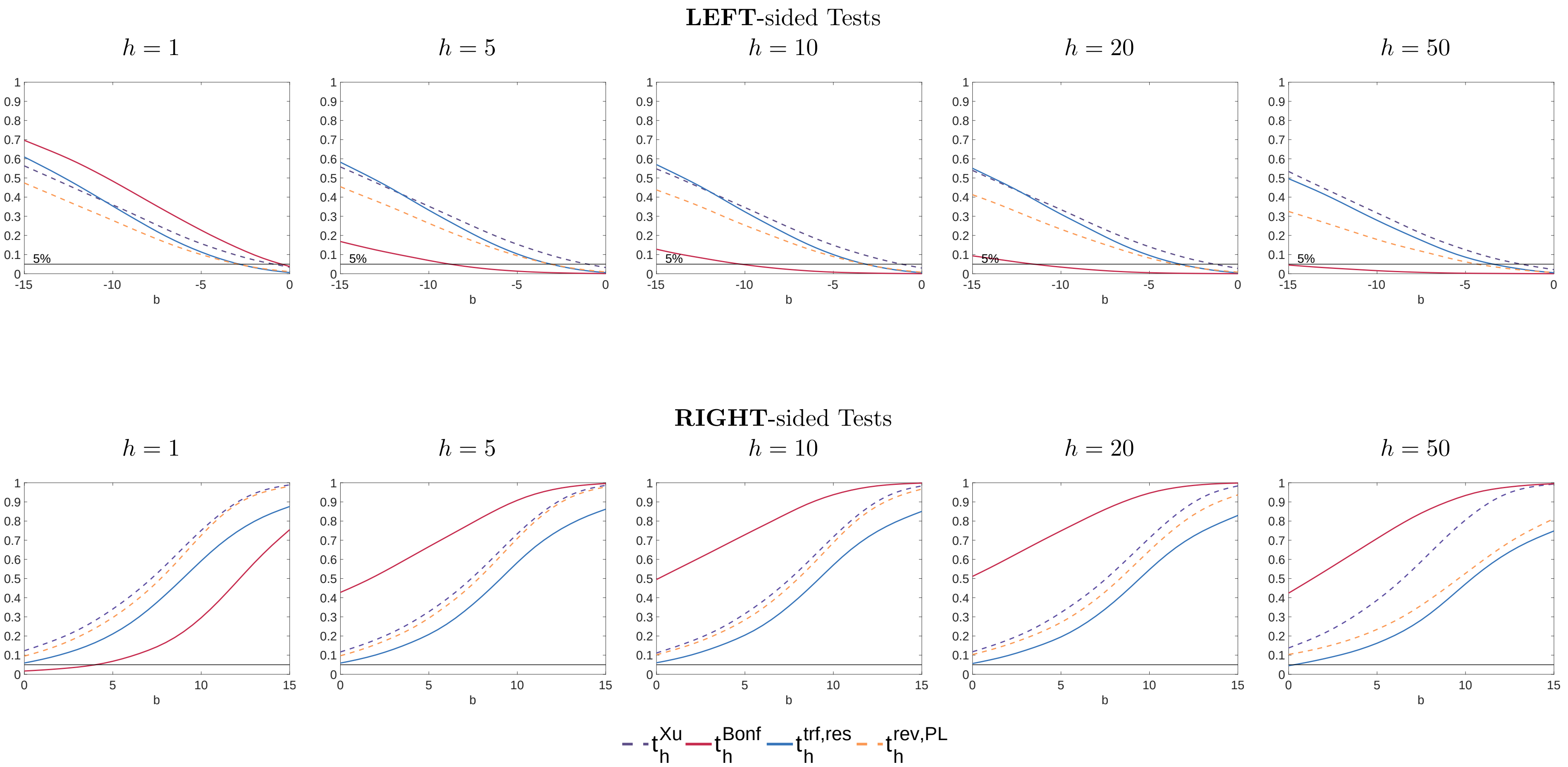


Figure S.9: Power curves of the t_h^{Xu}, t_h^{Bonf}, $t_{h,ivx}^{trf,res}$ and $t_{h,ivx}^{rev,PL}$ tests for prediction horizon $h = \{1, 5, 10, 20, 50\}$ and $T = 500$. **DGP:** $y_{t+1} = \beta x_t + u_{t+1}, x_{t+1} = \rho x_t + v_{t+1}$ and $v_{t+1} = \psi v_t + \nu_{t+1}$, where $\beta = b/T$ with $b \in \{-15, -14.5, ..., 0, ..., 15\}$, $\rho = 1 - 5/T$, $\psi = -0.5$ and $(u_{t+1}, \nu_{t+1})' \sim NIID(\mathbf{0}, \mathbf{\Sigma})$, with $\mathbf{\Sigma} = [1 \quad -0.95; \quad -0.95 \quad 1]$.

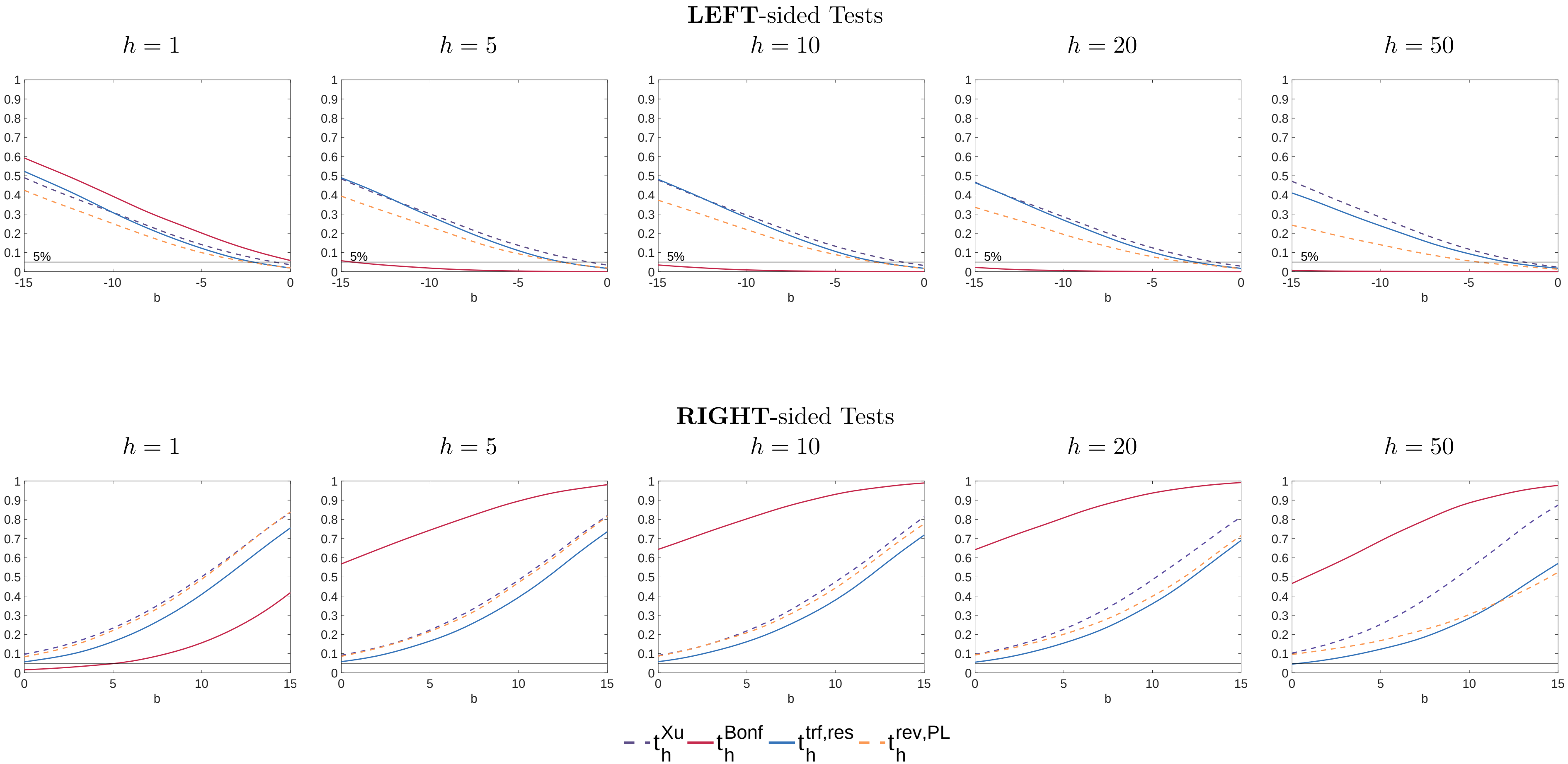


Figure S.10:Power curves of the t_h^{Xu}, t_h^{Bonf}, $t_{h,ivx}^{trf,res}$ and $t_{h,ivx}^{rev,PL}$ tests for prediction horizon $h = \{1, 5, 10, 20, 50\}$ and $T = 500$. **DGP:** $y_{t+1} = \beta x_t + u_{t+1}, x_{t+1} = \rho x_t + v_{t+1}$ and $v_{t+1} = \psi v_t + \nu_{t+1}$, where $\beta = b/T$ with $b \in \{-15, -14.5, ..., 0, ..., 15\}$, $\rho = 1 - 10/T$, $\psi = -0.5$ and $(u_{t+1}, \nu_{t+1})' \sim NIID(\mathbf{0}, \mathbf{\Sigma})$, with $\mathbf{\Sigma} = [1 \quad -0.95; \quad -0.95 \quad 1]$.

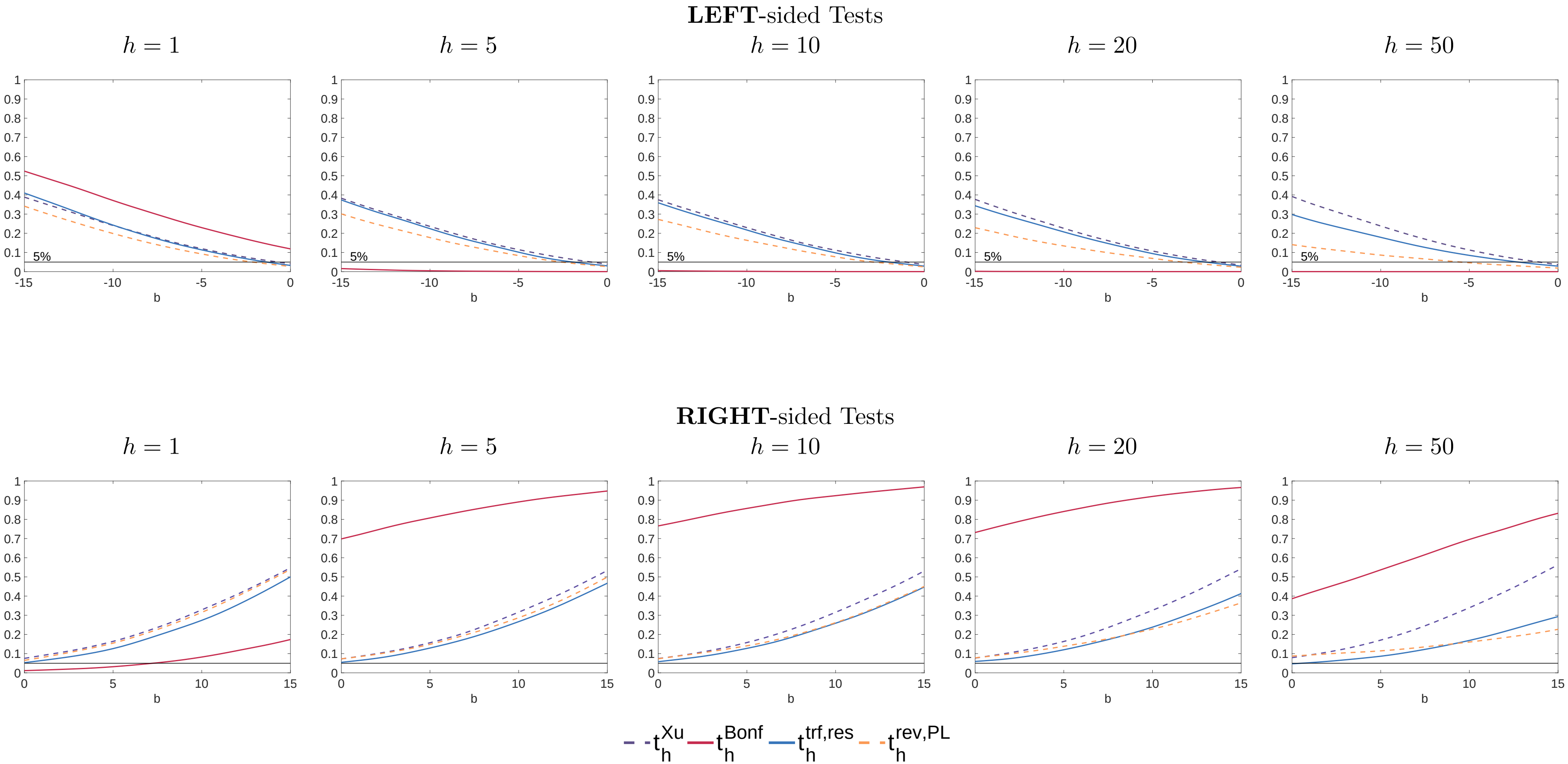


Figure S.11:Power curves of the t_h^{Xu}, t_h^{Bonf}, $t_{h,ivx}^{trf,res}$ and $t_{h,ivx}^{rev,PL}$ tests for prediction horizon $h = \{1, 5, 10, 20, 50\}$ and $T = 500$. **DGP:** $y_{t+1} = \beta x_t + u_{t+1}, x_{t+1} = \rho x_t + v_{t+1}$ and $v_{t+1} = \psi v_t + \nu_{t+1}$, where $\beta = b/T$ with $b \in \{-15, -14.5, ..., 0, ..., 15\}$, $\rho = 1 - 20/T$, $\psi = -0.5$ and $(u_{t+1}, \nu_{t+1})' \sim NIID(\mathbf{0}, \mathbf{\Sigma})$, with $\mathbf{\Sigma} = [1 \quad -0.95; \quad -0.95 \quad 1]$.

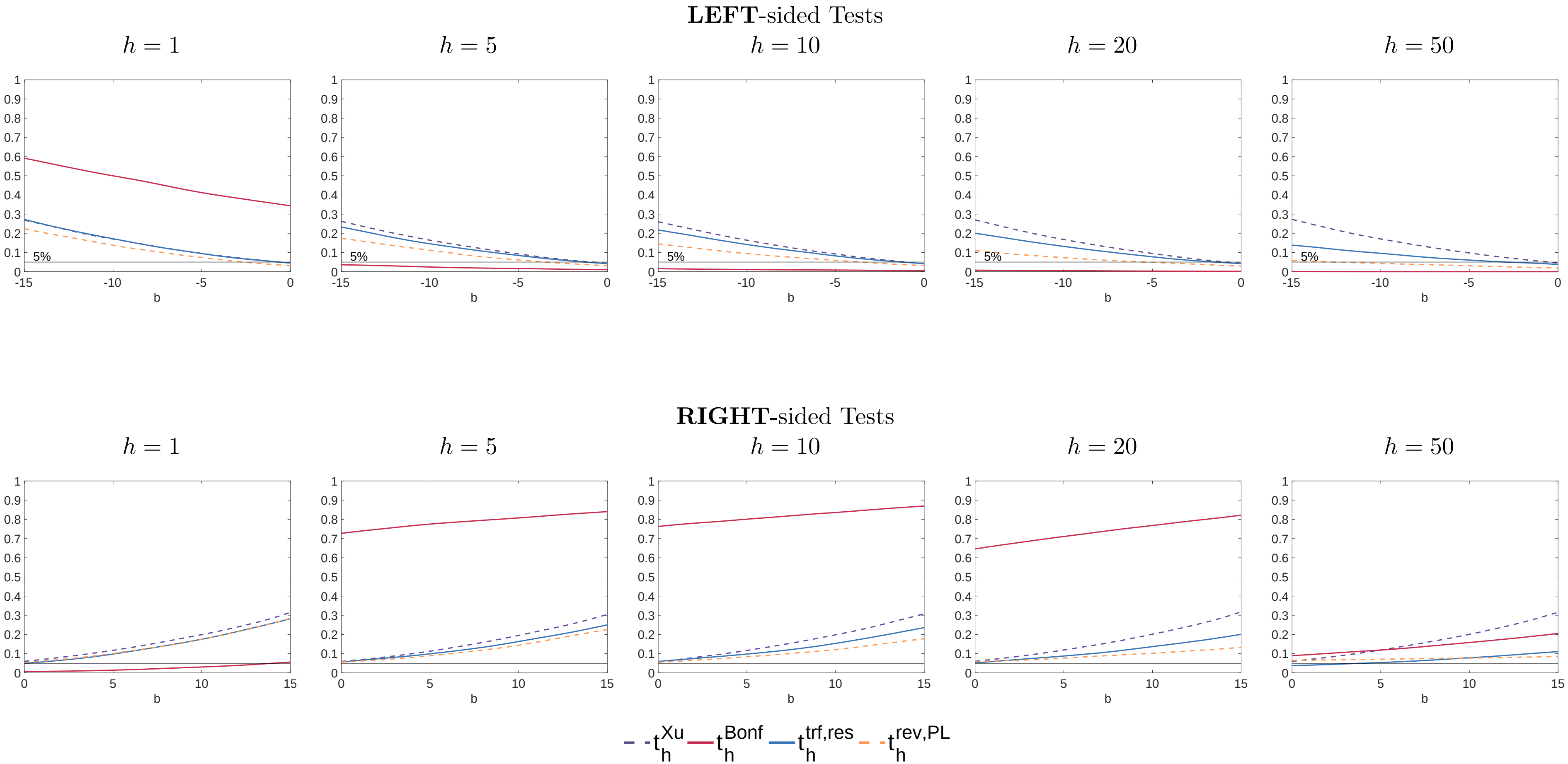


Figure S.11:Power curves of the t_h^{Xu}, t_h^{Bonf}, $t_{h,ivx}^{trf,res}$ and $t_{h,ivx}^{rev,PL}$ tests for prediction horizon $h = \{1, 5, 10, 20, 50\}$ and $T = 500$. **DGP:** $y_{t+1} = \beta x_t + u_{t+1}, x_{t+1} = \rho x_t + v_{t+1}$ and $v_{t+1} = \psi v_t + \nu_{t+1}$, where $\beta = b/T$ with $b \in \{-15, -14.5, ..., 0, ..., 15\}$, $\rho = 1 - 50/T$, $\psi = -0.5$ and $(u_{t+1}, \nu_{t+1})' \sim NIID(\mathbf{0}, \mathbf{\Sigma})$, with $\mathbf{\Sigma} = [1 \quad -0.95; \quad -0.95 \quad 1]$.

S.3 Derivations Relating to Section 4

S.3.1 Transformed Regression IVX based Tests

This appendix provides further details and derivations related to the transformed regression statistics presented in section 4. To simply our presentation, consider (2.8) in matrix notation; viz.,

$$\mathbf{A}_h\bar{\mathbf{y}}_{+1} = \bar{\mathbf{x}}_{-h}\beta_h + \mathbf{A}_h\mathbf{u}_{+1} + o_p(1) \tag{S.1}$$

where $\bar{\mathbf{y}}_{+1}$ is a $(T-1)\times 1$ vector of one period demeaned log excess returns, $\mathbf{A}_h$ is a $(T-h)\times(T-1)$ matrix with entries $a_{ij} = 1$ if $i \leq j \leq i+h-1$ and zero otherwise, $i = 1, ..., T-h$. Thus, $\mathbf{A}_h$ is a transformation matrix with ones on the main diagonal and the first $h-1$ right off-diagonals, and zero otherwise. Therefore, $\mathbf{A}_h\bar{\mathbf{y}}_{+1} := [\bar{y}^{(h)}_{1+h}, \bar{y}^{(h)}_{2+h}, ..., \bar{y}^{(h)}_T]'$ and the error term vector $\mathbf{A}_h\mathbf{u}_{+1} := [u^{(h)}_{1+h}, u^{(h)}_{2+h}, ..., u^{(h)}_T]'$. Finally, $\bar{\mathbf{x}}_{-h} := [\bar{x}_1, \bar{x}_2, ..., \bar{x}_{T-h}]'$ is a $(T-h)\times 1$ vector of demeaned predictor values.

The OLS estimator from (S.1), $\hat{\beta}_h := (\bar{\mathbf{x}}'_{-h}\bar{\mathbf{x}}_{-h})^{-1}\bar{\mathbf{x}}'_{-h}\mathbf{A}_h\bar{\mathbf{y}}_{+1}$, can equivalently be written as $\hat{\beta}^{trf}_h = (\bar{\mathbf{x}}'_{-h}\bar{\mathbf{x}}_{-h})^{-1}(\mathbf{A}'_h\bar{\mathbf{x}}_{-h})'\bar{\mathbf{y}}_{+1}$, which shows that the estimator can be equivalently computed from the original non-overlapping one period returns. As indicated by Britten-Jones et al. (2011), the transformed estimator can be obtained from a regression of $\bar{\mathbf{y}}_{+1}$ on $\tilde{\bar{\mathbf{x}}}$ where

$$\tilde{\bar{\mathbf{x}}} := \mathbf{A}'_h\bar{\mathbf{x}}_{-h}(\bar{\mathbf{x}}'_{-h}\mathbf{A}_h\mathbf{A}'_h\bar{\mathbf{x}}_{-h})^{-1}\bar{\mathbf{x}}'_{-h}\bar{\mathbf{x}}_{-h}$$

is a $(T-1)\times 1$ vector.

In order to derive transformed regression IVX estimators, we use the IVX instrument z_t constructed as described in (3.6). The resulting transformed regression IVX estimator is then given by

$$\hat{\beta}^{trf}_{h,ivx} := (\mathbf{z}'_{-h}\bar{\mathbf{x}}_{-h})^{-1}(\mathbf{A}'_h\mathbf{z}_{-h})'\bar{\mathbf{y}}_{+1} \tag{S.2}$$

which can be obtained from a transformed regression of $\bar{\mathbf{y}}_{+1}$ on $\tilde{\mathbf{z}}$ where $\mathbf{z}_{-h} := [z_1, z_2, ..., z_{T-h}]'$ and

$$\tilde{\mathbf{z}} := \mathbf{A}'_h\mathbf{z}_{-h}(\mathbf{z}'_{-h}\mathbf{A}_h\mathbf{A}'_h\mathbf{z}_{-h})^{-1}\mathbf{z}'_{-h}\bar{\mathbf{x}}_{-h} \tag{S.3}$$

is a $(T-1)\times 1$ vector.

Hence, we can test the null hypothesis, $H_0 : \beta_h = 0$, against one or two-sided alternatives using the transformed regression IVX based t-statistic with heteroskedasticity-robust standard errors,

$$t^{trf}_{h,ivx} := \frac{\hat{\beta}^{trf}_{h,ivx}}{s.e.\left(\hat{\beta}^{trf}_{h,ivx}\right)}. \tag{S.4}$$

where $s.e.\left(\hat{\beta}^{trf}_{h,ivx}\right) := \left[(\mathbf{z}'_{-h}\bar{\mathbf{x}}_{-h})^{-1}(\mathbf{A}'_h\mathbf{z}_{-h})'\ddot{\mathbf{u}}_{+1}\ddot{\mathbf{u}}'_{+1}(\mathbf{A}'_h\mathbf{z}_{-h})(\mathbf{z}'_{-h}\bar{\mathbf{x}}_{-h})^{-1}\right]^{1/2}$ and $\ddot{\mathbf{u}}_{+1} := \bar{\mathbf{y}}_{+1} - \tilde{\mathbf{z}}\hat{\beta}^{trf}_{h,ivx}$.

S.3.2 Residual Augmented Transformed Regression

A natural extension of the transformed regression approach discussed above is to consider a residual augmented transformed regression following for instance Demetrescu and Rodrigues (2020). This consists of regressing $(\bar{\mathbf{y}}_{+1} - \hat{\gamma}\hat{\boldsymbol{\nu}}_{+1})$ on $\tilde{\mathbf{z}}$, where $\tilde{\mathbf{z}}$ is defined in (S.3) and $\hat{\boldsymbol{\nu}}_{+1} = [\hat{\nu}_2, ..., \hat{\nu}_T]'$ is the vector of residuals $\hat{\nu}_t$ computed from an estimated autoregressive model of order p for the predictor x_t, *viz.*,

$$\hat{\nu}_t := \bar{x}_t - \sum_{k=1}^{p}\hat{\phi}_k\bar{x}_{t-k} = \nu_t - \sum_{k=1}^{p}\left(\hat{\phi}_k - \phi_k\right)\bar{x}_{t-k}, \tag{S.5}$$

where $\hat{\phi}_k$, $k = 1, ..., p$ are the OLS parameter estimates.

The residual augmented transformed regression IVX estimator is then defined as

$$\hat{\beta}_{h,ivx}^{trf,res} := (\mathbf{z}'_{-h}\bar{\mathbf{x}}_{-h})^{-1}(\mathbf{A}'_h\mathbf{z}_{-h})'(\bar{\mathbf{y}}_{+1} - \hat{\gamma}\hat{\boldsymbol{\nu}}_{+1}). \tag{S.6}$$

Hence, we can test the null hypothesis, $H_0 : \beta_h = 0$, against one or two-sided alternatives using the residual augmented transformed regression IVX based t-statistic with heteroskedasticity-robust standard errors,

$$t_{h,ivx}^{trf,res} := \frac{(\hat{\beta}_{h,ivx}^{trf,res} - \beta_h)}{s.e.\left(\hat{\beta}_{h,ivx}^{trf,res}\right)} \tag{S.7}$$

where $s.e.\left(\hat{\beta}_{h,ivx}^{trf,res}\right) := (\mathcal{H}_{zx})^{-1}\left[\mathcal{H}_{z\hat{\varepsilon}z\hat{\varepsilon}} + \hat{\gamma}^2\hat{Q}_{T,trf}^{(h)}\right]^{1/2}$; $\mathcal{H}_{zx} := (\mathbf{z}'_{-h}\bar{\mathbf{x}}_{-h})$;
$\mathcal{H}_{z\hat{\varepsilon}z\hat{\varepsilon}} := [(\mathbf{A}'_h\mathbf{z}_{-h})'\hat{\boldsymbol{\varepsilon}}_{+1}]'\,[(\mathbf{A}'_h\mathbf{z}_{-h})'\hat{\boldsymbol{\varepsilon}}_{+1}]$; and

$$\hat{Q}_{T,trf}^{(h)} := \mathcal{H}_{z^{(h)}\boldsymbol{X}}\mathcal{H}_{\boldsymbol{X}\boldsymbol{X}}^{-1}\mathcal{H}_{\boldsymbol{X}\boldsymbol{X}v}\mathcal{H}_{\boldsymbol{X}\boldsymbol{X}}^{-1}\mathcal{H}_{z^{(h)}\boldsymbol{X}} \tag{S.8}$$

with $\hat{\boldsymbol{\varepsilon}}_{+1}$ denoting the residuals from regressing $\mathbf{y}_{+1}$ on $\hat{\boldsymbol{\nu}}_{+1}$ and a vector of ones (i.e. under the null) and $\mathcal{H}_{z^{(h)}\mathbf{X}} := (\mathbf{A}'_h\mathbf{z}_{-h})'\mathbf{X}_{-p}$; $\mathcal{H}_{\boldsymbol{X}\boldsymbol{X}} := \mathbf{X}'_{-p}\mathbf{X}_{-p}$; and $\mathcal{H}_{\boldsymbol{X}\boldsymbol{X}v} := \mathbf{X}'_{-p}\hat{\mathbf{v}}\hat{\mathbf{v}}'\mathbf{X}_{-p}$, where $\mathbf{X}_{-p}$ is a $(T-p-1)\times p$ matrix of lags of the demeaned predictor, i.e., $\mathbf{X}_{-p} := [\bar{\mathbf{x}}_{-1}, \bar{\mathbf{x}}_{-2}, ..., \bar{\mathbf{x}}_{-p}]'$ and $\bar{\mathbf{x}}_{-k} = [\bar{x}_{p+1}, \bar{x}_{p+2}, ..., \bar{x}_{T-k}]'$, $k = 1, ..., p$.