BONFERRONI TYPE TESTS FOR RETURN PREDICTABILITY AND THE INITIAL CONDITION

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Outline of Presentation

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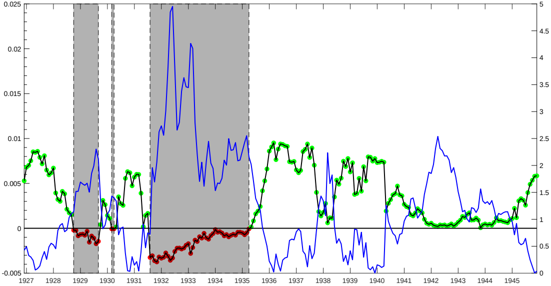
Introduction and Motivation

- We develop implementations of the popular Bonferroni Q test of Campbell and Yogo (2006) [CY] for return predictability that are designed to achieve much greater robustness to both the magnitude of the initial condition and the degree of persistence of the predictor than the original CY test.
- While the original CY test displays excellent power properties for strongly persistent predictors with an asymptotically negligible initial condition, we show that it can suffer from very severe size distortions and/or catastrophic power losses when either the initial condition is asymptotically non-negligible or the predictor is weakly persistent.

Motivation

- As an empirical illustration, when using the Bonferroni Q test CY find that the earnings-price ratio is a significant predictor for monthly returns of the NYSE/AMEX value-weighted index from the Center for Research in Security Prices (CRSP) for the period 1926M12-1994M12.
- We repeated this exercise, but applied the Bonferroni Q test sequentially by moving the start date along by one period (until we reached the end of 1945) thereby effecting a different initial condition in the dataset used to compute each of the Q tests in this sequence.
- For each candidate start date we recorded the lower bound of the confidence interval [CI] for the predictive regression [PR] coefficient β and an estimate of the magnitude of the initial condition of the predictor variable, $|\hat{\theta}|$ (subsequently defined in Equation (13)), using the method proposed by Harvey and Leybourne (2005).

Lower Bound of CI For β From Bonferroni Q Test Across Start Dates



Lower Bound of CI: —— (Left Axis), $|\hat{\theta}|$: —— (Right Axis)

Motivation

- Where the lines intercept the left axis corresponds to the full sample result of CY.
- For other start dates we see an inverse relationship between the lower bound of the the CI and $|\hat{\theta}|$ - ie "large" estimates of the initial condition are associated with CIs for β with much lower lower bounds and, hence, non-rejections of the no predictability null.
- Suggests that the empirical properties of the Q test may be highly sensitive to the magnitude of the initial condition of the predictor.
- Our simulations confirm this, showing the power of the Q test is severely diminished by large initial conditions.

Motivation

- We also find that while the Bonferroni Q test of CY is highly sensitive to the magnitude of the initial condition, the Bonferroni t-test of Cavanagh *et al.* (1995) is not.
- However, while the Bonferroni t-test is relatively robust to the magnitude of the initial condition, it is far less powerful than the Bonferroni Q test when the initial condition is asymptotically negligible.
- Accordingly, we develop hybrid testing strategies to exploit the best of these differing properties.
- Specifically, we consider a union-of-rejections of these two tests, and a strategy that takes a weighted average of the two tests, with the weight being a function of an estimate of the magnitude of the initial condition.
- In this presentation we focus on right-sided tests on β. We find in our paper that for left tailed tests one should, in fact, simply use the Bonferroni t-test.

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The Predictive Regression Model

- We consider the following predictive regression model

$$r_t = \alpha + \beta x_{t-1} + u_t, \qquad t = 1, ..., T \tag{1}$$

where r_t denotes the (excess) return in period t, and x_{t-1} denotes a (putative) predictor observed at time $t-1$.

- We assume the DGP for x_t is given by

$$x_t = \mu + w_t, \qquad t = 0, ..., T \tag{2}$$

$$w_t = \rho w_{t-1} + v_t, \qquad t = 1, ..., T. \tag{3}$$

- We focus on testing the null hypothesis of no predictability, $H_0 : \beta = 0$, against the right-sided alternative (positive predictability) $H_1 : \beta > 0$ (left-tailed testing is also covered in the paper).

The Predictive Regression Model

Assumption 1

We assume that $\psi(L)v_t = e_t$ *where* $\psi(L) := \sum_{i=0}^{p-1} \psi_i L^i$ *with* $\psi_0 = 1$ *and* $\psi(1) \neq 0$, *with the roots of* $\psi(L)$ *assumed to be less than one in absolute value.*

We assume that $z_t := (u_t, e_t)'$ *is a bivariate martingale difference sequence with respect to the natural filtration* $\mathcal{F}_t := \sigma\{z_s, s \leq t\}$ *satisfying the following conditions: (i)* $E[z_t z_t'] = \begin{bmatrix} \sigma_u^2 & \sigma_{ue} \\ \sigma_{ue} & \sigma_e^2 \end{bmatrix}$, *(ii)* $\sup_t E[u_t^4] < \infty$, *and (iii)* $\sup_t E[e_t^4] < \infty$.

We define $\omega_v^2 := \lim_{T\to\infty} T^{-1} E(\sum_{t=1}^{T} v_t)^2 = \sigma_e^2/\psi(1)^2$ *to be the long run variance of the error process* $\{v_t\}$, *and* $\delta := \sigma_{ue}/\sigma_u\sigma_e$ *as the correlation between the innovations* $\{u_t\}$ *and* $\{e_t\}$.

Potential Assumptions for x_t

Assumption 2

The predictor $\{x_t\}$ is strongly persistent, with the autoregressive parameter ρ in (3) given by $\rho = 1 - c/T$ with $c = 0$. The initial condition w_0 is unrestricted.

Assumption 3

The predictor $\{x_t\}$ is strongly persistent, with the autoregressive parameter ρ in (3) given by $\rho = 1 - c/T$ with c a finite non-zero constant. The initial condition is given by $w_0 = o_p(T^{1/2})$.

Assumption 4

The predictor $\{x_t\}$ is strongly persistent, with the autoregressive parameter ρ in (3) given by $\rho = 1 - c/T$ with c a finite positive constant. The initial condition is given by $w_0 = \theta\sigma_w$ where σ_w^2 denotes the short run variance of the process $\{w_t\}$ and $\theta \sim N(\mu_\theta I(\sigma_\theta^2 = 0), \sigma_\theta^2)$. When $\sigma_\theta^2 > 0$ we further assume that the random variable θ is independent of z_t for all t.

The Bonferroni Q test

- The Bonferroni Q test of CY is constructed under the assumption that either Assumption 2 or 3 holds.
- The CY test is based around estimates of β and its standard error from a version of (1) augmented by the (infeasible) additional regressor $(x_t - \rho x_{t-1})$.
- To make the test feasible, an initial $100(1-\alpha_1)\%$ (asymptotic) confidence interval for ρ is calculated by inverting some unit root test statistic, with this confidence interval denoted $[\underline{\rho}, \bar{\rho}]$.
- An equal tailed $100(1-\alpha_2)\%$ confidence interval for β given ρ is obtained by running regression (1), but with r_t replaced by $r_t - \hat{\sigma}_{ue}(\hat{\sigma}_e\hat{\omega}_v)^{-1}(x_t - \underline{\rho}x_{t-1})$ and $r_t - \hat{\sigma}_{ue}(\hat{\sigma}_e\hat{\omega}_v)^{-1}(x_t - \bar{\rho}x_{t-1})$, respectively.
- By Bonferroni's inequality this CI for β will have coverage of at least $100(1-\alpha)\%$ where $\alpha := \alpha_1 + \alpha_2$.

The Refined Bonferroni Q test

- CY find this method can be very be conservative so, for a given value of δ, they propose a refined method where the value of α_1 is chosen to give a one-sided test for predictability with maximum asymptotic size of 5% when either Assumption 2 or 3 hold.
- We omit details for the Bonferroni t test of Cavanagh *et al.* (1995), which is based around the simple OLS estimates of β and its standard error from (1), but a similar refined Bonferroni strategy can be employed to deliver a test with maximum asymptotic size of 5% when either Assumption 2 or 3 holds.

Limit Distribution of Test Statistics Under Assumption 2 or 3

Theorem 1

Let data be generated according to (1)-(3). Let $(W_u(s), W_e(s))$ be a two-dimensional Weiner process with correlation parameter δ, and let $W_{e,c}(s)$ be the Ornstein-Uhlenbeck process defined by the stochastic differential equation $dW_{e,c}(s) = cW_{e,c}(s)ds + dW_e(s)$ with initial condition $W_{e,c}(0) = 0$. If Assumption 2 or 3 holds, then under the local alternative $H_b : \beta = T^{-1}b$,

$$(a) \qquad t \xrightarrow{w} \frac{b\omega_v\kappa_c}{\sigma_u} + \delta\frac{\tau_c}{\kappa_c} + (1-\delta^2)^{1/2}Z \tag{4}$$

$$(b) \qquad Q(\tilde{c}) \xrightarrow{w} \frac{b\omega_v\kappa_c}{\sigma_u(1-\delta^2)^{1/2}} + \frac{\delta(\tilde{c}-c)\kappa_c}{(1-\delta^2)^{1/2}} + Z \tag{5}$$

where $\kappa_c := (\int_0^1 W_{e,c}^{\mu}(s)^2 ds)^{1/2}$ and $\tau_c := \int_0^1 W_{e,c}^{\mu}(s)dW_e(s)$ with $W_{e,c}^{\mu}(s) := W_{e,c}(s) - \int_0^1 W_{e,c}(r)dr$, and where Z is a standard normal random variable independent of $W_e(s)$.

Limit Distribution of Test Statistics Under Assumption 4

Theorem 2

Let data be generated according to (1)-(3). Let $W_u(s)$, $W_e(s)$ and $W_{e,c}(s)$ be as defined in Theorem 1. If Assumption 4 holds then under the local alternative $H_b : \beta = T^{-1}b$,

$$(a) \quad t \xrightarrow{w} \frac{b\omega_v \kappa_c^\theta}{\sigma_u} + \delta \frac{\tau_c^\theta}{\kappa_c^\theta} + (1-\delta^2)^{1/2} Z \tag{6}$$

$$(b) \quad Q(\tilde{c}) \xrightarrow{w} \frac{b\omega_v \kappa_c^\theta}{\sigma_u (1-\delta^2)^{1/2}} + \frac{\delta(\tilde{c}-c)\kappa_c^\theta}{(1-\delta^2)^{1/2}} + Z \tag{7}$$

where $\kappa_c^\theta := (\int_0^1 K_c^\mu(s)^2 ds)^{1/2}$ and $\tau_c^\theta := \int_0^1 K_c^\mu(s) dW_e(s)$ with $K_c^\mu(s) := K_c(s) - \int_0^1 K_c(r)dr$ and

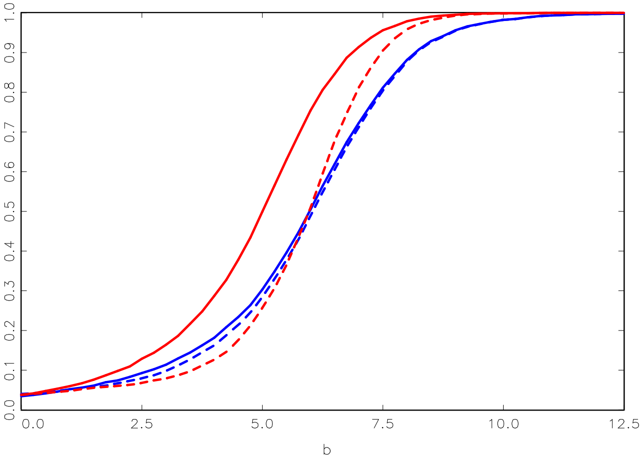
$$K_c(r) := \theta(e^{-rc} - 1)(2c)^{-1/2} + W_{e,c}(r) \tag{8}$$

and where $Z \sim N(0,1)$ is a standard normal random variable that is independent of $W_e(s)$.

Limit Distribution Under Assumption 4

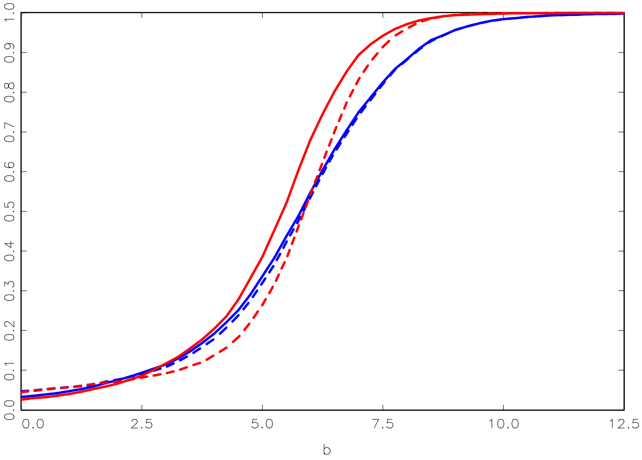
- The limit distributions of the Q and t tests when Assumption 4 holds are different to those found by CY and Cavanagh *et al.* (1995), displaying dependence on the initial condition.
- The same is true for the limit distributions of the unit root test statistics used to construct the initial CI for ρ. And the behaviour of these unit root statistics will be crucial in determining the behaviour of the Bonferroni tests.
- We now explore the impact of a non-negligible initial condition. We concentrate on fixed initial conditions ($\sigma_\theta^2 = 0$), so that $\theta = \mu_\theta$, but quantitatively similar results were found for random initial conditions.
- We report results for when either the DF-OLS or DF-GLS unit root test statistic is used to construct the initial CI for ρ. A superscript OLS/GLS indicates the former/latter.

Local Asymptotic Power. $c = 2$, $\theta = 0$, $\delta = -0.95$



t^{GLS}: ——— , t^{OLS}: - - - , Q^{GLS}: ———, Q^{OLS}: - - -

Local Asymptotic Power. $c = 2$, $\theta = 1$, $\delta = -0.95$



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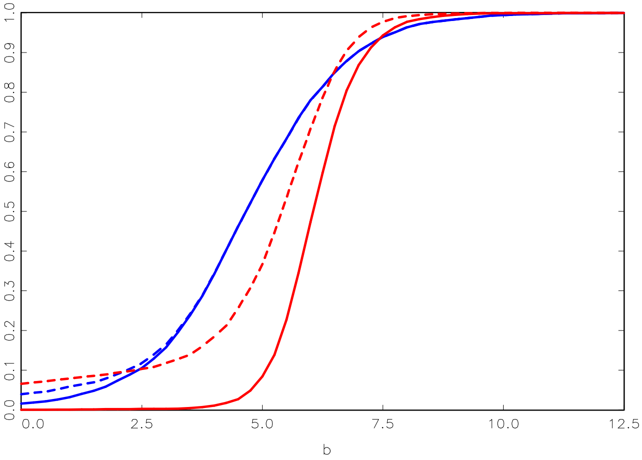
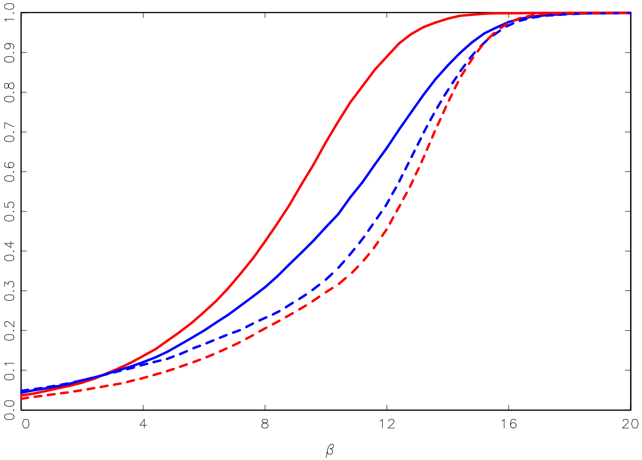
Local Asymptotic Power. $c = 2$, $\theta = 3$, $\delta = -0.95$



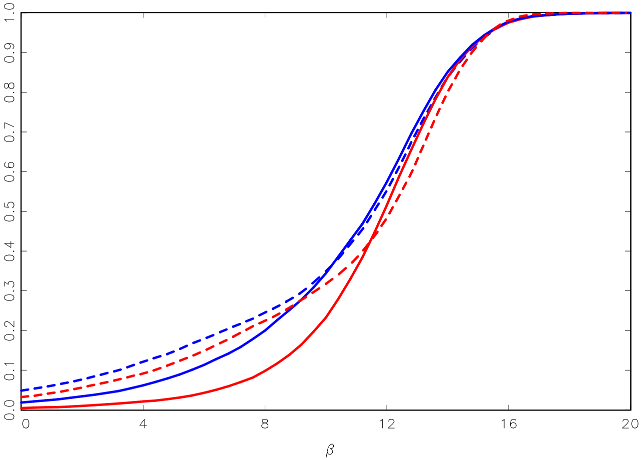
t^{GLS}: —— , t^{OLS}: - - - , Q^{GLS}: ——, Q^{OLS}: - - -

Local Asymptotic Power. $c = 10$, $\theta = 0$, $\delta = -0.95$



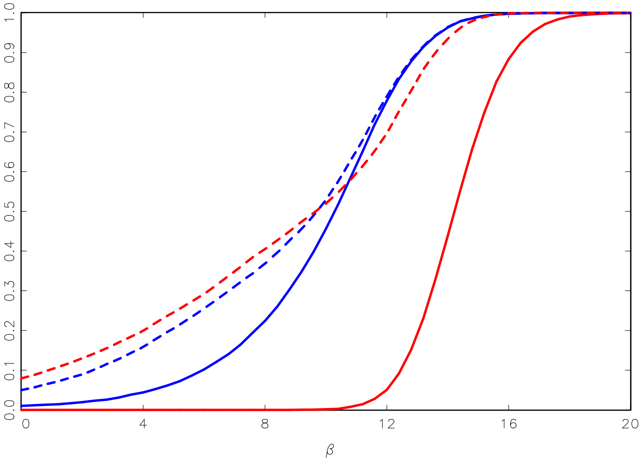
t^{GLS}: ——— , t^{OLS}: - - - , Q^{GLS}: ———, Q^{OLS}: - - -

Local Asymptotic Power. $c = 10$, $\theta = 1$, $\delta = -0.95$



t^{GLS}: ——— , t^{OLS}: - - - , Q^{GLS}: ——— , Q^{OLS}: - - -

Local Asymptotic Power. $c = 10$, $\theta = 3$, $\delta = -0.95$



t^{GLS}: —— , t^{OLS}: - - - , Q^{GLS}: ——, Q^{OLS}: - - -

Local Asymptotic Power

- We see that the Q^{GLS} test of CY is severely impacted by a non-zero initial condition, being badly undersized and severely lacking in power. t^{GLS} shows similar, though less extreme, patterns, while Q^{OLS} has a tendency to over-sizing for a large initial condition.
- The t^{OLS} test of Cavanagh *et al.* (1995), however, retains size control regardless of the value of θ and power is not impacted too much by large initial conditions.
- Therefore, we ideally want to use Q^{GLS} when the initial condition is small, and t^{OLS} when the initial condition is large.
- We next outline the hybrid testing strategies we propose to do exactly this.

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A Union of Rejection Strategy

Our first proposed testing strategy is a union-of-rejections test, U, defined by the decision rule

$$U: \text{ Reject } H_0 \text{ if } \underline{U} > 0 \tag{9}$$

where

$$\underline{U} := \max\left(\underline{\beta}^{Q^{GLS}}, \underline{\beta}^{t^{OLS}}\right). \tag{10}$$

and where $\underline{\beta}^{Q^{GLS}}$ and $\underline{\beta}^{t^{OLS}}$ denote the lower bound of the CI for β from the Q^{GLS} and t^{OLS} tests, respectively.

The significance level at which the initial unit root tests are performed are, for a given value of δ, chosen such that the asymptotic size of U is no greater than 5% across a specified range of values of c and initial conditions.

This testing strategy exploits the superior power of Q^{GLS} when θ is small and t^{OLS} when θ is large.

A Weighting Strategy

Our second proposed testing strategy is a weighting strategy, W_γ, defined by the decision rule

$$W_\gamma : \text{ Reject } H_0 \text{ if } \underline{W}_\gamma > 0 \tag{11}$$

where $\underline{W}_\gamma$ is a weighted average of the lower confidence interval bounds for β from the Q^{GLS} and t^{OLS} tests, viz:

$$\underline{W}_\gamma := \lambda_\gamma(|\hat{\theta}|)\underline{\beta}^{Q^{GLS}} + (1 - \lambda_\gamma(|\hat{\theta}|))\underline{\beta}^{t^{OLS}}, \tag{12}$$

and where $|\hat{\theta}|$ is an estimate of the magnitude of the initial condition of the predictor, and $\lambda_\gamma(|\hat{\theta}|)$ is a function of $|\hat{\theta}|$ that is designed to be large (small) when $|\hat{\theta}|$ is small (large).

A Weighting Strategy

We follow Harvey and Leybourne (2005) and use the following estimate of the magnitude of the initial condition

$$|\hat{\theta}| := |x_0 - \hat{\mu}|/\hat{\sigma}_w, \tag{13}$$

where $\hat{\mu} := T^{-1}\sum_{t=1}^{T} x_t$ and $\hat{\sigma}_w^2 := T^{-1}\sum_{t=1}^{T}(x_t - \hat{\mu})^2$.

We also make use of the same weight function, $\lambda_\gamma(|\hat{\theta}|)$, used by Harvey and Leybourne (2005) in the context of testing for a unit root, given by:

$$\lambda_\gamma(|\hat{\theta}|) := exp(-\gamma|\hat{\theta}|) \tag{14}$$

For a given value of γ, we once again select the significance level at which the initial unit root tests are performed for the constituent Q^{GLS} and t^{OLS} tests such that the W_γ test has size no greater than 5% across a specified range of values of c and initial conditions.

We will report results for this testing strategy using $\gamma = 1, 2$.

Hybrid Testing Strategies

- Up until now we have assumed that the predictor, x_t, is strongly persistent. The Q^{GLS} and t^{OLS} tests, along with the U and W_γ tests we propose, are all constructed under this assumption.
- When the predictor is weakly persistent we will show that the Q^{GLS} test in particular suffers from severe distortions.
- We therefore propose a simple modification to the U and W_γ tests whereby they switch to using the standard t-test, comparing with standard normal critical values, if there is sufficient evidence that the predictor is weakly stationary.
- To ensure that the standard t-test is used asymptotically when the predictor is weakly stationary we apply the standard t-test instead of the U and W_γ test whenever the normalised bias ADF statistic applied to the predictor is less than $-4.5T^{1/2}$.
- We will denote these hybrid testing strategies as U^{hyb} and W_γ^{hyb}.

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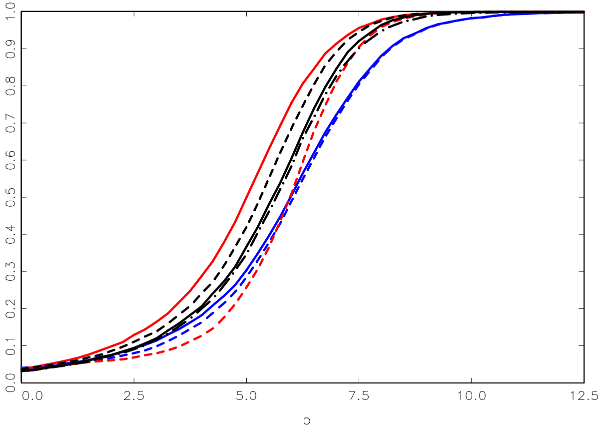
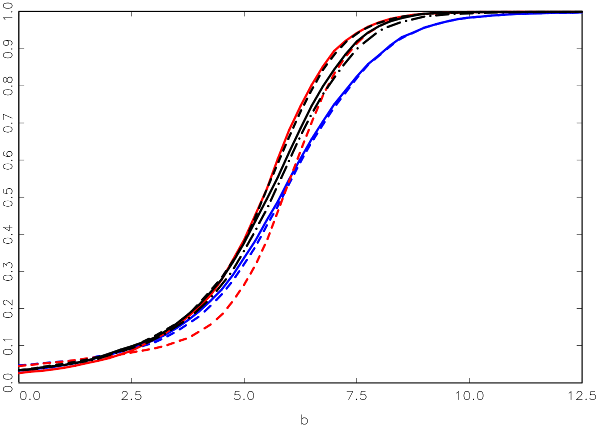
Local Asymptotic Power of Proposed Tests. $c = 2$, $\theta = 0$, $\delta = -0.95$



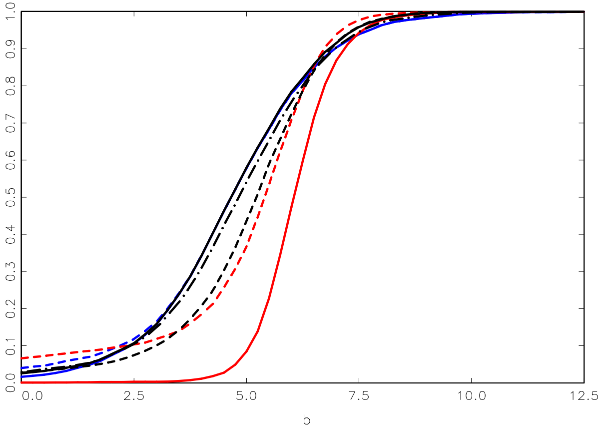
t^{GLS}: ——— , t^{OLS}: - - - , Q^{GLS}: ———, Q^{OLS}: - - -
U^{hyb}———, W_1^{hyb}: – –, W_2^{hyb}: –·–

Local Asymptotic Power of Proposed Tests. $c = 2$, $\theta = 1$, $\delta = -0.95$



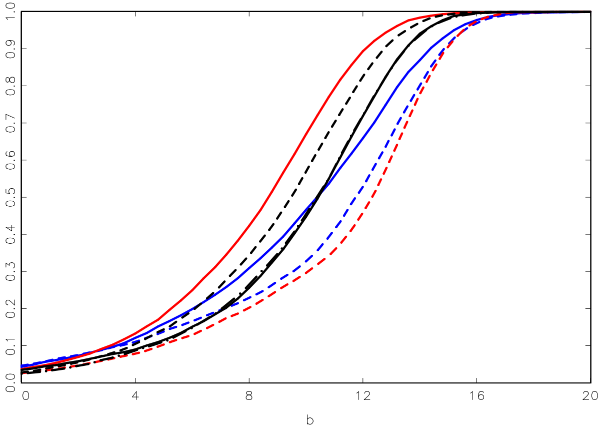
t^{GLS}: ——— ,t^{OLS}: - - - , Q^{GLS}: ———, Q^{OLS}: - - -
U^{hyb}———, W_1^{hyb}: – –, W_2^{hyb}: –·–

Local Asymptotic Power of Proposed Tests. $c = 2$, $\theta = 3$, $\delta = -0.95$



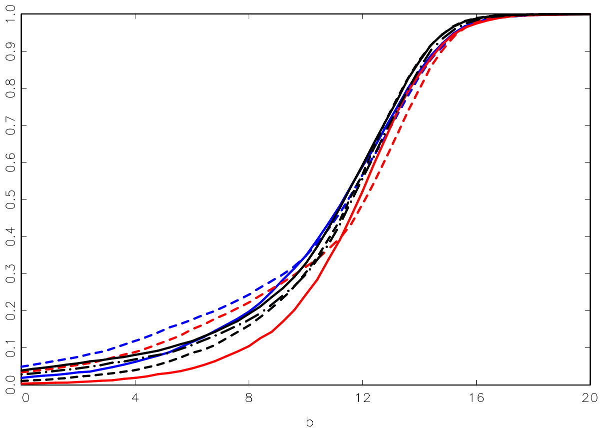
t^{GLS}: ——— ,t^{OLS}: - - - , Q^{GLS}: ———, Q^{OLS}: - - -
U^{hyb}———, W_1^{hyb}: – –, W_2^{hyb}: –·–

Local Asymptotic Power of Proposed Tests. $c = 10$, $\theta = 0$, $\delta = -0.95$



t^{GLS}: ——— , t^{OLS}: - - - , Q^{GLS}: ———, Q^{OLS}: - - -
U^{hyb}———, W_1^{hyb}: – –, W_2^{hyb}: –·–

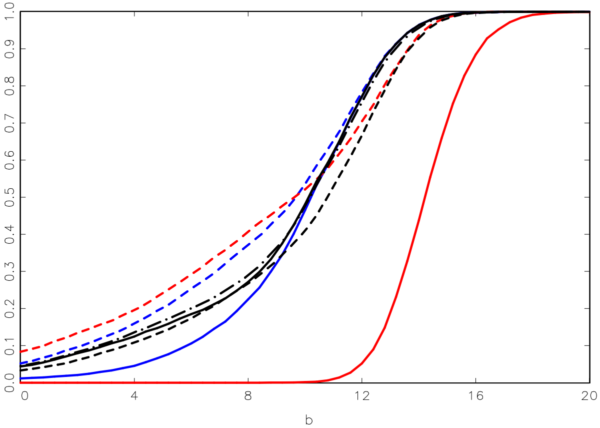
Local Asymptotic Power of Proposed Tests. $c = 10$, $\theta = 1$, $\delta = -0.95$



t^{GLS}: ——— , t^{OLS}: - - - , Q^{GLS}: ———, Q^{OLS}: - - -

U^{hyb}———, W_1^{hyb}: – –, W_2^{hyb}: –·–

Local Asymptotic Power of Proposed Tests. $c = 10$, $\theta = 3$, $\delta = -0.95$



t^{GLS}: ——— , t^{OLS}: - - - , Q^{GLS}: ——— , Q^{OLS}: - - -

U^{hyb}——, W_1^{hyb}: – –, W_2^{hyb}: –·–

Local Asymptotic Power of Proposed Tests

- Our proposed hybrid testing strategies offer an attractive overall power profile relative to extant tests.
- For strongly persistent predictors, when the initial condition is small they offer power close to that of the Q^{GLS} test.
- For strongly persistent predictors, when the initial condition is large they offer power close to that of the t^{OLS} test.
- We will now show that this also manifests in finite samples.

Finite Sample Simulations

Data were generated according to

$$\begin{aligned} r_t &= \alpha + \beta x_{t-1} + u_t, & t = 1, ..., T \\ x_t &= \mu + w_t, & t = 0, ..., T \\ w_t &= \rho w_{t-1} + v_t, & t = 1, ..., T \end{aligned}$$

with $\alpha = \mu = 0$ (without loss of generality), $\rho = 1 - c/T$ and a sample size of $T = 250$.The innovations $\{u_t\}$, $\{v_t\}$ have correlation parameter δ and are drawn from a bivariate normal distribution.

We considered $c = 0$ (i.e. Assumption 2) and a range of $c > 0$ values, $c \in \{2, 5, 10, 20, 50, 100, 250\}$, with the values of $c \leq 50$ chosen to demonstrate the behaviour of the tests when the predictor is a strongly persistent process, and the values of $c \geq 100$ used to illustrate the behaviour of the tests when the predictor is weakly persistent given that $c \geq 100$ implies that $\rho \leq 0.6$.

Finite Sample Simulations

When $c > 0$, the initial condition w_0 is either generated as a $N(0, 1)$ [tables and graphs labelled $\mu_\theta = 0$], such that Assumption 3 holds, or as fixed according to Assumption 4, with $\theta = \mu_\theta \in \{1, 3\}$.

The DF-OLS and DF-GLS unit root test statistics used to construct the tests for predictability were estimated using a lag length chosen by the Bayes Information Criterion with $p_{\max} = 5$, with this lag length selection method also used for the Dickey-Fuller normalised bias coefficient unit root test statistic used in the switching mechanism to the standard t-test in the hybrid tests.

Finite Sample Size. $\mu_\theta = 0$, $\delta = -0.95$

c	t^{GLS}	t^{OLS}	Q^{GLS}	Q^{OLS}	U^{hyb}	W_1^{hyb}	W_2^{hyb}
0	0.049	0.052	0.044	0.052	0.048	0.054	0.046
2	0.039	0.042	0.048	0.041	0.036	0.043	0.036
5	0.040	0.046	0.045	0.034	0.034	0.041	0.035
10	0.042	0.047	0.044	0.035	0.039	0.039	0.038
20	0.046	0.051	0.036	0.037	0.048	0.035	0.036
50	0.044	0.048	0.032	0.048	0.056	0.052	0.050
100	0.038	0.041	0.048	0.115	0.060	0.060	0.060
250	0.035	0.036	0.275	0.899	0.053	0.053	0.053

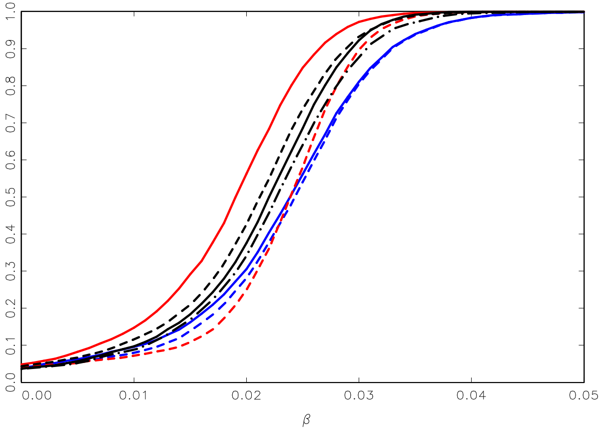
Finite Sample Size. $\mu_\theta = 1$, $\delta = -0.95$

c	t^{GLS}	t^{OLS}	Q^{GLS}	Q^{OLS}	U^{hyb}	W_1^{hyb}	W_2^{hyb}
0	0.049	0.052	0.044	0.052	0.048	0.054	0.046
2	0.027	0.037	0.028	0.039	0.027	0.034	0.027
5	0.020	0.039	0.014	0.034	0.026	0.028	0.028
10	0.019	0.045	0.006	0.036	0.036	0.025	0.034
20	0.027	0.049	0.003	0.037	0.045	0.019	0.035
50	0.035	0.048	0.000	0.049	0.053	0.043	0.046
100	0.037	0.040	0.000	0.116	0.061	0.061	0.061
250	0.035	0.036	0.000	0.898	0.053	0.053	0.053

Finite Sample Size. $\mu_\theta = 3$, $\delta = -0.95$

c	t^{GLS}	t^{OLS}	Q^{GLS}	Q^{OLS}	U^{hyb}	W_1^{hyb}	W_2^{hyb}
0	0.049	0.052	0.044	0.052	0.048	0.054	0.046
2	0.018	0.040	0.001	0.065	0.030	0.034	0.031
5	0.010	0.046	0.000	0.089	0.035	0.037	0.038
10	0.004	0.047	0.000	0.094	0.041	0.039	0.043
20	0.003	0.043	0.000	0.085	0.040	0.036	0.041
50	0.005	0.044	0.000	0.097	0.048	0.044	0.048
100	0.012	0.042	0.000	0.184	0.060	0.060	0.060
250	0.021	0.034	0.000	0.931	0.053	0.053	0.053

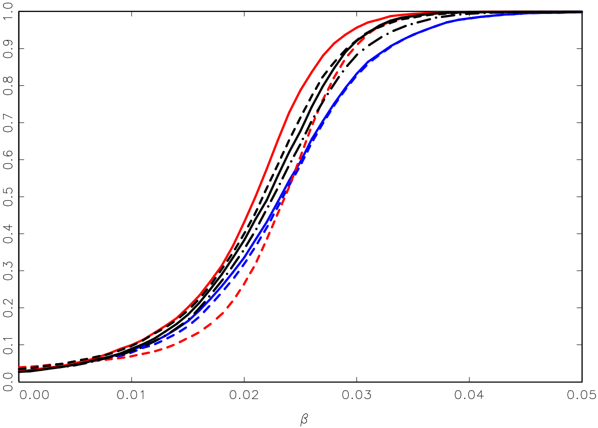
Finite Sample Power of Proposed Tests. $c = 2$, $\mu_\theta = 0$, $\delta = -0.95$



t^{GLS}: ——— , t^{OLS}: - - - , Q^{GLS}: ——— , Q^{OLS}: - - -

U^{hyb}———, W_1^{hyb}: – –, W_2^{hyb}: –·–

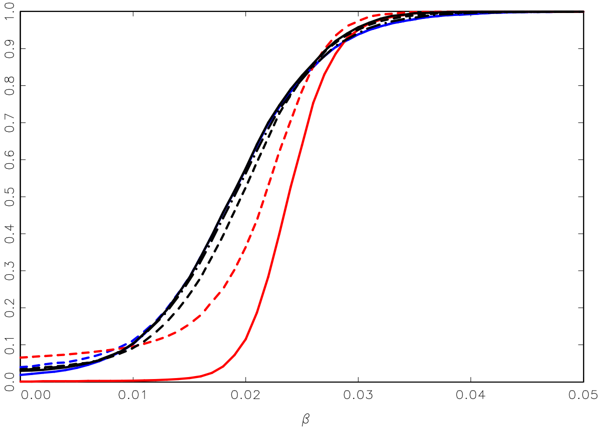
Finite Sample Power of Proposed Tests. $c = 2$, $\mu_\theta = 1$, $\delta = -0.95$



t^{GLS}: ——— , t^{OLS}: - - - , Q^{GLS}: ——— , Q^{OLS}: - - -

U^{hyb}———, W_1^{hyb}: – –, W_2^{hyb}: –·–

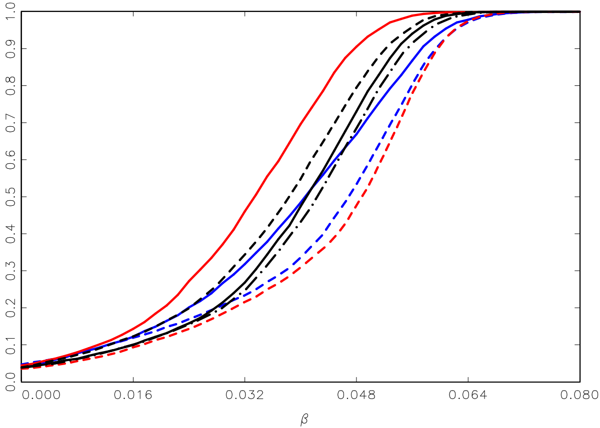
Finite Sample Power of Proposed Tests. $c = 2$, $\mu_\theta = 3$, $\delta = -0.95$



t^{GLS}: ——— , t^{OLS}: - - - , Q^{GLS}: ———, Q^{OLS}: - - -

U^{hyb}———, W_1^{hyb}: – –, W_2^{hyb}: –·–

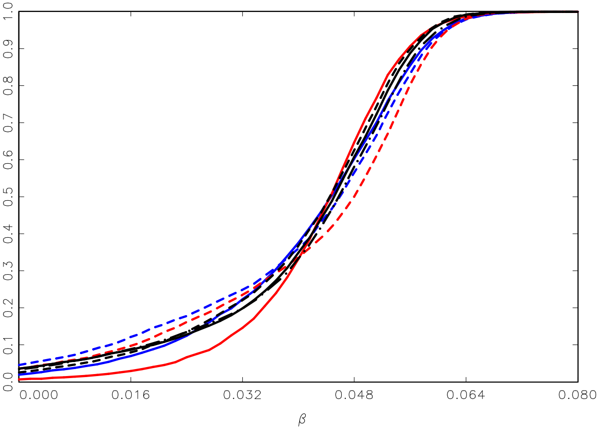
Finite Sample Power of Proposed Tests. $c = 10$, $\mu_\theta = 0$, $\delta = -0.95$



t^{GLS}: —— , t^{OLS}: - - - , Q^{GLS}: ——, Q^{OLS}: - - -

U^{hyb}——, W_1^{hyb}: – –, W_2^{hyb}: –·–

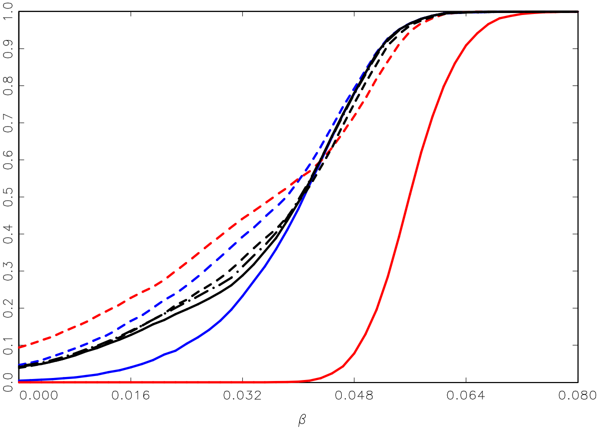
Finite Sample Power of Proposed Tests. $c = 10$, $\mu_\theta = 1$, $\delta = -0.95$



t^{GLS}: ——— , t^{OLS}: - - - , Q^{GLS}: ———, Q^{OLS}: - - -

U^{hyb}———, W_1^{hyb}: – –, W_2^{hyb}: –·–

Finite Sample Power of Proposed Tests. $c = 10$, $\mu_\theta = 3$, $\delta = -0.95$



t^{GLS}: ——— , t^{OLS}: - - - , Q^{GLS}: ———, Q^{OLS}: - - -

U^{hyb}———, W_1^{hyb}: – –, W_2^{hyb}: –·–

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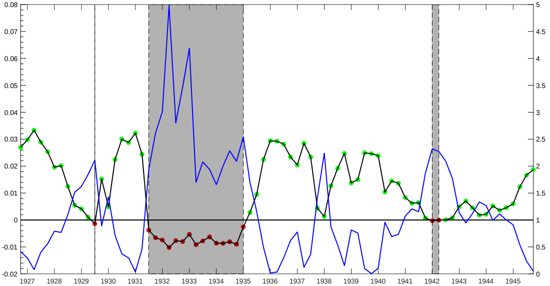
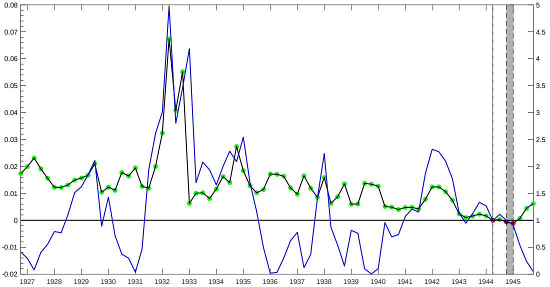
- We now revisit the dataset of CY to further illustrate the sensitivity of their Q^{GLS} test to the value of the initial condition of the predictor, and to explore to what extent our proposed U^{hyb} and W_{γ}^{hyb} hybrid procedures are able to overcome this.
- We applied all test procedures to the same empirical returns/predictor pairings considered in CY, but rather than applying the procedures to only the full sample of data, we applied them sequentially across all possible start dates, t_s up until we reached the end of 1945.
- The following figures display plots of the lower bound of the CI for β and the estimated magnitude of the initial condition, along with the rejection frequency across start dates, for each test.
- We report results for both quarterly and monthly frequency CRSP returns from 1926-1994 using the earnings-price ratio as a predictor. Results for all return/predictor pairings considered by CY are available in the paper.

Quarterly CRSP 1926-1994. $x = e - p$, Q^{GLS} Test. (Rejection Rate 77%)



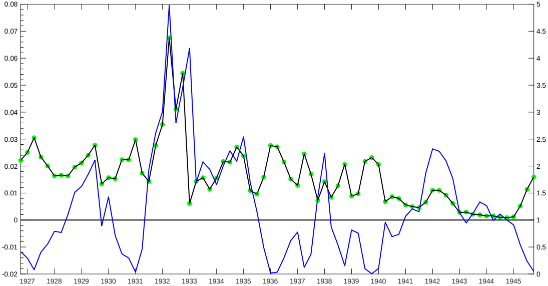
Lower Bound of CI: —— (Left Axis), $|\hat{\theta}|$: —— (Right Axis)

Quarterly CRSP 1926-1994. $x = e - p$, U Test. (Rejection Rate 96%)



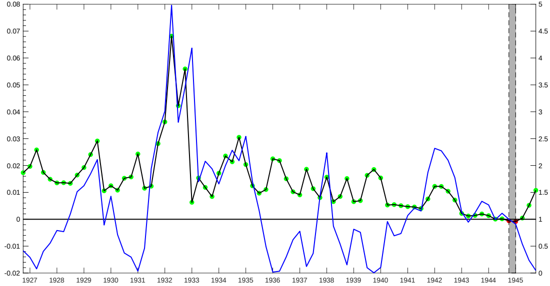
Lower Bound of CI: —— (Left Axis), $|\hat{\theta}|$: —— (Right Axis)

Quarterly CRSP 1926-1994. $x = e - p$, W_1^{hyb} Test. (Rejection Rate 100%)



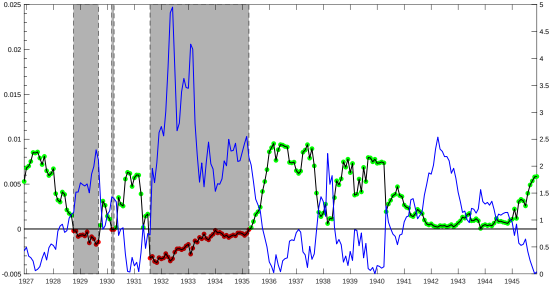
Lower Bound of CI: —— (Left Axis), $|\hat{\theta}|$: —— (Right Axis)

Quarterly CRSP 1926-1994. $x = e - p$, W_2^{hyb} Test. (Rejection Rate 97%)



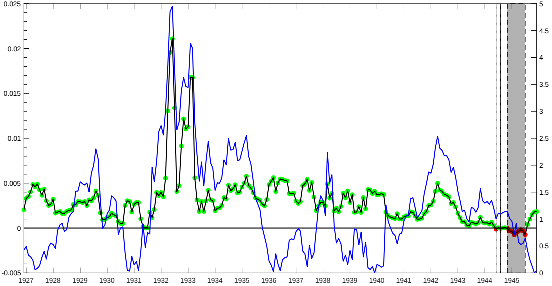
Lower Bound of CI: —— (Left Axis), $|\hat{\theta}|$: —— (Right Axis)

Monthly CRSP 1926-1994. $x = e - p$, Q^{GLS} Test. (Rejection Rate 74%)



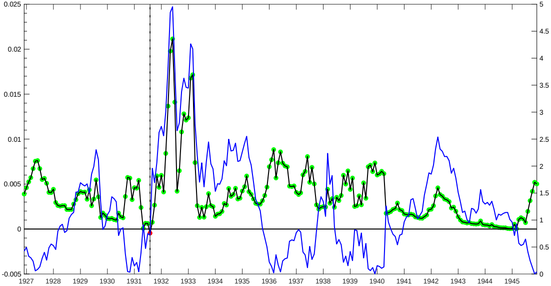
Lower Bound of CI: —— (Left Axis), $|\hat{\theta}|$: —— (Right Axis)

Monthly CRSP 1926-1994. $x = e - p$, U Test. (Rejection Rate 95%)



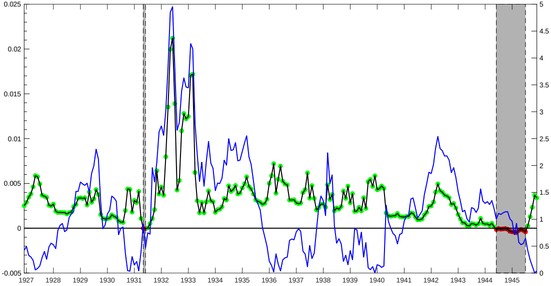
Lower Bound of CI: —— (Left Axis), $|\hat{\theta}|$: —— (Right Axis)

Monthly CRSP 1926-1994. $x = e - p$, W_1^{hyb} Test. (Rejection Rate 100%)



Lower Bound of CI: —— (Left Axis), $|\hat{\theta}|$: —— (Right Axis)

Monthly CRSP 1926-1994. $x = e - p$, W_2^{hyb} Test. (Rejection Rate 93%)



Lower Bound of CI: —— (Left Axis), $|\hat{\theta}|$: —— (Right Axis)

Empirical Application

- We see that our proposed hybrid tests reject with greater frequency across start dates than the Q^{GLS} test.
- While the Q^{GLS} test is more likely to reject for small values of $|\hat{\theta}|$, it often fails to reject for larger values of $|\hat{\theta}|$.
- Our hybrid tests, on the other hand, display a much more consistent pattern of rejections across the estimated values of $|\hat{\theta}|$.

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- We propose hybrid testing strategies that are functions of the Q^{GLS} test of CY and the t^{OLS} test of Cavanagh *et al.* (1995).
- We show that when the predictor is local to unity these hybrid tests deliver good size control and power regardless of the magnitude of the initial condition.
- The Q^{GLS} test, on the other hand, displays significant under-size and poor power when the predictor is local-to-unity and the initial condition is large.
- The hybrid tests are also made robust to a weakly persistent predictor by having them switch into the standard t-test when there is sufficient evidence that the predictor is weakly stationary.
- Revisiting the empirical application of CY show that our proposed hybrid tests reject much more consistently than the Q^{GLS} test across start dates for this dataset, giving further evidence of their robustness to the magnitude of the initial condition.

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